Chapter 1
Review Exercises

1–4 Find the domain and range of the given relation and determine whether the relation is a function.

1. $R = \{(-2,9),(-3,-3),(-2,2),(-2,-9)\}$
2. $3x - 4y = 17$
3. $x = y^2 - 6$
4. $x = \sqrt{y-4}$

5–8 Identify the domain, codomain, and range of the given function.

5. $f:\mathbb{N}\to\mathbb{R},\quad f(x)=\frac{3x}{4}$
6. $g:\mathbb{R}\to\mathbb{R},\quad g(x)=5x+1$
7. $h:\mathbb{R}\to\mathbb{R},\quad h(x)=\frac{1}{x^2+1}$
8. $k:\mathbb{N}\to\mathbb{R},\quad k(x)=2+\sqrt{x-1}$

9–12 Find the value of the given function for **a.** $f(x-1)$, **b.** $f(x^2)$, and **c.** $\frac{f(x+h)-f(x)}{h}$.

9. $f(x)=(x+5)(2x)$
10. $f(x)=\sqrt[3]{x}+6(x+4)$
11. $f(x)=\frac{3}{x+2}$
12. $f(x)=\sin 2x$

13–14 Find all open intervals of monotonicity (intervals where the function is increasing or decreasing) for the given function.

13. $f(x)=(x-2)^4-6$
14. $R(x)=\begin{cases}(x+2)^2 & \text{if } x<-1\\ -x & \text{if } x\ge -1\end{cases}$

15–18 Determine if the function is even, odd, or neither and then graph it.

15. $f(x)=\frac{1}{3}x^3$
16. $f(x)=\sqrt{x}$
17. $f(x)=-2\sin x$
18. $g(x)=-2\sin^2 x$

19–20 Discuss the symmetry of the given equation and then graph it.

19. $y=|5x|$
20. $x^2+y^2=25$

21–34 Graph the given function. Locate the *x*- and *y*-intercepts, if any.

21. $f(x)=7x-2$
22. $f(x)=\frac{2x-6}{3}$
23. $f(x)=(x-1)^2-1$
24. $f(x)=-x^2-6x-11$
25. $f(x)=4x^3$
26. $f(x)=-\frac{2}{x^2}$
27. $f(x)=\frac{-x^3+7x+6}{2}$
28. $f(x)=\frac{x+1}{x^2-4}$
29. $f(x)=\frac{\sqrt[3]{x}}{2}$
30. $f(x)=5|-x|$
31. $f(x)=\left[\!\left[\frac{2x}{3}\right]\!\right]$
32. $f(x)=\begin{cases}x^2 & \text{if } x<1\\ \frac{1}{x} & \text{if } x\ge 1\end{cases}$
33. $f(x)=-\cot x$
34. $f(x)=\log_{1/2} x$

35–36 Sketch the graph of the given function by first identifying the more basic function that has been shifted, reflected, stretched, or compressed. Then determine the domain and range of the function.

35. $f(x)=(x-1)^3+2$
36. $f(x)=4|x+3|$

37–38 Write an equation for the function described.

37. Use the function $f(x)=1/x$. Move the function 2 units to the right and 3 units up.
38. Use the function $f(x)=\sqrt[3]{x}$. Move the function 1 unit to the left and reflect across the *y*-axis.

39–40 Find the formula and domain for **a.** $f+g$ and **b.** f/g.

39. $f(x)=x^2;\quad g(x)=\sqrt{x}$
40. $f(x)=\frac{1}{x-2};\quad g(x)=\sqrt[3]{x}$

41–42 Use the information given to determine **a.** $(f\circ g)(x)$, **b.** $(g\circ f)(x)$, and **c.** $(f\circ g)(3)$.

41. $f(x)=-x+1;\quad g(x)=-x-1$
42. $f(x)=\frac{x^{-1}}{18}-3;\quad g(x)=\frac{x-4}{x^3}$

43–44 Write the given function as a composition of two functions. (Answers will vary.)

43. $f(x)=\dfrac{\sqrt{x+3}+2}{x^2+6x+9}$

44. $f(x)=|x+2|+x^2+4x+4$

45–46 Graph the inverse of the given relation, and state its domain and range.

45. $R=\{(-3,5),(2,1),(0,-5),(-1,-2)\}$

46. $y=\dfrac{1}{3}x^2$

47–50 Find the inverse of the given function.

47. $f(x)=\dfrac{2}{7x-1}$

48. $f(x)=\dfrac{4x-3}{x}$

49. $f(x)=x^{1/5}-6$

50. $f(x)=\dfrac{6x-7}{2-x}$

51. Show that $f^{-1}(f(x))=x$ and that $f(f^{-1}(x))=x$ for the functions given in Exercises 49 and 50.

52–53 Rewrite the given function as a purely algebraic function.

52. $\cos(\sin^{-1} x)$

53. $\tan\left(\sec^{-1}\dfrac{x}{2}\right)$

54–55 Use the properties of logarithms to expand the given expression as much as possible; that is, decompose the expression into sums or differences of the simplest possible terms.

54. $\ln\dfrac{x^2y-xz}{5y^3}$

55. $\log\sqrt[3]{\dfrac{yz^2}{x^4}}$

56. If a pebble is shot upward with an initial (vertical) velocity of 56 ft/s, how high does it go? (**Hint:** Use the height function $h(t)=-16t^2+56t$.)

57. The half-life of actinium-225 is 10 days. Assuming that A_0 is the mass at time $t=0$, find the function $A(t)$ that gives the mass remaining after t days. What percentage of the original mass of a sample of Ac-225 remains after 25 days?

58. Prove that the product of an odd function and an even function is odd.

59. Among all the pairs of numbers with a sum of 15, find the pair whose product is maximum.

60. The January 2010 Haiti earthquake was initially reported as a 7.2-magnitude quake on the Richter scale, while the March 2011 earthquake off the eastern shores of Japan was magnitude 9.0. According to these numbers, about how many times more intense was the Japanese earthquake? (**Hint:** See the directions preceding Exercise 138 in Section 1.4.)

61–62 Determine if the given complex number is in the Mandelbrot set. (**Hint:** See the directions preceding Exercises 101–108 in Section 1.3.)

61. $c=-i$

62. $c=2$

Concept Check

63–76 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

63. The graph of any function can always be represented by a curve that passes the vertical line test.

64. Any linear function whose graph is a line with negative slope always has an inverse.

65. If f is a function, and $f(a)=f(b)$, then $a=b$.

66. If $(2,5)$ is a point on the graph of an odd function, then $(-2,-5)$ is also on the graph.

67. The graph of a quadratic function is a parabola, so quadratic functions are even functions.

68. In general, $(f\circ g)(x)=(g\circ f)(x)$ holds for all x if both compositions can be formed.

69. If a function can be decomposed into three functions, it can only be done in one way.

70. The graph of $f(x)+a$ is that of $f(x)$ translated vertically by $|a|$ units.

71. If $c\in\mathbb{R}$ is a constant, and f is any function, then the graphs of $f(cx)$ and $cf(x)$ are identical.

72. When the graph of any function $f(x)$ is reflected about the line $y=x$, the graph of the inverse is obtained.

73. The graph of any function must pass the horizontal line test.

74. If (a,b) is a point on the graph of an invertible function $f(x)$, then (b,a) is on the graph of its inverse.

75. The common logarithm and $y=10^x$ are inverses.

76. The function $f(x)=\tan^{-1}x$ has no asymptotes.

Chapter 1
Technology Exercises

77–79 Mentally sketch the graph of the given function by identifying the basic shape that has been shifted, reflected, stretched, or compressed. Then use a graphing utility to graph the function and check your reasoning.

77. $f(x)=\ln(x+1)+2$

78. $f(x)=-\frac{2}{x-3}+1$

79. $f(x)=\sin \pi x-1$

80. The annual expenditures (in millions of dollars) for a corporation are given in the table below.

Annual Expenditures

Year	2017	2018	2019	2020	2021	2022
Expenditures (in millions)	\$16.2	\$17.1	\$18.8	\$19.6	\$21.1	\$22.9

a. Find the least-squares line of best fit for the data. (Let $x = 0$ correspond to the year 2017.)

b. Estimate the expenditures for 2023.

81–82 Use a graphing utility to approximate the solution(s) of the given equation, rounded to four decimal places. (**Hint:** Zoom in on the *x*-intercepts or points of intersection as appropriate for each equation.)

81. $x^5-x^3-3=0$

82. $x^2+6=2^{x+1}$

83–84 Use a graphing utility to graph the given function, and describe the characteristics of the graph as c varies. Use different viewing windows.

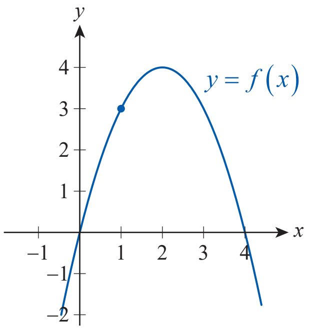
83. $u(x)=\frac{1-e^{c/x}}{1+e^{c/x}}$

84. $v(x)=\frac{x}{c^2}\sqrt[4]{c^4-x^4}$

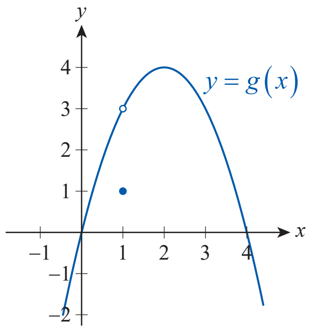
Chapter 2
Review Exercises

1–4 Use the graph of the function to find the indicated (possibly one-sided) limit, if it exists. Also examine the continuity of the function at the indicated point and classify any discontinuities.

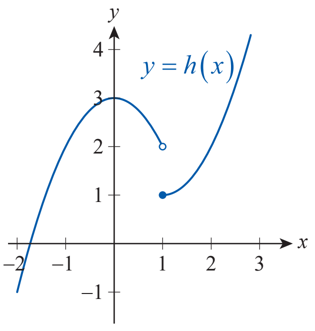
1. $\lim_{x\to 1} f(x)$



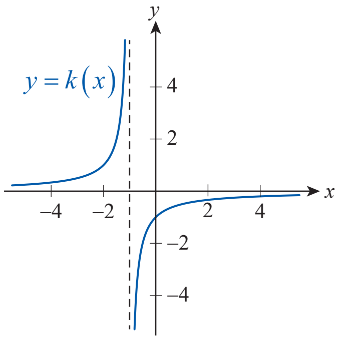
2. $\lim_{x\to 1} g(x)$



3. $\lim_{x\to 1^-} h(x)$



4. $\lim_{x\to -1^+} k(x)$



5–8 Use difference quotients to approximate the slope of the tangent to the graph of the function at the given point. Use at least five different *h*-values that are decreasing in magnitude. (Answers will vary.)

5. $f(x) = 3x + 2;\quad (0,2)$

6. $g(x) = 2 - x^2;\quad (1,1)$

7. $h(x) = \sqrt{x-1};\quad (2,1)$

8. $k(x) = \sin x;\quad (0,0)$

9. A pellet is shot vertically upward from an initial height of 6 feet. Its height after t seconds is given by $h(t) = 6 + 608t - 16t^2$ feet. Use difference quotients to answer the questions below.

a. What will be the pellet's height at the end of the first second?

b. What is the average velocity of the pellet during the first two seconds?

c. Estimate the instantaneous velocity at $t = 0$ seconds.

d. Estimate the instantaneous velocity at $t = 2$ seconds.

e. When will the velocity be 0?

10–11 Approximate the area of the region between the graph of $f(x)$ and the *x*-axis on the given interval. Use $n = 4$, as in Example 5a of Section 2.1. (Round your answer to four decimal places.)

10. $f(x) = x^3$ on $[0,1]$ **11.** $f(x) = \ln x$ on $[1,3]$

12–13 Approximate the arc length of the graph of $g(x)$ on the given interval. Use $n = 5$, as in Example 5b of Section 2.1. (Round your answer to four decimal places.)

12. $g(x) = x^{2/3}$ on $[3,8]$ **13.** $g(x) = e^x$ on $[-2,3]$

14–17 Create a table of values to estimate the value of the indicated limit without graphing the function. Choose the last x-value so that it is no more than 0.001 units from the given c-value.

14. $\lim_{x\to 1} \frac{x^3-1}{x-1}$ **15.** $\lim_{x\to 0} x^x$

16. $\lim_{x\to 0} \frac{\sin 2x}{4x}$ **17.** $\lim_{x\to 0} \left(2x \sin \frac{1}{4x}\right)$

18. Use one-sided limit notation to describe the behavior of $f(x) = \frac{1}{x-1}$ near $x = 1$.

19–20 Find a $\delta > 0$ that satisfies the limit claim corresponding to $\varepsilon = 0.01$, that is, such that $0 < |x - c| < \delta$ would imply $|f(x) - L| < 0.01$.

19. $\lim_{x\to 0}(3-2x) = 3$ **20.** $\lim_{x\to 4}\sqrt{x} = 2$

21–24 Give the formal definition of the limit claim. Then use the definition to prove the claim.

21. $\lim_{x\to 1}(3x+1) = 4$ **22.** $\lim_{x\to 1} x^2 = 1$

23. $\lim_{x\to 1}\sqrt{x} = 1$ **24.** $\lim_{x\to 2}\frac{2}{x} = 1$

25–41 Use algebra and/or appropriate limit laws to evaluate the given limit (one-sided limit where indicated). If the limit is unbounded, use the symbol ∞ or $-\infty$ in your answer.

25. $\lim_{x\to 3}\left(2x^2 - 3x + 5\right)$

26. $\lim_{x\to -2}\left(\frac{x^3}{4} + 2x^2 - x + 1\right)$

27. $\lim_{x\to 3}\sqrt{x^3 + 2x^2 + 4}$ **28.** $\lim_{x\to -2}\frac{2x+1}{x^2 - x}$

29. $\lim_{t\to 1}\left(\frac{3t + 5t^3}{t^2+1}\right)^{3/2}$ **30.** $\lim_{x\to 4}\frac{x^2-16}{x-4}$

31. $\lim_{x\to -5}\frac{x+5}{x^2-25}$ **32.** $\lim_{x\to 5^-}\frac{x+5}{x^2-25}$

33. $\lim_{x\to 1^+}\frac{x^2+1}{x^4-1}$ **34.** $\lim_{x\to 1}\frac{x^2-1}{x^4-1}$

35. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$ **36.** $\lim_{x\to 0}\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

37. $\lim_{x\to 0^-}\frac{2|x|}{x}$ **38.** $\lim_{x\to -2^+}\sqrt{4 - x^2}$

39. $\lim_{x\to 2^-}\left([\![x]\!] + 2x\right)$ **40.** $\lim_{x\to 1^+}[\![x]\!]x$

41. If $f(x) = x^2$, find $\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$.

42–43 Use the Squeeze Theorem to prove the limit claim.

42. $\lim_{x\to 0} x\cos\frac{1}{x} = 0$ **43.** $\lim_{x\to\infty}\frac{\sin x}{\ln x} = 0$

44. Sketch a graph of a function (a formula is not necessary) that is not continuous at $x = 0$ from either direction, but both of its one-sided limits exist at $x = 0$. (Answers will vary.)

45. Sketch a graph of a function that is left-continuous at $x = 0$, but its right-hand limit at $x = 0$ doesn't exist. (Answers will vary.)

46–51 Find and classify the discontinuities (if any) of the given function as removable or nonremovable.

46. $f(x) = \frac{x-9}{\sqrt{x}-3}, \quad x \ge 0$

47. $g(x) = \frac{\sqrt{x}+2}{x-4}, \quad x \ge 0$

48. $h(x) = \frac{1}{\sqrt{x^2+1}}$

49. $t(x) = 2 + 2[\![x]\!]$

50. $G(x) = \frac{x}{\sqrt{x+1}-1}, \quad x \ge -1$

51. $k(x) = |x-3| + |x+1|$

52–53 Use the ε-δ definition to prove that the function is continuous.

52. $f(x) = 3x - 1$ **53.** $g(x) = 2x^2$

54. Find the values of a and b such that f is continuous on the entire real line.

$$f(x) = \begin{cases} -1 & \text{if } x \le -3 \\ ax + b & \text{if } -3 < x < 2 \\ x^2 & \text{if } x \ge 2 \end{cases}$$

55. Use the Intermediate Value Theorem to prove that the equation $2x^5 + x + 1 = 0$ has a solution on the interval $[-1,1]$.

56. Use the Intermediate Value Theorem to show that the graphs of $f(x)=x^3$ and $g(x)=e^{-x}$ intersect.

57–58 Find the equation of the tangent line to the graph of $f(x)$ at the given point.

57. $f(x)=x^2+x;\quad (1,2)$

58. $f(x)=\sqrt{x};\quad (4,2)$

59–60 Use the definition (also called the limit process) to find the derivative function f' of the given function f. Find all x-values (if any) where the tangent line is horizontal.

59. $f(x)=2x-x^2$

60. $f(x)=\dfrac{3}{x-2}$

61–62 Sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

61. f is continuous at 0, $f(0)=1$, $f'(x)<0$ for $x<0$, $f'(x)>0$ for $x>0$, and $f'(0)$ does not exist

62. $g(1)<0$, $g'(1)>0$, and $g(2)>0$, but $g'(2)<0$

63. Prove that if $f(x)$ is a quadratic function, then $f'(x)$ is linear.

64. A small object is thrown upward with an initial velocity of 12 m/s from the top of a 15 m high building.

a. How high does it go and when does it reach the ground?

b. What is the speed of impact?

(**Hint:** Use $h(t)=-5t^2+12t+15$ as the position function, where h is in meters, t in seconds.)

65. The owner of a small toy manufacturer has determined that he can sell x toys if the price is $p=D(x)=0.2x+30$ dollars. The total cost as a function of x is given by $C(x)=0.1x^2+15x+247.5$ dollars.

a. Find the profit function $P(x)$.

b. Find any break-even points.

c. Find the marginal profit function.

Concept Check

66–73 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

66. Instantaneous velocity can be interpreted as the slope of a tangent line.

67. If $\lim_{x\to c} f(x)$ doesn't exist, then $f(x)$ has a vertical asymptote at $x=c$.

68. Any rational function has at least one vertical asymptote.

69. If $\lim_{x\to c} f(x)=A$ and $\lim_{x\to c} g(x)=B$, then $\lim_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{A}{B}$.

70. If f is defined on $[a,b]$, L is a real number between $f(a)$ and $f(b)$, and $\lim_{x\to c} f(x)$ exists for all $x\in(a,b)$, then there is a c in the interval (a,b) such that $f(c)=L$.

71. If f is continuous at c, then $f(c)$ is equal to both one-sided limits at c.

72. If both one-sided limits of f exist at c, and if f is defined at c, then f is continuous at c.

73. If $g(x)\le f(x)\le h(x)$ for all x in some open interval containing c, and if $\lim_{x\to c} g(x)=\lim_{x\to c} h(x)=L$, then by the Squeeze Theorem $f(c)=L$ as well.

Chapter 2 Technology Exercises

74. Use a computer algebra system to find approximations for the areas in Exercises 10 and 11 by using $n=100$. (Round your answers to four decimal places.)

75. Use a computer algebra system to find approximations for the arc lengths in Exercises 12 and 13 by using $n=100$. (Round your answers to four decimal places.)

76. Use a graphing utility to verify your answers given for Exercises 14–17.

77. Use a graphing utility to approximate the solutions for Exercises 55 and 56. Round your answers to four decimal places.

78–81 Use a graphing utility to graph the function, and estimate from the graph the value of the given limit.

78. $\lim_{x\to\infty} x^{1/x}$

79. $\lim_{x\to 0}\dfrac{\arcsin x}{x}$

80. $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{2x}$

81. $\lim_{x\to 1}\dfrac{\ln(x^3)}{x-1}$

Chapter 3
Review Exercises

1–2 Find the derivative of the given function at the specified point and express your answer using the differential notation due to Leibniz.

1. $f(x)=x^3+x;\quad x=1$

2. $g(x)=\frac{2}{x};\quad x=2$

3–4 Find the derivative of the function and use the differentiation operator D_x to express your answer.

3. $s(x)=\sqrt{x-2}$

4. $t(x)=\frac{1}{x^2+1}$

5–6 Find the first, second, and third derivatives of the function.

5. $f(x)=x^2-1$

6. $g(x)=x^4$

7–10 Find all points where the function is not differentiable. For each of those points, find the one-sided derivatives (if they exist).

7. $f(x)=\sqrt[3]{x}$

8. $g(x)=|x+1|+|x-3|$

9. $h(x)=[\![x]\!]+x$

10. $F(t)=\begin{cases} t^2\sin\frac{1}{t} & \text{if } t\neq 0 \\ 0 & \text{if } t=0 \end{cases}$

11–20 Use differentiation rules to find the derivative of the function.

11. $f(x)=0.2x^5-2x^4+x^3+0.5x^2+2^{3/4}$

12. $g(x)=\sqrt{3}x+\sqrt{3x}+\frac{1}{\sqrt{3x}}$

13. $h(x)=(2x-1)(x^2+4)$

14. $k(x)=\frac{x-2}{x}e^{x-2}$

15. $F(t)=(t-1)\left(t^2+\frac{1}{t}\right)\left(\sqrt{t}+t\right)$

16. $G(s)=\frac{1}{2s+s^2}$

17. $u(x)=\frac{2e^x+1}{3e^x+5}$

18. $t(x)=\frac{\frac{2}{x}+\frac{1}{x^2}+4}{\frac{2}{x^2}-\frac{1}{x}}$

19. $v(x)=\ln(x^2+2)$

20. $w(x)=\sin(\sin(\sin x))$

21–24 Find the first, second, and third derivatives of the function.

21. $f(x)=\frac{x}{x+1}$

22. $f(x)=3\sqrt{x}$

23. $f(x)=\tan x$

24. $f(x)=\arctan x$

25–26 Find a function f that satisfies the given conditions. (**Hint:** A polynomial is the most natural choice. Answers will vary.)

25. $f(0)=0$, $f'(1)=1$, and $f''(2)=4$.

26. $f(0)=2$ and $y=2x+1$ is tangent to the graph at $x=-1$.

27–28 Find the equation of the line tangent to the graph of the function at the given point.

27. $f(x)=\frac{1}{\sqrt{3x^2+1}};\quad \left(1,\frac{1}{2}\right)$

28. $f(x)=\tan(\sin x);\quad (\pi,0)$

29. Find the equation(s) of the line(s) tangent to the graph of $f(x)=x^2+3x+1$ through the point $(2,2)$, which is not on the graph of f.

30. Assuming f is differentiable, find the derivative of $y=\ln\sqrt{[f(x)]^2+1}$.

31–32 Find the indicated limit.

31. $\lim\limits_{x\to 0}\frac{-\sin 2x}{4x}$

32. $\lim\limits_{x\to 0^+}\frac{x\cos x}{1-\cos x}$

33. The position function of a moving particle is given by $x(t)=\frac{50t}{t+1}$ feet at t seconds. Find its velocity and acceleration at $t=1$ second.

34. An object is moving along a straight line so that its distance from the start at t seconds is given by $d(t)=12t-t^3$ meters. Find its position and acceleration at the instant when its velocity changes directions.

35. The radius of a spherical balloon being inflated increases according to the function $r(t)=3+4\sqrt[3]{t}$, where r is measured in centimeters and t in seconds. Find the rate of change of the balloon's volume and surface area with respect to time at $t=1$ second.

36–37 Find dy/dx by implicit differentiation.

36. $x^3 + y^3 = 2$

37. $6(x^2 + y^2) = 15xy$

38–39 Find dx/dy by implicit differentiation.

38. $x\sin(x+y) = y^2 + 6$

39. $6(y^2 - x^2) = y^4$

40–43 Use implicit differentiation to find the equation of the line tangent to the curve at the indicated point.

40. $\frac{1}{x^3} + \frac{1}{y^3} = 2;\quad (1,1)$

41. $x^3 + y^2 = 2x + 1;\quad (0,1)$

42. $\frac{3(x+y)}{xy} = 16\sqrt{x+y};\quad \left(\frac{3}{4}, \frac{1}{4}\right)$

43. $3\sqrt{x} + \frac{2}{\sqrt{y}} = xy;\quad (1,4)$

44. Find all points on the lemniscate

$$(x^2 + y^2)^2 = 2(x^2 - y^2)$$

where its graph has horizontal tangent lines.

45. Use implicit differentiation to find d^2y/dx^2 for $x^{2/3} + y^{2/3} = 1$.

46–49 Use the Derivative Rule for Inverse Functions to determine $(f^{-1})'(a)$ for the indicated value of *a*. (The domain of *f* is assumed to have been restricted so that the inverse exists and is differentiable, whenever appropriate.)

46. $f(x) = x^3 + x;\quad a = 10$

47. $f(x) = \sqrt[4]{x+1};\quad a = 2$

48. $f(x) = \frac{2}{x^2};\quad a = \frac{1}{2}$

49. $f(x) = e^x + x;\quad a = 1$

50–53 Determine the derivative of the given function.

50. $f(x) = \log\sqrt{x^2+1}$

51. $f(x) = \tan^{-1}\sqrt{x}$

52. $f(x) = e^{\arcsin x}$

53. $f(x) = \sec^{-1}(\ln x)$

54–55 Use logarithmic differentiation to find y'.

54. $y = \frac{\sqrt[3]{x^2+1}\,(x+3)}{x^{2/3}\sqrt{2x^2+3}}$

55. $y = (\sqrt{x})^{\ln x}$

56. A fast-growing population of bacteria doubles every half hour. If the initial count is 1000, how many bacteria are there in 100 minutes?

57. A 350 °F pizza is left on the counter and cools to 250 °F in 4 minutes. If the room temperature is 70 °F, determine the total time it takes for the pizza to cool down from 350 °F to 185 °F. (**Hint:** See Exercise 9 of Section 3.7.)

58. Find the rate of change of the distance from the origin of a point moving on the graph of $f(x) = x^3$ when $x = 1$ and $dx/dt = 2$ units per second.

59. A spherical balloon is being filled with helium at a rate of $20 \text{ in.}^3/\text{s}$. How fast is the radius increasing at the instant when the radius is 4 in.?

60. A small plane, flying at an altitude of 0.1 miles at a ground speed of 85 miles per hour, passes directly over an observer. How fast is the distance between the observer and the plane increasing a minute later?

61. Radar is tracking a rocket that was launched vertically upward. It is found that the rocket's distance from the radar is increasing at a rate of 1200 km/h at the instant when that distance is 5 km. If the radar station is 4 km from the launch site, find the speed of the rocket.

62. A ship sailing west at 9 miles per hour passes a buoy 20 minutes before another ship sailing due north at 12 miles per hour passes the same buoy. How fast will they be separating an hour later?

63. Tiffany walks toward a light source that is 8 feet above ground. If the speed of the tip of her shadow is three times that of her walking speed, how tall is Tiffany?

64–65 Find the linearization of the function at the given value.

64. $f(x) = \frac{1}{(x-1)^2};\quad x = 2$

65. $f(x) = \sin x;\quad x = \frac{\pi}{4}$

66–67 Use linear approximation to approximate the given number. Round your answer to four decimal places.

66. $\sqrt[3]{8.2}$

67. $\arctan 0.9$

68. The diameter of a large bouncy ball was measured to be 65 cm with a possible error of 1 mm. Approximate the propagated errors in the calculated volume and surface area of the ball, respectively. Express your answers as percentage errors.

69. The proper dosage d of a certain over-the-counter medicine for children depends on body weight w according to the function $d(w)=\frac{5}{4}w^{3/5}$, where d is measured in milligrams and w in pounds. Use differentials to estimate how accurately (in terms of percentage error) we need to know a 32-pound child's weight if we cannot stray from the proper dosage by more than 6 percent.

70. A manufacturing business found its daily revenue to be $R(x)=150x-\frac{1}{4}x^2$ dollars when x units are produced and sold.

a. Use linearization and marginal revenue to estimate the extra revenue when production is increased from 100 to 102 units.

b. Use the revenue function to calculate the actual revenue increase. Compare your answers.

71. Use the concept of the derivative function to explain why the graph of $y=x^a$, $a>1$ curves upward, while the graph of $y=x^b$, $0<b<1$ curves downward.

Concept Check

72–82 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

72. If both one-sided derivatives of $f(x)$ either exist or are equal to $\pm\infty$ at c, then f is continuous at c.

73. If $p(x)$ is a polynomial of degree n, then all k^{th}-order derivatives of $p(x)$ for $k>n$ are 0.

74. If $y=\pi^n\sin x$, then $y'=n\pi^{n-1}\sin x+\pi^n\cos x$.

75. If $y=1/(x^2-3x+1)$, then $y'=1/(2x-3)$.

76. If $y=\ln(3x+1)$, then $y'=1/(3x+1)$.

77. If $y=x^x$, then $y'=x\cdot x^{x-1}$.

78. Since $(e^x)'=e^x$, therefore $\left(e^{e^x}\right)'=e^{e^x}$.

79. If $f(x)=x$, then $df=dx$.

80. If $f(x)$ is linear, then its linearization at any point is itself.

81. If $x\to 0$, then $\Delta x\to dx$ and $\Delta y\to dy$.

82. If $\Delta x\to 0$, then $\Delta y/\Delta x\to dy/dx$.

Chapter 3
Technology Exercises

83–85 Use a graphing utility to graph the function and identify all points where the function is not differentiable. Explain.

83. $f(x)=\left|x^2-x\right|$

84. $f(x)=|x|(x+2)$

85. $f(x)=\sqrt[4]{x^2-1}$

86. Use the differentiation capabilities of a graphing utility to find the derivative of $f(x)=2\cos^2 x-\cos 2x$. Then find the derivative by hand, applying a trigonometric identity before differentiating. Does your answer agree with that of your technology? If not, what do you think is the reason? Can you "force" your graphing utility to represent its answer in a simpler form?

87. Repeat Exercise 86 for the function $f(x)=2\sin(x/2)\cos(x/2)$.

88. Use a graphing utility to graph the functions $y=\ln x$, $y=a^x$, $a>1$ and $y=x^b$, $0<b<1$ for various values of the parameters a and b. By zooming out appropriately, compare their relative growth rates; that is, conjecture "who wins the race toward infinity" in general among these three types of functions. Use the concept of the derivative to support your conjecture.

89. The displacement of a mass attached to a spring is given by the function $h(t)=e^{-t/6}\cos 2t$.

a. Use a graphing utility to graph the function and explain why it is realistic.

b. Use a graphing utility to graph the velocity and acceleration functions together with $h(t)$ on the same screen. What seems to be the position of the mass when velocity is maximum? When is velocity 0? When is acceleration maximum, and when is it 0?

Chapter 4
Review Exercises

1–4 Sketch by hand the graph of a function f on the specified domain, with the specified properties. (Answers will vary.)

1. Defined on $(0,2)$, absolute minimum at 1, no absolute maximum
2. Defined on $[-2,2]$, absolute maximum occurs twice, no absolute minimum
3. Defined on $(-1,1)$, no absolute or relative extrema
4. Differentiable on $(0,1)$, no critical points, no extrema

5–14 Find all absolute extrema of the function on the indicated domain.

5. $f(x)=x^2-\frac{4}{3}x^3;\quad D=[-1,1]$
6. $f(x)=-x^4+8x^3-16x^2;\quad D=[-1,5]$
7. $f(x)=\left|x^2-2x-8\right|;\quad D=[-3,5]$
8. $f(x)=(x+2)|x|;\quad D=[-2,2]$
9. $f(x)=\sqrt{x}(1-x);\quad D=[0,2]$
10. $f(x)=\frac{x^2+1}{x+1};\quad D=[0,1]$
11. $f(x)=3x^5-2x^3+1;\quad D=\mathbb{R}$
12. $f(x)=\sqrt{1-x^4};\quad D=(-1,1)$
13. $f(x)=\csc\frac{x}{2};\quad D=(0,2\pi)$
14. $f(x)=x^2-x[\![x]\!];\quad D=[1,2]$

15–16 Prove that the equation has exactly one real solution on the given interval.

15. $3x^3-x^4=1$ on $(0,1)$
16. $x\arcsin x=e^{-x}$ on $(0,1)$

17–18 Determine whether Rolle's Theorem applies to the function on the given interval. If so, find all possible values of c as in the conclusion of the theorem. If the theorem does not apply, state the reason.

17. $f(x)=x^3+x^2-8x-12$ on $[-2,3]$
18. $g(x)=x^4+2x^2-2$ on $[0,1]$

19–20 Determine whether the Mean Value Theorem applies to the function on the given interval. If so, find all possible values of c as in the conclusion of the theorem. If the theorem does not apply, state the reason.

19. $f(x)=\left|x^4-3x\right|$ on $[2,3]$
20. $f(x)=\left|x^4-3x\right|$ on $[0,2]$
21. If $\left|f'(x)\right|\le 3$ for all x, prove that $\left|f(10)-f(2)\right|\le 24$.
22. If $g(-5)=-1$ and $g'(x)\le 4$ for all x, what is the largest possible value of $g(1)$?
23. Find the function f that passes through $(0,3)$ and whose derivative is $\cos x+e^x$.
24. An object is moving along the x-axis, starting at $x_0=-4$ with velocity function $v(t)=3t^2-2t$ $(0\le t\le 5)$. Find the time t when it reaches the origin.
25. A car driving at 70 mph passes a mile marker, and then exactly 48 seconds later, still driving at 70 mph, passes the next mile marker.
    a. Prove that there was at least one instant when the car traveled at 75 mph between the markers.
    b. Prove that there was at least one instant when the car's acceleration was zero.

26–29 Use the first derivative to determine where the function is increasing and decreasing.

26. $f(x)=|2x-2|$
27. $g(x)=-x^2+6x+7$
28. $h(x)=x^3-6x^2+9x+1$
29. $k(x)=x^4-4x^3-20x^2+96x$

30–33 Determine the intervals of concavity of the given function.

30. $f(x)=\frac{x^3}{6}-x^2-2$
31. $g(x)=\frac{e^x}{x}$
32. $h(x)=(x-2)\sqrt[3]{x}$
33. $k(x)=\frac{x+3}{x^2-1}$

34–35 Use the first and second derivatives to identify the intervals of monotonicity, extrema, intervals of concavity, and inflection points of the given function.

34. $f(x)=3x^5-20x^3$
35. $g(x)=\arctan\left(x^2\right)$

36–37 The function $p(t)$ gives the position, relative to its starting point, of an object moving along a straight line. Identify the time intervals when the object is moving in the positive versus negative direction, as well as those intervals when it is accelerating or slowing down. Find the times when the object changes direction as well as when its acceleration is zero.

36. $p(t)=t^3-3t^2, \quad 0\le t\le 5$

37. $p(t)=3t^2-\dfrac{t^4}{2}, \quad 0\le t\le 3$

38–49 Check whether L'Hôpital's Rule applies to the given limit. If it does, use it to determine the value of the limit. If it does not, find the limit some other way. (When necessary, apply L'Hôpital's Rule several times.)

38. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$

39. $\lim\limits_{x\to 0}\dfrac{x-\tan x}{\sec x-1}$

40. $\lim\limits_{x\to \infty}\dfrac{e^{-x}}{\ln x}$

41. $\lim\limits_{x\to 0^+}\cot x\csc x$

42. $\lim\limits_{x\to 0}\left(1+4x^2\right)^{1/x^2}$

43. $\lim\limits_{x\to (1/2)^+}\left(\dfrac{1}{4x-2}-\dfrac{1}{\ln 2x}\right)$

44. $\lim\limits_{x\to 0^+}x^{\sqrt{x}}$

45. $\lim\limits_{x\to 0^+}\left(\sqrt{x}\right)^{\ln x}$

46. $\lim\limits_{x\to 0^-}x\cot x$

47. $\lim\limits_{x\to 0}\dfrac{\arctan x}{\arctan 2x}$

48. $\lim\limits_{x\to 0^+}\sin x\ln x$

49. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\csc x\right)$

50. By examining the limits $\lim\limits_{x\to\infty}\dfrac{\ln x}{x^a}$ and $\lim\limits_{x\to\infty}\dfrac{x^a}{b^x}$ using L'Hôpital's Rule, compare the relative growth rates of the functions $y=\ln x$, $y=x^a$, and $y=b^x$ $(a>0,\ b>1)$. (See Exercise 88 in the Chapter 3 Review.)

51–60 Use the curve-sketching strategy to construct a graph of the function.

51. $f(x)=\dfrac{x^3}{3}-x^2-15x$

52. $g(x)=-x^3+12x-16$

53. $h(x)=x^4-2x^3$

54. $m(x)=3x^3-4x^5$

55. $f(x)=\dfrac{2x}{x^2+1}$

56. $f(x)=\dfrac{2x}{x^2-1}$

57. $f(x)=|x|(x-1)$

58. $f(x)=x\sqrt{9-x^2}$

59. $f(x)=\dfrac{x^3}{x^2-4}$

60. $f(x)=\sin^2 x\cos x$

61–62 Use Newton's method to approximate the given number to five decimal places.

61. $\sqrt[5]{30}$

62. $\log 11$

63–64 Use Newton's method to solve the equation on the given interval. Approximate the root to six decimal places.

63. $2x^5=1-x^2$ on $(0,1)$

64. $\ln x=\cos x$ on $(0,\infty)$

65. Use Newton's method to approximate to four decimal places the fixed point(s) of $f(x)=1-\tan x$ on $(0,\pi/2)$. (See Exercise 62 of Section 4.2.)

66. Find a positive number that is greater than its own cube by the greatest possible amount.

67. Generalize Exercise 66 to the n^{th} power of a number $(n\ge 2)$.

68. Find a number a so that for given $a_1, a_2, a_3\in\mathbb{R}$, the quantity $S_3=(a-a_1)^2+(a-a_2)^2+(a-a_3)^2$ is minimal.

69. Generalize Exercise 68 for n given numbers to minimize the following quantity: $S_n=(a-a_1)^2+(a-a_2)^2+\cdots+(a-a_n)^2$.

70. A wire of length l is bent into an L shape. Where should it be bent in order to minimize the distance between the two endpoints?

71. Find the length l and width w of the rectangle inscribed in the unit circle for which l^2w is maximal.

72. Find the dimensions of the rectangle whose diagonal is d units and whose area is maximum.

73. A book page of area 500 cm^2 is required to have 1 cm margins on the sides, while the margins on the top and bottom are to be 2 cm. Find the dimensions of the page that maximize the printed area.

74. Among all isosceles triangles whose legs are l units long, find the base angle that maximizes the area.

75. A vertex of a rectangle is at the origin and the opposite vertex sits in the first quadrant and on the graph of $y=\dfrac{2-x}{x+1}$. Find the maximum possible area for such a rectangle.

76. Find the point on the graph of $y=1-x^2$ that is closest to the point $(-3,1)$.

77. Among all isosceles triangles that can be inscribed in the circle of radius R, find the one with maximum area.

78. A vending machine sells 500 bars of a certain type of candy when the price is \$1.50. It was discovered that 10 fewer customers will buy the candy bar for each 5¢ increase in price. What is the price that will bring maximum revenue from the sales of this type of candy bar?

79. Maximize the surface area of the can in Example 3 of Section 4.6. Explain your findings.

80. Minimize the cost of producing the can in Example 3 of Section 4.6 if the top and bottom are produced using a material that is 50% more expensive than the material used for the side.

81. Nate needs to reach a restaurant that is 600 ft upstream on the other side of a 150 ft wide river. Find the point where he has to reach the other side in order to make the best time if he can swim at 5 ft/s and walk at 9 ft/s. (Ignore the flow of the river.)

82–89 Find the general antiderivative of the given function, and check your answer by differentiation. (If necessary, rewrite the function before antidifferentiation.)

82. $f(x) = 2x^3 - 6x^2 + 3x$

83. $f(x) = 5x^4 - 4.8x^3 + e^2$

84. $f(x) = x(x+2)(2x-3)$

85. $f(x) = 0.4x\sqrt{x} - \dfrac{2}{\sqrt{x}}$

86. $f(x) = \dfrac{x^4 - 4x}{x^2}$

87. $f(x) = 2(x + \sec^2 2x)$

88. $f(x) = 6e^{3x}$

89. $f(x) = \dfrac{3}{4x^2+1}$

90–91 Find $f(x)$ that satisfies the specified conditions.

90. $f''(x) = x, \quad f'(1) = 1, \quad f(1) = 0$

91. $f'''(x) = 2, \quad f''(2) = -1, \quad f'(2) = 2, \quad f(2) = 3$

92. A tennis ball is thrown upward from an initial height of 4 feet with an initial velocity of 56 feet per second. How high will it go and for how long is it rising? (Ignore air resistance.)

93. With what initial velocity do we need to throw a golf ball vertically upward in order for it to rise 100 feet high? (Ignore the initial height and air resistance.)

94. A pebble is shot horizontally using a slingshot at 10 meters per second from the top of a building that is 20 meters high. If the terrain around the building is nearly flat, approximately how far from the building will the pebble hit the ground? (Use the approximation $g \approx 10 \text{ m/s}^2$ and ignore air resistance.)

Concept Check

95–101 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

95. A continuous function on a finite interval always attains its maximum and minimum.

96. If $f(x)$ has a relative maximum or minimum at $x = c$, then $f'(c) = 0$.

97. If $f(x)$ has a relative maximum or minimum at $x = c$, then c is a critical point of f.

98. A cubic polynomial has exactly one inflection point.

99. If $f(x)$ is a polynomial, then between two consecutive local extrema there must be an $x = c$ so that $f''(c) = 0$.

100. If $f(x)$ is a polynomial and c is a critical point, then there is a relative maximum or minimum at $x = c$.

101. If $f'''(c) = 0$, then $f'(x)$ has a point of inflection at $x = c$.

Chapter 4
Technology Exercises

102–111. Use a graphing utility to verify the answers you obtained for Exercises 51–60.

112–113. Use a graphing utility to verify the conclusions of Exercises 15 and 16.

Chapter 5
Review Exercises

1–2 Use $(O_4 + U_4)/2$ to estimate the area under the graph of the function and above the x–axis on the given interval.

1. $f(x) = \frac{x^2}{2}$ on $[0,2]$
2. $f(x) = \sin x$ on $\left[0, \frac{\pi}{2}\right]$

3–6 Write the given sum using sigma notation.

3. $\frac{1}{2} - \frac{1}{9} + \frac{1}{28} - \frac{1}{65} + \cdots - \frac{1}{1{,}000{,}001}$
4. $a_1 + a_5 + a_9 + a_{13} + \cdots + a_{97}$
5. $f\left(\frac{2}{n^2}\right) + f\left(\frac{4}{n^2}\right) + f\left(\frac{6}{n^2}\right) + \cdots + f\left(\frac{100}{n^2}\right)$
6. $g(t_{-2}^*)\Delta t + g(t_{-1}^*)\Delta t + g(t_0^*)\Delta t + \cdots + g(t_{2n}^*)\Delta t$

7–8 Assuming that $\sum_{i=0}^{n} a_i = 50$ and $\sum_{i=0}^{n} b_i = 80$, find the sum.

7. $\sum_{i=0}^{n} (a_i + 2b_i + 2)$
8. $\sum_{i=0}^{n} \left(\frac{a_i}{5} - \frac{b_i}{4}\right)$

9–10 Use summation formulas to find the value of the sum.

9. $\sum_{i=1}^{10} (3i^3 - 1)$
10. $\sum_{j=1}^{n} \frac{(2j+1)(j-2)}{2}$

11. Find the value of the sum $\sum_{i=1}^{n} \left[\frac{1}{i^2} - \frac{1}{(i+1)^2}\right]$.
(**Hint:** Write out the first few terms as well as the last few terms.)

12–13 Evaluate the geometric sum using the formula proven in Exercise 53 of Section 5.1.

12. $\sum_{i=0}^{10} \frac{3}{2^i}$
13. $\sum_{j=0}^{6} (-1)^j (0.3)^j$

14–15 Evaluate the given double sum.

14. $\sum_{i=3}^{11} \sum_{j=1}^{4} (2i - j)$
15. $\sum_{i=1}^{n} \sum_{j=1}^{m} i^2 j$

16–18 Find the area under the graph of $f(x)$ and above the x–axis on the given interval, by taking the limit of the associated Riemann sums.

16. $f(x) = x^2 + 1$ on $[0,2]$
17. $f(x) = x^3$ on $[0,1]$
18. $f(x) = \sqrt{x}$ on $[0,4]$
(**Hint:** Choose $x_i^* = \frac{4(i-1)^2}{n^2}$.)
19. Identify the region whose area is the limit given by $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[\frac{4i}{n} - \left(\frac{2i}{n}\right)^2\right]$. Then use summation formulas to evaluate the limit.

20–21 Prove that the given function is not integrable on the interval $[0,1]$.

20. $f(x) = \frac{1}{x^2}$
21. $g(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$

22–25 Suppose that f is an even function, g is odd, both are integrable on $[-a,a]$, and we know that $\int_0^a f(x)\,dx = 2$, while $\int_0^a g(x)\,dx = 0.5$ $(a > 0)$. If possible, find the integral.

22. $\int_{-a}^{a} [5f(x) + 4g(x)]\,dx$
23. $\int_{-a}^{a} [f(x)]^2 g(x)\,dx$
24. $\int_{-a}^{a} f(x)[g(x)]^2\,dx$
25. $\int_{-a}^{0} [f(x) + g(x)]\,dx$

26–27 Find the average value of $f(x)$ over the given interval and identify all points in the domain where $f(x)$ assumes its average value.

26. $f(x) = 4x - x^2$ on $[0,4]$
27. $f(x) = |x-2| - 1$ on $[1,5]$

28. Use the Fundamental Theorem of Calculus to evaluate the limit

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\frac{i}{n}}-\frac{i}{n}\right)$$

by recognizing it as a Riemann sum of a function over an interval.

29–30 Use Part I of the Fundamental Theorem of Calculus to find the derivative of the given function.

29. $F(x)=\int_0^x \sqrt{1+t^2}\,dt$ **30.** $G(x)=\int_0^{x^2} e^{t^2}\,dt$

31–38 Use Part II of the Fundamental Theorem of Calculus to evaluate the definite integral.

31. $\int_1^2 (2x^4+3x^2-2)\,dx$ **32.** $\int_0^2 (3x+2)(5-x)\,dx$

33. $\int_1^4 \left(\frac{1}{t}-\frac{2}{t^2}+1\right)dt$ **34.** $\int_1^9 \frac{x^2-2\sqrt{x}+2}{x}\,dx$

35. $\int_0^1 \frac{2}{\sqrt{1-x^2}}\,dx$

36. $\int_{\pi/4}^{3\pi/4} (2\csc^2 x-\cos x)\,dx$

37. $\int_2^3 \frac{x+2}{x-1}\,dx$ **38.** $\int_0^1 \frac{2x^2-1}{x^2+1}\,dx$

39–40 Find the area of the region between the graph of the given function and the x-axis on the indicated interval.

39. $y=\frac{1}{2x^2}$ on $[1,10]$

40. $y=2\sqrt{x}-x^2$ on $[0,2]$

41. Find a formula for $f(x)$ if $\int_0^{x^3} f(t)\,dt=\sin(x^3)$.

42. The velocity function of a particle moving along the x–axis is $v(t)=3t-t^2$ units per second. If it started at the origin, find **a.** the position of the particle at $t=5$ seconds and **b.** the total distance traveled by the particle in the time interval $[0,5]$.

43–54 Use an appropriate substitution (when necessary) to evaluate the indefinite integral.

43. $\int \frac{-2}{\sqrt{1-x^2}}\,dx$ **44.** $\int \frac{-2x}{\sqrt{1-x^2}}\,dx$

45. $\int \sec x(\sec x+\tan x)\,dx$

46. $\int \sec^2 x\tan x\,dx$

47. $\int 6x^2(2x^3-7)^9\,dx$ **48.** $\int x^4\sqrt{x^5-3}\,dx$

49. $\int \frac{4x}{(x^2+1)^2}\,dx$ **50.** $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$

51. $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$ **52.** $\int \frac{1}{x\ln(x^2)}\,dx$

53. $\int \frac{e^{2x}}{e^x+1}\,dx$ **54.** $\int \frac{1}{x^2}\sin\left(\frac{x+1}{x}\right)dx$

55–56 Find $y(x)$ that satisfies the given conditions.

55. $\frac{dy}{dx}=\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$; $y(9)=1$

56. $y''(x)=1-\sin x$; $y'(0)=2$; $y(0)=0$

57. A particle is moving along the x-axis in the positive direction with a velocity function of $v(t)=\frac{t}{t^2+1}$ units per second. If it started at the point $(1,0)$, what is the particle's position at $t=4$ seconds?

58–63 Evaluate the definite integral.

58. $\int_0^1 3x^5(x^6-1)^{12}\,dx$

59. $\int_0^4 (x+1)\sqrt{x^2+2x}\,dx$

60. $\int_1^2 \frac{x^3}{x^4+1}\,dx$ **61.** $\int_{-1/2}^{0} \frac{2^t}{\sqrt{1-4^t}}\,dt$

62. $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx$ **63.** $\int_1^4 \frac{dx}{\sqrt{x}(\sqrt{x}+1)^2}$

64–69 Find the area of the region bounded by the graphs of the given equations. (If convenient or necessary, integrate with respect to y rather than x.)

64. $y=x^3-4x,\quad 3y=15x,\quad x\ge 0$

65. $y=1-x^2,\quad y=1-x^6,\quad x\ge 0$

66. $y=2\sqrt[4]{x},\quad y=4-2x,\quad y=0$

67. $y=\ln x,\quad (e-1)y=x-1$

68. $y=\frac{1}{1+x^2},\quad 2y=1$

69. $y=\sin x,\quad y=\sin x\cos x,\quad 0\le x\le\pi$

70. Consider the region bounded by the graph of $y=1/x$ and the x-axis over the interval $[1,a]$ $(a>1)$. Find the vertical line $x=c$ that bisects the region in two subregions of equal area.

71. Consider the function $f(x)=1/x^2$ defined on some interval $[a,b]$. Partition $[a,b]$ and in each subinterval $[x_{i-1},x_i]$ choose the sample point $x_i^*=\sqrt{x_{i-1}x_i}$ (the geometric mean of the endpoints). Show that

$$\frac{1}{\left(x_i^*\right)^2}\Delta x_i=\frac{1}{x_{i-1}}-\frac{1}{x_i}$$

and use this observation to prove the following formula.

$$\int_a^b \frac{1}{x^2}\,dx=\frac{1}{a}-\frac{1}{b}$$

72. Prove that if the conditions of Part I of the Fundamental Theorem of Calculus are satisfied and $F(x)=\int_{g(x)}^{h(x)} f(t)\,dt$, where $g(x)$ and $h(x)$ are differentiable, then $F'(x)=f(h(x))h'(x)-f(g(x))g'(x)$. (**Hint:** See Example 3 of Section 5.3.)

73. Prove that if f is a linear function, then its definite integral on an interval $[a,b]$ is the average of its left and right Riemann sums, that is,

$$\int_a^b f(x)\,dx=\frac{L_n+R_n}{2}.$$

What is your expectation regarding the integral and the average above if f is concave up? Concave down?

Concept Check

74–81 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

74. If $n_1<n_2$, then the Riemann sum R_{n_2} is always a better approximation of the integral than R_{n_1}.

75. If f is piecewise continuous on a closed interval, then the limit of its Riemann sums always exists.

76. When applying the Fundamental Theorem of Calculus, we must choose the antiderivative with $C=0$.

77. $\int \frac{1}{e^x}\,dx=\ln\left(e^x\right)+C=x+C$

78. The definite integral of the velocity function of a moving object on $[t_1,t_2]$ is equal to the total distance traveled by the object from time $t=t_1$ to $t=t_2$.

79. $\int_a^b f(x)\,dx>0$ if and only if $f(x)>0$ on $[a,b]$.

80. $\int \sec x\,dx=\sec x\tan x+C$

81. $\int_{-1}^{1}\frac{1}{x^3}\,dx=\left.\frac{-1}{2x^2}\right]_{-1}^{1}=-\frac{1}{2}-\left(-\frac{1}{2}\right)=0$

Chapter 5 Technology Exercises

82. Use the summation feature of a graphing utility to verify your answers for Exercises 9–15.

83. Write a program for a graphing calculator or computer algebra system that calculates the n^{th} Riemann sum for a given function on a given interval, using subintervals of equal width and sample points of your choice. Use your program to verify your answers for Exercises 16–18.

84. Use a graphing utility to evaluate the limit of Exercise 28. What do you find? (Answer will vary depending on the capabilities of the particular software used.)

Chapter 6
Review Exercises

1–2 The base of a solid *S* is described in the *xy*-plane along with its cross-sections in a certain direction. Find the volume of *S*.

1. The base of S is the first-quadrant region of the unit disk centered at the origin; the cross-sections perpendicular to the y-axis are squares.

2. The base of S is the region bounded by the graph of $y = \sqrt{2-x}$ and the coordinate axes; the cross-sections perpendicular to the x-axis are equilateral triangles.

3–6 Use the disk/washer method to find the volume of the solid generated by rotating the region *R* about the indicated axis.

3. R is bounded by the graph of $y = \sqrt{1-x}$ and the coordinate axes, rotated about the x-axis.

4. R is the region of Exercise 3, rotated about the y-axis.

5. R is bounded by the graphs of $y = \operatorname{arcsec} x$, $x = \sqrt{2}$, and $y = 0$, rotated about the y-axis.

6. R is the first-quadrant portion of the region bounded by the graphs of $y = 4/x^2$ and $y = 5 - x^2$, rotated about $x = -2$.

7–10 Use the shell method to find the volume of the solid generated by rotating the region *R* about the indicated axis.

7. R is bounded by the graph of $y = 2x - x^2$ and the x-axis, rotated about the y-axis.

8. R is bounded by the graphs of $y = x$, $y = 3 - 2x$, and the y-axis, rotated about $x = -1$.

9. R is the first-quadrant region bounded by the graphs of $y = 4x$ and $y = x^3$, rotated about the x-axis.

10. R is bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$, rotated about $y = -2$.

11–14 The given integral represents the volume of a solid of revolution. Describe the solid. (Do not evaluate the integral.)

11. $\int_0^3 \pi x^2\,dx$

12. $\pi\int_0^2 \left[1-(y-1)^2\right]dy$

13. $\int_0^2 2\pi x\left(4-x^2\right)dx$

14. $2\pi\int_1^3 y\sqrt{1-(y-2)^2}\,dy$

15–18 Use the shell method or the disk/washer method to find the volume of the solid obtained by revolving the region bounded by the graphs of the equations about the given axis. Choose the method that seems to work best.

15. $x = \sqrt{\sin y}, \quad x = 0, \quad 0 \le y \le \pi;$ about the y-axis

16. $y = x^3 - 2x^2 + x, \quad y = 0, \quad x = 0, \quad x = 1;$ about the x-axis

17. $y = \sqrt{x}, \quad y = 2 - x, \quad x = 0;$ about the y-axis

18. $y = \dfrac{e^x}{x}, \quad y = 0, \quad x = 1, \quad x = 2;$ about the y-axis

19. Suppose the region bounded by the graph of $y = \sqrt{c^2 - x}$ $(c \ne 0)$ and the coordinate axes is revolved about the y-axis. Find the resulting solid's volume, and then consider the volume of the inscribed circular cone that results from rotating the line segment $y = c - \dfrac{x}{c}$, $0 \le x \le c^2$, about the y-axis. Show that the ratio of these two volumes is $\frac{8}{5}$. (This result is due to Pierre de Fermat, the great 17th-century mathematician.)

20–23 Set up, but do not evaluate, an integral defining the arc length of the graph of the equation over the given interval.

20. $y = x^2 - x + 1; \quad 0 \le x \le 3$

21. $x + 2y^2 = 3 - y; \quad 1 \le y \le 5$

22. $y = \tan x; \quad -\pi/4 \le x \le \pi/4$

23. $e^x = \cos y; \quad 0 \le y \le \pi/3$

24–29 Determine the arc length *L* of the curve defined by the equation over the given interval.

24. $y = \sqrt{3}x + 1; \quad 0 \le x \le 4$

25. $y = \dfrac{4x^{3/2} + 1}{6}; \quad 0 \le x \le 3$

26. $y = \dfrac{x^4 + 3}{6x}; \quad 1 \le x \le 3$

27. $y = \dfrac{x^2}{4} - \ln\sqrt{x}; \quad 1 \le x \le e^2$

28. $y = \dfrac{\sqrt{x}}{6} - 2x^{3/2}; \quad 0 \le x \le 1$

29. $y = e^{x/2} + e^{-x/2}$; $0 \le x \le 2$

30–33 Set up, but do not evaluate, an integral defining the surface area of the solid obtained by revolving the given curve about the indicated axis.

30. $y = \sin x$; $0 \le x \le \pi$; about the x-axis

31. $y = \sqrt{\ln x}$; $1 \le x \le e$; about the x-axis

32. $y = \sqrt{\ln x}$; $1 \le x \le e$; about the y-axis

33. $y = x^2$; $0 \le x \le 2$; about $x = -1$

34–39 Find the surface area of the solid generated by revolving the given curve about the indicated axis.

34. $y = \frac{1}{3}x + 2$; $0 \le x \le 3$; about the x-axis

35. $y = \frac{x^3}{4}$; $0 \le x \le 1$; about the x-axis

36. $3y = 3x^{3/2} - \sqrt{x}$; $1 \le x \le 2$; about the x-axis

37. $x = \frac{y^3}{12} + \frac{1}{y}$; $1 \le y \le 2$; about the y-axis

38. $x = 1.5y^{5/3} - 0.3\sqrt[3]{y}$; $0 \le y \le 1$; about the y-axis

39. $12xy = 3y^4 + 4$; $1 \le y \le 2$; about $x = -\frac{1}{2}$

40–43 Find the centroid of the plane region bounded by the given curves. If possible, use symmetry to simplify your calculations.

40. $y = 5x - x^2$, $y = 0$

41. $y = 2\sqrt{x}$, $y = 2x$

42. $y = x^{3/5}$, $x = 0$, $y = 8$

43. $y = x$, $y(x+1)^2 = 4$, $x = 0$

44–47 Find the center of mass of the plane region of varying density that is bounded by the given curves.

44. $2y = 4 - x$, $x = 0$, $y = 0$; $\rho(x, y) = x^2$

45. $2y = 4 - x$, $x = 0$, $y = 0$; $\rho(x, y) = \sqrt{y}$

46. $y = x$, $y = \sqrt{x}$; $\rho(x, y) = 1 + x$

47. $x = 9 - y^2$, $y = x - 3$; $\rho(x, y) = y + 3$

48. Suppose the density of a baseball bat lying along the positive x-axis is given by the function $\rho(x) = (0.033x + 0.5)^2$ ounces per inch $(0 \le x \le 30)$. Find the center of mass of the bat.

49. Find the centroid of the upper semielliptical region bounded by the x-axis and the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (See Exercise 58 of Section 6.4.)

50. Use Pappus' Theorem for volumes to find the volume of the solid generated by revolving the region of Exercise 43 about the y-axis.

51.* Use Pappus' Theorem for volumes to find a second solution to Exercise 49 using the fact that the area of the semiellipse is $A = \pi ab/2$.

52. A 10 cm long unstressed spring requires a force of 2 N to stretch to 14 cm. How much work will be done in stretching the spring an additional 4 cm?

53. A 100 ft cable is lifting a 30 lb weight by 20 ft, so that the cable is wound on a cylindrical drum. Find the total work done if the cable weighs $2\ \text{lb/ft}$.

54. A 20 m cable is being wound up on a cylindrical drum at a rate of $25\ \text{cm/s}$. If the cable weighs $10\ \text{N/m}$, find the work done from $t = 4$ s to $t = 32$ s.

55. A leaky container is being lifted by a crane at $0.5\ \text{m/s}$ for 20 s. If the full container originally weighed 300 kg, but is leaking at a rate of $0.1\ \text{kg/s}$, find the work done by the crane. Use $g \approx 9.81\ \text{m/s}^2$.

56. The shape of an underground gasoline tank is an inverted circular cone with a top radius of 6 ft and a depth of 18 ft. Find the work done in pumping the full content of the tank to 1 ft above the top of the tank. The weight density of gasoline is $44\ \text{lb/ft}^3$.

57. Find the total work done in Exercise 56 if the tank initially is filled to only half of its capacity.

58. The shape of a water tank can be approximated by rotating the graph of $y = x^2$, $-2 \le x \le 2$, about the y-axis (units in meters). Find the work required to fill up the empty tank through an opening at its lowest point. Use $9810\ \text{N/m}^3$ for the weight density of water.

59.* A cup of soda of height 5 inches can be approximated by a frustum of a circular cone, standing on its smaller base of radius 1.25 inches, with a top opening of radius 2 inches. Find the work done in drinking a full cup of soda through a straw, supposing that the end of the straw is 2 inches above the rim of the cup. Use the weight density of water, $62.4\ \text{lb/ft}^3$, for the soda.

60. A cylindrical underground tank of radius 3 ft is positioned horizontally, and is half full of gasoline. Find the force exerted by the gasoline on one end of the tank. (See Exercise 56 for the weight density of gasoline.)

61. Use the centroid approach (Example 8 in Section 6.5) to provide a second solution to Exercise 60. (**Hint:** See Exercise 52 of Section 6.4.)

62. Use the centroid approach to provide a second solution to Exercise 56. (**Hint:** See the discussion preceding Exercises 58–64 of Section 6.5 and use the fact that the centroid of a right circular cone is on the axis of the cone, one-fourth of the way from the base toward the vertex.)

63. One end of a dam is an isosceles trapezoid, standing vertically on its shorter base of 15 meters. The top width is 30 meters, while the depth of the dam is 20 meters. Find the force exerted by the water on one end of the dam. Use $9810\ \text{N/m}^3$ for the weight density of water.

64.* The water depth in an 18 ft by 35 ft pool increases uniformly from the shallow end of 3 ft to the deep end of 8 ft. Find the fluid force exerted on the bottom of the pool. Use $62.4\ \text{lb/ft}^3$ for the weight density of water.

65. Find the energy expended in moving a 1.5-ton satellite to an altitude of 220 miles above the surface of Earth. Assume the radius of Earth is approximately 4000 miles. Express your answer in the units foot-pounds. (**Hint:** Ignore the work done in accelerating the satellite, air resistance, as well as the weight of the launching vehicle and fuel.)

66–67 Find the indicated function value. If it doesn't exist, say so.

66. **a.** $\sinh 0$ **b.** $\operatorname{csch} 0$

67. **a.** $\coth^{-1} 0$ **b.** $\operatorname{sech} 0$

68. Use the definition to show that $\tanh x > 0$ if $x > 0$ and $\tanh x < 0$ if $x < 0$.

69. Prove that if $f(x) = \tanh x$ or $f(x) = \coth x$, then $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = -1$.

70. Suppose that $e^x = f(x) + g(x)$, where $f(x)$ is even and $g(x)$ is odd. Prove that in this case, we must necessarily have $f(x) = \cosh x$ and $g(x) = \sinh x$. (**Hint:** Start by expressing e^{-x} in terms of $f(x)$ and $g(x)$.)

71. Verify the following identity.

$$\cosh x + \cosh y = 2\cosh\frac{x+y}{2}\cosh\frac{x-y}{2}$$

72–81 Find the derivative or integral as indicated.

72. $\frac{d}{dx}\left[\sinh^2\left(\frac{6x-1}{2}\right)\right]$ **73.** $\frac{d}{dx}\left(e^{2x}\tanh x\right)$

74. $\frac{d}{dx}\left[\operatorname{csch}\left(x^2+2\right)\right]$

75. $\frac{d}{dx}\left[\ln\left(\sinh(5x-2)\right)\right]$

76. $\int \frac{\tanh\sqrt{x}}{2\sqrt{x}}\,dx$ **77.** $\int \frac{\sinh x}{\cosh^2 x}\,dx$

78. $\int \coth x \operatorname{csch}^2 x\,dx$ **79.** $\int \frac{dx}{\sqrt{x^2-4x+5}}$

80. $\int_0^1 \frac{2x}{\sqrt{1+9x^4}}\,dx$ **81.** $\int \frac{e^x}{\sqrt{e^{2x}-1}}\,dx;\quad x>0$

82. Prove the following formula for the derivative of the inverse hyperbolic cosecant function.

$$\frac{d}{dx}\left(\operatorname{csch}^{-1} x\right) = \frac{-1}{|x|\sqrt{1+x^2}}$$

83–85 In the 16th century, Galileo observed the additive property of velocities. For example, if a boat travels at 5 mph in the direction of flow on a river that in turn flows at 2 mph, then the velocity of the boat relative to the shore will be $u = 5 + 2 = 7$ mph. However, according to Albert Einstein's theory of special relativity, the relative velocity between any two objects can never exceed c, the speed of light. This contradicts Galileo's observation. (On a theoretical level, just think of a "river" flowing at $\frac{3}{4}c$, with a "boat" on it traveling at, say, $\frac{1}{2}c$. The velocity relative to shore would be $u = \left(\frac{3}{4}+\frac{1}{2}\right)c = \frac{5}{4}c$!) Instead of the Galilean $u = v_1 + v_2$, Einstein discovered the following relativistic addition formula for velocities.

$$u = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

In Exercises 83–85, refer to the above formula.

83. Use the addition formula for $\tanh x$ (Exercise 33 of Section 6.6) to verify the following identity.

$$\tanh^{-1}\left(\frac{u}{c}\right) = \tanh^{-1}\left(\frac{v_1}{c}\right) + \tanh^{-1}\left(\frac{v_2}{c}\right)$$

84. Suppose a fighter plane fires a missile at 500 mph in the forward direction at a moment when the plane itself is flying at 900 mph. Use Einstein's relativistic formula to find the missile's velocity relative to Earth and compare it with Galileo's prediction of 500 + 900 = 1400 mph. Approximate the speed of light by 3×10^8 m/s.

85. In this exercise, we are going to up the numbers of Exercise 84 significantly. Suppose a rocket is traveling away from Earth at a speed of $0.7c$ and fires another rocket at $0.5c$. Use Einstein's formula to calculate the velocity of the second rocket relative to Earth.

Concept Check

86–92 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

86. The disk method is based on the idea of integrating slices.

87. When both the disk method and the shell method are applied to calculate the volume of a solid of revolution, the variable of integration is always the same.

88. If the area of the region bounded by $y=f(x)$ and $y=g(x)$ is A, then the volume of the solid obtained by revolving the same region about the x-axis is $V=\pi A^2$.

89. If the area of the region bounded by $y=f(x)$ and $y=g(x)$ is A, then the volume of the solid obtained by revolving the same region about the x-axis never equals πA^2.

90. $\lim\limits_{x\to-\infty} \tanh x = -1$

91. $\cosh 2x = 2\sinh^2 x + 1$

92. The work needed to pump fluid out of a tank through an opening on its top equals the total weight of the fluid multiplied by the distance traveled by its center of mass.

Chapter 6 Technology Exercises

93–96 Use a graphing utility to find (or approximate) the volume of the solid generated by rotating the region bounded by the graphs of the given equations about the indicated axis.

93. $y=\sin\left(x^2\right),\quad y=0,\quad x=0,\quad x=\sqrt{\pi};$ about the x-axis

94. $y=\arccos x,\quad y=0,\quad x=0,\quad x=1;$ about the x-axis

95. $y=\sinh^{-1} x,\quad y=0,\quad x=4;$ about the y-axis

96. $y=x^2\sin^2 x,\quad y=0,\quad x=0,\quad x=\pi;$ about the y-axis

97–98 Use a graphing utility to find the arc length of the graph of the equation over the given interval.

97. $y=\dfrac{1}{x^2+1};\quad -1\le x\le 1$

98. $y=\sin x;\quad 0\le x\le \pi$

99–100 Use a graphing utility to find the surface area of the solid generated by revolving the given curve about the indicated axis.

99. $y=\sin x;\quad 0\le x\le\pi;\quad$ about the x-axis

100. $y=\sqrt{\ln x};\quad 1\le x\le e;\quad$ about the y-axis

Chapter 7
Review Exercises

1–8 Use integration by parts to evaluate the given indefinite or definite integral.

1. $\int \sqrt{x}\ln x\,dx$
2. $\int x\sec^2 x\,dx$
3. $\int \frac{x}{\sqrt{x-1}}\,dx$
4. $\int \operatorname{arccot} x\,dx$
5. $\int t^2\cos t\,dt$
6. $\int 3t^2 e^{t+1}\,dt$
7. $\int_0^{\pi} x^3\sin x\,dx$
8. $\int_0^{\pi/2} e^{2t}\cos t\,dt$

9–10 Combine the method of integration by parts with substitution to evaluate the integral.

9. $\int \sin t\cos t\ln(\sin t)\,dt$
10. $\int \frac{\arctan\sqrt{t}}{\sqrt{t}}\,dt$

11. Use integration by parts to find the area of the region bounded by the graph of $y=\cos(\ln x)$ and the x-axis over the interval $\left[e^{-\pi/2}, e^{\pi/2}\right]$.

12. Consider the region bounded by the graph of $y = xe^x$ and the x-axis over the interval $[0,1]$.
    - **a.** Find the centroid of the region.
    - **b.** Use the shell method to find the volume of the solid generated by revolving the region about the y-axis.

13. Repeat Exercise 12 for the region bounded by the graph of $y = 3x^2\ln x$ and the x-axis over the interval $[1,e]$.

14. Use the disk method to find the volume of the solid generated by revolving the region bounded by the graph of $y=\cos x$, $-\pi/2\le x\le \pi/2$, about the x-axis.

15. Use integration by parts to prove the following formula for $a>0$, $a\ne 1$.

$$\int xa^x\,dx = a^x\left[\frac{x}{\ln a}-\frac{1}{(\ln a)^2}\right]+C$$

16–21 Use the partial fractions method to evaluate the given integral.

16. $\int \frac{4}{x^2-4}\,dx$
17. $\int \frac{x-7}{(x+1)(x-3)}\,dx$
18. $\int \frac{x+3}{x(x+1)(x+2)}\,dx$
19. $\int \frac{3x-4}{x^3(x-1)}\,dx$
20. $\int \frac{4}{2x^3+2x^2+x}\,dx$
21. $\int \frac{x^5+6x^3+7x}{\left(x^2+2\right)^3}\,dx$

22–23 Use the Heaviside cover-up method to evaluate the given integral.

22. $\int \frac{4x^2+4x-2}{x\left(x^2+3x+2\right)}\,dx$
23. $\int \frac{3x^2+16x+29}{(x+5)(x+3)(x-1)}\,dx$

24–25 Find the definite integral of the rational function over the given interval.

24. $\int_{-1}^{1} \frac{x^3+x^2-x-2}{x^2-4}\,dx$
25. $\int_0^{1} \frac{2x^4+x^3-2x^2-3}{(x+1)\left(x^2+1\right)}\,dx$

26–27 Combine integration by substitution and the partial fractions method to evaluate the given integral.

26. $\int \frac{\sec^2 x}{\tan^2 x-\tan x}\,dx$
27. $\int \frac{dx}{\sqrt{x}\left(\sqrt{x}-2\right)\left(\sqrt{x}+3\right)}$

28. Use the disk method to find the volume of the solid generated by revolving the graph of $f(x)=2/\sqrt{x^2+8x+12}$, $0\le x\le 2$, about the x-axis.

29. Use the shell method to find the volume of the solid obtained by revolving about the y-axis the region bounded by the graph of $g(x)=4/\left(x^2+6x+5\right)$, the coordinate axes, and the line $x=5$.

30. If a and b are constants such that $a\ne b$, use the partial fractions method to prove the following formula.

$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b}\ln\left|\frac{x-a}{x-b}\right|+C$$

31–40 Evaluate the given indefinite or definite integral involving powers of sines and cosines.

31. $\int_0^{\pi} 4\sin^3 x\,dx$
32. $\int_0^{\pi/4} \frac{\sin x}{\sqrt{\cos x}}\,dx$
33. $\int \cos^2 x\sin^3 x\,dx$
34. $\int x^2\cos^2 x\,dx$
35. $\int_0^{\pi/4} \sin x\cos^{-5} x\,dx$
36. $\int_0^{\pi} \sin^9 x\,dx$

37. $\int \frac{1}{\sin^2 x \cos^2 x}\,dx$ **38.** $\int (\cos x - \sin x)^2\,dx$

39. $\int \sin 3x \sin x\,dx$ **40.** $\int \cos 2x \sin 5x\,dx$

41–48 Evaluate the given indefinite or definite integral involving powers of tangents and secants (or their cofunctions).

41. $\int_0^{\pi/4} \tan^3 x \sec^4 x\,dx$ **42.** $\int_0^{\pi/4} \tan^2 t \sec^3 t\,dt$

43. $\int \cot^3 x\,dx$ **44.** $\int \csc^4 x\,dx$

45. $\int \tan^4 x \sec^2 x\,dx$ **46.** $\int \frac{\csc^4 x}{\sqrt{\cot x}}\,dx$

47. $\int \frac{\tan^2 v}{\sec^3 v}\,dv$ **48.** $\int \frac{\sec^2 \theta}{\sqrt{\tan \theta}}\,d\theta$

49. Find the area of the region between the x-axis and the graph of $f(x) = 1/(\cos x + 1)$ from $x = -\pi/2$ to $x = \pi/2$.

50. Find the area of the region bounded by the graphs of $y = 16 \sin^2 x$ and $y = \csc^2 x$. (Restrict both functions to the interval $(0, \pi)$).

51. Find the volume of the solid obtained by revolving about the x-axis the region bounded by the graph of $y = \csc x + \sin x$ and the x-axis over the interval $(\pi/4, 3\pi/4)$.

52. Find the volume of the solid obtained by revolving about the x-axis the region bounded by the graph of $y = (\tan x + 1)/\sec x$ and the x-axis over the interval $(0, \pi/4)$.

53. For positive integers m and n that are not equal, prove the following identities.

a. $\int_0^{2\pi} \sin(mx)\sin(nx)\,dx = 0$

b. $\int_0^{2\pi} \sin(mx)\cos(nx)\,dx = 0$

c. $\int_0^{2\pi} \cos(mx)\cos(nx)\,dx = 0$

54. Use substitution to establish the following integration formula.

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

(**Hint:** Substitute $u = \sec x + \tan x$. Compare with Example 5d of Section 7.3.)

55–60 Use trigonometric substitution to evaluate the given indefinite or definite integral.

55. $\int \frac{2x^2}{\sqrt{4 - x^2}}\,dx$ **56.** $\int \frac{\sqrt{1 - x^2}}{x^2}\,dx$

57. $\int \frac{2}{(t^2 + 4)^{3/2}}\,dt$ **58.** $\int \frac{\sqrt{t^2 - 4}}{t^2}\,dt$

59. $\int_0^3 \frac{2x^2}{\sqrt{x^2 + 9}}\,dx$ **60.** $\int_1^2 \frac{dx}{\sqrt{16x^2 - 9}}$

61–64 Use the methods of this chapter to evaluate the given integral.

61. $\int \sqrt{20 + 16x - 4x^2}\,dx$ **62.** $\int \frac{\cos x}{4 \sin^2 x + 9}\,dx$

63. $\int \frac{e^x}{\sqrt{3 + 2e^x - e^{2x}}}\,dx$ **64.** $\int 4x \sinh^{-1} x\,dx$

65. Find the area of the region enclosed by the graphs of $y = \sqrt{x^2 + 1}$ and $y = 2$.

66. Rotate the region bounded by the graph of $y = \sqrt{x^2 - 4}/x^2$ and the x-axis, $2 \le x \le 4$, about the y-axis. Use the shell method to find the volume of the resulting solid.

67. Use the method of disks to determine the volume of the solid obtained by revolving the graph of $y = x^2/\sqrt[4]{4 - x^2}$, $0 \le x \le 1$, about the x-axis.

68. Find the surface area of the solid that results from rotating the graph of $y = x^2$ about the x-axis over the interval $[0, 2]$. (Round your answer to four decimal places.)

69. Find the surface area of the solid generated by rotating the graph of $y = \cos x$, $-\pi/2 \le x \le \pi/2$, about the x-axis.

70.* Find the length of the graph of $y = e^{-x}$, $0 \le x \le 2$. (Round your answer to four decimal places.)

71.* Find the surface area of the solid generated by rotating the graph of Exercise 70 about the x-axis. (Round your answer to four decimal places.)

72–73 Use substitution and a table of integrals to evaluate the given integral.

72. $\int \frac{e^x}{1 + \cos e^x}\,dx$ **73.** $\int \frac{\sin 2x}{2\sqrt{2 \cos x - \cos^2 x}}\,dx$

74–77 Use the Trapezoidal Rule with $n = 6$ to approximate the integral, and compare the result to the exact value of the integral by determining the absolute value of the error E_T.

74. $\int_1^4 x^{3/2}\,dx$ **75.** $\int_1^2 \frac{1}{x^2}\,dx$

76. $\int_0^4 \sqrt{x^2+1}\,dx$ **77.** $\int_0^{\pi/3} \tan x\,dx$

78–81. Use the error estimate for the Trapezoidal Rule to estimate $|E_T|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 74–77.

82–85. Use Simpson's Rule to approximate the integrals from Exercises 74–77 with $n = 6$. Determine the absolute value of the error E_S.

86–89. Use the error estimate for Simpson's Rule to estimate $|E_S|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 82–85.

90. Prove that if $f(x) = ax + b$ is a linear function on a closed interval $[a,b]$, then for any n, $T_n = \int_a^b f(x)\,dx$.

91–96 Identify the type of the improper integral and determine whether it is convergent or divergent. If it is convergent, find its value.

91. $\int_2^4 \frac{dx}{(x-2)^4}$ **92.** $\int_{-\infty}^0 x^2 e^x\,dx$

93. $\int_2^6 \frac{dx}{\sqrt{x-2}}$ **94.** $\int_0^\infty \frac{2}{(x+3)^{2/3}}\,dx$

95. $\int_{-\infty}^\infty \frac{2e^x}{e^{2x}+4}\,dx$ **96.** $\int_e^\infty \frac{dx}{x\ln x}$

97–98 Use the Direct Comparison Test to determine whether the integral converges.

97. $\int_0^\infty \frac{dx}{\sqrt{x^3+1}}$ **98.** $\int_1^\infty \frac{\ln x}{\sqrt{x}}\,dx$

99. Use substitution to turn the improper integral $\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ into a proper one and evaluate it.

100. Rotate about the x-axis the region bounded by the graph of $y = \sqrt{\ln x}/x^2$ and the x-axis over the interval $[1,\infty)$. Use the disk method to determine if the resulting unbounded solid has finite volume. If so, find the volume.

101.* Rotate about the x-axis the region bounded by the x-axis and the graph of $y = e^{-x}$ over the infinite interval $[0,\infty)$. Determine if the resulting infinite solid has finite volume or surface area. If so, find their values.

102. Prove that the improper integral $\int_2^\infty \frac{dx}{x(\ln x)^a}$ converges if and only if $a > 1$.

103–106 Find the Laplace transform. (See Exercises 88–93 in Section 7.7.)

103. $L\{te^{at}\}$ **104.*** $L\{t^2e^{at}\}$

105.* $L\{t\sin kt\}$ **106.*** $L\{t\cos kt\}$

Concept Check

107–113 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

107. If $\lim_{x\to\infty} f(x) = L$ and $L \neq 0$, then $\int_0^\infty f(x)\,dx$ diverges.

108. If $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

109. Let $f(x)$ be defined on $[a,\infty)$ and S be the solid generated by rotating the graph of $f(x)$ about the x-axis. If the surface area of S is infinite, then the volume of S is also infinite.

110. If $f(x)$ is an odd function, then $\int_{-\infty}^\infty f(x)\,dx = 0$.

111. If $f(x)$ has a vertical asymptote at $x = 0$ and $a > 0$, then $\int_{-a}^a f(x)\,dx$ diverges.

112. Any rational function is integrable on any finite interval that doesn't include a zero of the denominator.

113. If an integrand contains the expression $\sqrt{a^2 \pm x^2}$, a trigonometric substitution must be used to evaluate the integral.

Chapter 7 Technology Exercises

114. Use a graphing utility to find the integral from Exercise 33. If the answer appears different from what you obtained by hand, prove that the answers are equivalent.

115. Write a program for a computer algebra system or programmable calculator that evaluates the trapezoidal approximation T_n for a given input function on a specified interval and positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal.

116. Use the program you wrote for Exercise 115 for the integral from Exercise 75.

117. Write a program for a computer algebra system or programmable calculator that evaluates the Simpson approximation S_n for a given integral and an even positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal. Compare this with your answer for Exercise 115. What n ensures that the answer is correct to at least five decimal places?

118. Use the program you wrote for Exercise 117 for the integral from Exercise 75.

119–120 Use the programs you wrote for Exercises 115 and 117 to approximate the given nonelementary integral with $n = 50$. Which method (the Trapezoidal Rule or Simpson's Rule) do you expect to be more accurate? Use the built-in numerical integration command of your technology to verify your conjecture.

119. $\int_0^1 \sqrt{1+x^4}\, dx$

120. $\int_0^1 e^{e^x}\, dx$

Chapter 8
Review Exercises

1–4 Determine whether the differential equation is separable, linear, or autonomous; and find its order. (Note that more than one description may be applicable.)

1. $\dfrac{y'}{y+1} = y^3 - 2$

2. $xy\,dy = \sqrt{1-x^2}\,dx$

3. $xy' = 2y + x^4e^x - y'$

4. $2y'' = y - 3y'$

5–6 Solve the differential equation with the given initial condition.

5. $(x^2+1)y' = x;\quad y(0) = 1$

6. $y'\sec x = \sin x;\quad y(0) = 0$

7–12 Solve the separable differential equation.

7. $xy' = 2y$

8. $y' = e^{-y}\cos x$

9. $x^2y' = 2\sqrt{y}(x+1)$

10. $x^2\,du = u^2\,dx$

11. $x\,du = (u^2-1)\,dx$

12. $\dfrac{y'}{3} = e^{3x-y}$

13–16 Solve the given initial value problem.

13. $y' = \dfrac{9x^2+2x}{3y^2};\quad y(0) = 2$

14. $x' = x\sin t;\quad x(\pi/2) = 5$

15. $\dfrac{dy}{dt} = 4t\sqrt{y-2};\quad y(1) = 6$

16. $\dfrac{dy}{dx} = 2xy + y - 4x - 2;\quad y(0) = 0$

17–22 Solve the linear differential equation. (**Hint:** In some cases, *x* has to be the dependent variable in order for the equation to be linear.)

17. $y' - \dfrac{y}{x} = 0$

18. $xy' + y = x^2\sin x$

19. $xy' + 6y = 2$

20. $y' - 4xy = 2x$

21. $(y^2+1)\,dx + 2xy\,dy = 4y\,dy$

22. $dx + x\cot y\,dy = \cos y\,dy$

23–24 Find a first-order linear differential equation in standard form that has the given general solution. (**Hint:** Identify the integrating factor and "reverse" the solution technique discussed in this chapter.)

23. $y = Ce^{x^3} - \dfrac{1}{3}$

24. $y = \dfrac{C}{\sqrt{x^2-1}} - 1$

25–28 Solve the given initial value problem.

25. $xy' + (x-1)y = x^3;\quad y(1) = 1$

26. $\dfrac{dy}{dx} - 2xy = 2xe^{x^2};\quad y(0) = 2$

27. $(x^2+1)y' + xy = \dfrac{1}{x^2+1};\quad y(0) = -3$

28. $(\cos t)\dfrac{dy}{dt} + (\sin t)y = 1;\quad y(0) = 1$

29–32 Match the differential equation with its slope field (labeled A–D). Classify each equilibrium solution as stable or unstable.

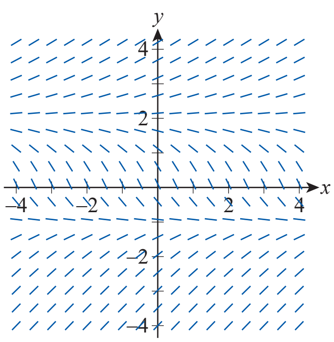
29. $y' = y^2 + 2y$

30. $y' = 9y - y^3$

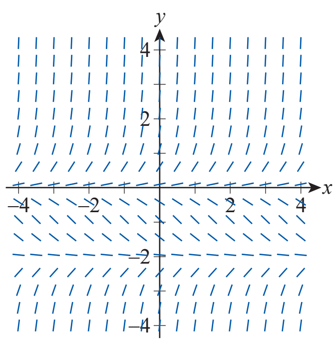
31. $y' = \dfrac{y^2 - y - 2}{y^2+1}$

32. $y' = y^4 - 8y$

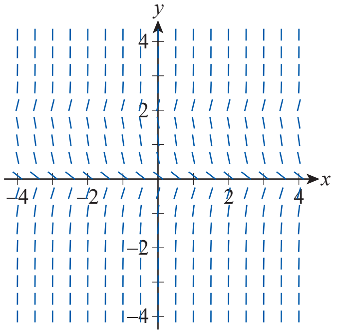
A.



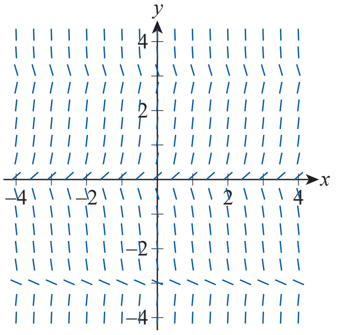
B.



C.



D.

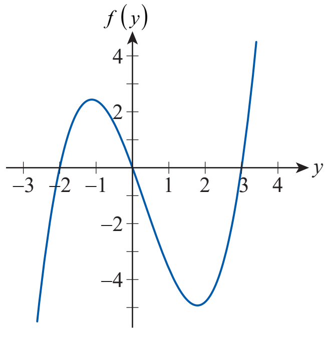


33–34 Graph by hand the slope field of the given differential equation. If applicable, find and classify each equilibrium solution as stable or unstable.

33. $y' = y^2 - 9$

34. $y' = y^3 + y^2 - 2y$

35. Create a rough sketch of the slope field of the differential equation $y' = f(y)$, where the graph of f is given below. Classify equilibria as stable or unstable.



36–37 For the initial value problem, **a.** use Euler's method with the indicated step sizes to approximate the given value of y and **b.** solve the IVP by conventional methods and compare your approximations with the exact value.

36. $y' = 2y + 1;\quad y(0) = 1;$
approximate $y(1)$ with (i) $h = 0.2$ (ii) $h = 0.1$

37. $y' = y - x;\quad y(0) = 2;$
approximate $y(2)$ with (i) $h = 0.5$ (ii) $h = 0.25$

38–43 Find the general solution of the second-order homogeneous linear equation.

38. $2y'' + 3y' - 2y = 0$

39. $y'' - 16y = 0$

40. $y'' - 2y' + 2y = 0$

41. $y'' - 5y' = 0$

42. $y'' + 9y = 0$

43. $4y'' - 4y' + y = 0$

44–47 Solve the given second-order initial value problem.

44. $9y'' - 6y' + y = 0;\quad y(0) = 3;\quad y'(0) = 2$

45. $y'' - y' - 6y = 0;\quad y(0) = 3;\quad y'(0) = -1$

46. $y'' + 2y' + 3y = 0;\quad y(0) = 0;\quad y'(0) = 2$

47. $2y'' - 3y' = 0;\quad y(2) = -5;\quad y'(2) = 3$

48–49 Solve the boundary value problem, if possible.

48. $2y'' + y = 0;\quad y(0) = 2;\quad y\left(\pi/\sqrt{2}\right) = -4$

49. $y'' + 4y = 0;\quad y(0) = 0;\quad y(\pi) = 1$

50. Find the general solution of the third-order equation $y''' - 3y'' - y' + 3y = 0$. (**Hint:** See Exercises 35–38 in Section 8.4.)

51. Find the general solution of the nonhomogeneous equation $2y'' - 7y' + 3y = \cos x$. As your initial guess, use $y_p = A\cos x + B\sin x$. (**Hint:** See Exercises 39–42 in Section 8.4.)

52–53 Determine the orthogonal trajectories of the family of curves, where a is an arbitrary nonzero constant.

52. $y = ax^4$

53. $y = \dfrac{ax}{\sqrt{x^2 + 1}}$

54. A 500-liter tank is filled with water holding 5 kilograms of salt in the solution. Through an inlet, a stronger solution with salt concentration of 0.05 kilograms per liter is being added at a rate of 16 liters per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the same rate. What is the amount of salt in the tank after 25 minutes?

55. Answer the question of Exercise 54 if the contents of the tank are drained out at a rate of 20 liters per minute.

56.* A container in a lab contains 14 gallons of pure distilled water. 10% and 25% acid solutions are pumped into the container through two respective inlets. The 10% solution is flowing in at a rate of 0.2 gallons per minute, while the 25% solution is being allowed in by the second inlet at a rate of 0.5 gallons per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the rate of 0.7 gallons per minute. How long does it take to form 14 gallons of 14% solution in this way?

57. A hailstone is melting so that its volume $V(t)$ decreases at a rate proportional to its surface area.

a. Assuming that the hailstone is nearly spherical, find a differential equation satisfied by $V(t)$.

b. If a hailstone of diameter 1 inch loses 20% of its volume in half an hour, predict how long it takes for it to completely melt away. (Consider it melted away when your model predicts less than 1 percent remaining).

58. If a vertical cylindrical tank of radius $\frac{1}{2}$ meters and height 4 meters is initially full of water but is draining through a circular orifice of diameter 2 centimeters that is on the bottom of the tank, what is the water level in the tank 2 minutes later? (**Hint:** See Exercise 59 in Section 8.1.)

59. Find the charge $q(t)$ of the 10^2-farad capacitor in an RC circuit if the impressed voltage on the circuit is $V(t)=t$ and the resistance is 25 ohms. Assume $q(0)=0.$ (**Hint:** See Exercise 62 in Section 8.1.)

60. Suppose that the impressed voltage in a simple RL circuit is $V(t)=2t,\ \ I(0)=0,$ the inductance is 0.1 henries, and the resistance 0.5 ohms. Find the electric current I at time $t=4$ seconds. (**Hint:** See Example 5 in Section 8.2.)

61. A baking dish is removed from a 210 °C oven and left at 20 °C room temperature. Two and a half minutes later the dish's temperature is 155 °C. Find the bakeware's temperature 10 minutes after it was removed from the oven. (**Hint:** See Example 2 in Section 8.3.)

62. A snapping turtle population grows logistically with a carrying capacity of 200 turtles and constant of proportionality $k=0.2$ per year.

a. Find the population size $P(t)$ as a function of time if initially 50 turtles are present in the habitat. (**Hint:** See Example 3 in Section 8.3.)

b. How long does it take for the population to reach 100 turtles?

63. Suppose that an object of mass 200 grams stretches a spring by 10 centimeters. If it is pulled upward to a position of 5 centimeters above equilibrium and released with a downward velocity of $1\ \text{m/s},$ find and graph the resulting displacement function, assuming that the surrounding medium offers resistance with a damping constant of $c=0.5\ \text{kg/s}.$ (**Hint:** See Example 5 in Section 8.4.)

Concept Check

64–70 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

64. The equation $(y')^2+xy'-3y=0$ is a first-order differential equation.

65. The equation $y'=-y$ is not linear.

66. If $y_1(x)$ and $y_2(x)$ are solutions of a homogeneous linear differential equation, then so is $3y_1(x)-2y_2(x).$

67. Only autonomous equations have slope fields.

68. The logistic equation discussed in this text is autonomous.

69. The equations $y=2e^{x/2}$ and $y=xe^{x/2}$ are linearly independent solutions of $4y''-4y'+y=0.$

70. A second-order BVP with two boundary conditions always has a solution.

Chapter 8 Technology Exercises

71–72. Use a graphing utility to display the slope fields of the differential equations in Exercises 33 and 34. Compare the graphs to your original sketches.

73. Write a program for a computer algebra system that accepts a spring constant, a damping constant, and the mass of an oscillating object as inputs, and graphs the displacement function as output. Use it to check your answer for Exercise 63.

Chapter 9
Review Exercises

1–4 Sketch the curve defined by the parametric equations by eliminating the parameter.

1. $x = \dfrac{1}{36t}, \quad y = t^2$

2. $x = t + 5, \quad y = |t - 2|$

3. $x = \dfrac{3}{4t-2}, \quad y = 2t - 2$

4. $x = 4\sin\theta, \quad y = \cos\theta + 1$

5–6 Construct parametric equations defining the graph of the given equation.

5. $y^2 = x^2 + 4$

6. $6x = 2 - y$

7–8 Find parametric equations to represent the graph described. (Answers will vary.)

7. Line, passing through $(14,4)$ and $(-3,-8)$

8. Circle, center $(1,1)$, radius 1

9–10 Find the equations of any horizontal or vertical tangent lines to the given curve.

9. $x = 2t + 1, \quad y = t^2 - 4$

10. $x = -2t^2, \quad y = \dfrac{3}{t-2}, \quad t < 2$

11–14 Find the values of dy/dx and d^2y/dx^2 for the given curve at the indicated point.

11. $x = 2t^2 + 1, \quad y = \sqrt{t-1}; \quad (9,1)$

12. $x = e^t, \quad y = t^2 e^{-t}; \quad (1,0)$

13. $x = \dfrac{1}{t}, \quad y = t^2 + t; \quad (1,2)$

14. $x = \sin t, \quad y = \cos 2t; \quad \left(\dfrac{1}{2}, \dfrac{1}{2}\right)$

15–16 Find the value(s) of the parameter for any inflection point(s) of the given curve.

15. $x = t - t^3, \quad y = 3t + 1$

16. $x = 5t - 1, \quad y = t^3\left(2 - t^2\right)$

17–18 Find the area enclosed by the given curve.

17. $x = \sin 3t, \quad y = \cos 2t, \quad -\dfrac{\pi}{3} \le t \le \dfrac{\pi}{3}$

18. $x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \le t \le 2\pi$

19–20 Find the arc length of the given curve on the indicated interval.

19. $x = \dfrac{4}{3}t^{3/2}, \quad y = 4t, \quad 0 \le t \le 5$

20. $x = t\sin t + \cos t, \quad y = t\cos t - \sin t, \quad 0 \le t \le \pi$

21–23 Find the area of the surface generated by revolving the parametric curve about the indicated axis.

21. $x = \dfrac{t^3}{2}, \quad y = 2t + 1, \quad 0 \le t \le 1;$ about the y-axis

22. $x = \dfrac{1-t}{2}, \quad y = \sqrt{t}, \quad 0 \le t \le 1;$ about the x-axis

23. $x = 3t - \dfrac{t^3}{3}, \quad y = \sqrt{3}\left(9 - t^2\right), \quad 0 \le t \le 3;$ about the y-axis

24–25 Convert the point from polar to Cartesian coordinates.

24. $\left(-3.45, \dfrac{\pi}{3}\right)$

25. $\left(7, \dfrac{7\pi}{6}\right)$

26–27 Convert the point from Cartesian to polar coordinates.

26. $\left(-\sqrt{3}, -1\right)$

27. $(10,12)$

28–29 Rewrite the rectangular equation in polar form.

28. $x^2 + y^2 = 16a^2$

29. $x^2 + y^2 = 9ax$

30–31 Rewrite the polar equation in rectangular form.

30. $r = 4\cos\theta$

31. $r = \dfrac{16}{4\cos\theta + 4\sin\theta}$

32–33 Sketch a graph of the given polar equation.

32. $r = 4\sin 3\theta$

33. $r^2 = 25\cos 2\theta$

34–35 Find the slope of the line tangent to the given polar curve at the indicated point.

34. $r = 4\cos 3\theta; \quad \theta = \frac{\pi}{12}$

35. $r = \frac{1}{\theta^2+1}; \quad \theta = \frac{\pi}{2}$

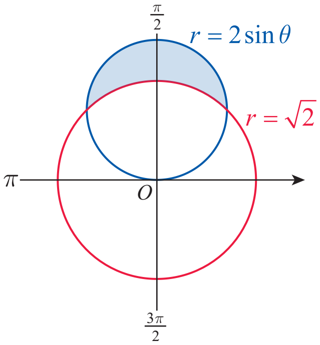
36–37 Find all points where the given polar curve has a horizontal or vertical tangent line.

36. $r = 2\sin\theta, \quad 0 \le \theta \le 2\pi$

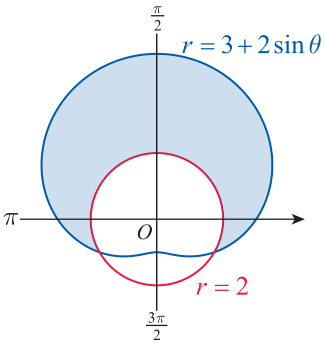
37. $r = 1 + \cos 2\theta, \quad 0 \le \theta \le \pi$

38–39 Find the area of the shaded region.

38.



39.



40–41 Find the area of the specified region.

40. A large loop of $r = 1 + 2\cos 2\theta$

41. The portion of the rose $r = 4\cos 2\theta$ outside the circle $r = 2$

42–45 Use polar coordinates to find the arc length of the curve.

42. The circle $r = 2\cos\theta$

43. The line segment $r = \sec\theta, -\pi/4 \le \theta \le \pi/4$

44. The spiral $r = 2\theta, 0 \le \theta \le 2\pi$

45. The cardioid $r = 2 + 2\cos\theta$

46. Find the area of the surface generated by revolving the curve $r = 2\cos\theta, 0 \le \theta \le \pi$ about $\theta = \pi/2$. (**Hint:** See Exercise 58 in Section 9.4)

47–52 Sketch the graph of the given conic section, and determine the coordinates of the foci and the equations of the directrix or asymptotes as appropriate.

47. $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{16} = 1$

48. $x^2 + 9y^2 - 6x + 18y = -9$

49. $(y+1)^2 = -12(x+3)$

50. $x^2 - 8x + 2y + 14 = 0$

51. $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

52. $9x^2 - 4y^2 + 54x - 8y + 41 = 0$

53–63 Find the equation, in standard form, of the conic section with the given properties.

53. Ellipse, center at $(-1,4)$, major axis is vertical and of length 8, foci $\sqrt{7}$ units from the center

54. Ellipse, foci at $(1,2)$ and $(7,2)$, $e = \frac{1}{2}$

55. Ellipse, vertices at $\left(\frac{7}{2},-1\right)$ and $\left(\frac{1}{2},-1\right)$, $e = 0$

56. Ellipse, vertices at $(1,-8)$ and $(1,2)$, minor axis of length 6

57. Parabola, vertex at $(-2,3)$, directrix is the line $y = 2$

58. Parabola, vertex at $(5,-3)$, focus at $(5,1)$

59. Parabola, focus at $(3,-1)$, directrix is the line $x = 2$

60. Hyperbola, vertices at $(4,-1)$ and $(-2,-1)$, foci at $(5,-1)$ and $(-3,-1)$

61. Hyperbola, asymptotes of $y = \pm\frac{5}{2}(x+1) - 2$, vertices at $(-3,-2)$ and $(1,-2)$

62. Hyperbola, foci at $(-1,-2)$ and $(-1,8)$, asymptotes of $y = \pm\left(\frac{3}{4}x + \frac{3}{4}\right) + 3$

63. Hyperbola, asymptotes of $y = 3x - 4$, $y = 8 - 3x$, vertices at $(2,-1)$ and $(2,5)$

64–70 Identify the given conic section and find the equation for its directrix.

64. $r = \dfrac{8}{1+2\sin\theta}$

65. $r = \dfrac{5}{4-8\sin\theta}$

66. $r = \dfrac{3}{7+6\sin\theta}$

67. $r = \dfrac{6}{9-9\cos\theta}$

68. $r = \dfrac{7}{4+4\sin\theta}$

69. $r = \dfrac{4}{6-3\cos\theta}$

70. $r = \dfrac{7}{5+2\cos\theta}$

71–76 Construct a polar equation for the conic section with the focus at the origin and the given eccentricity and directrix.

71. Eccentricity: $e = 1$; directrix: $x = -3$

72. Eccentricity: $e = 4$; directrix: $y = 3$

73. Eccentricity: $e = \frac{1}{5}$; directrix: $y = -15$

74. Eccentricity: $e = \frac{1}{4}$; directrix: $x = 16$

75. Eccentricity: $e = 1$; directrix: $y = -7$

76. Eccentricity: $e = 9$; directrix: $x = \frac{1}{3}$

77. A motorcycle headlight is made by placing a strong light bulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 5y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

78.* Prove that the graph of $r = \cos\theta + \sin\theta$ is a circle.

79. Show that the two limaçons $r = 1 + \sin\theta$ and $r = 1 - \sin\theta$ are orthogonal, that is, they intersect at right angles. (Ignore their intersection at the pole.)

Concept Check

80–90 Determine whether each of the following statements is true or false. In case of a false statement, explain or provide a counterexample.

80. Two different sets of parametric equations may describe the same curve.

81. If $x = f(t)$ and $y = g(t)$ both pass through $(0,0)$, the parametric curve $x = f(t)$, $y = g(t)$ also passes through the origin.

82. The line $y = 2x$ can be parametrized as $x = t^2$, $y = 2t^2$.

83. If a parametric curve C is differentiable on (a,b) then its graph has a tangent line at any of its points.

84. Let C be the curve $x = t^5$, $y = t^4$. Since $\left.\dfrac{dy}{dt}\right|_{t=0} = 0$, the graph of C has a horizontal tangent at the origin.

85. The polar coordinates (r,θ) and $(-r,\theta-\pi)$ describe the same point.

86. If the polar coordinates (r_1,θ_1) and (r_2,θ_2) describe the same point, then $r_1 = \pm r_2$.

87. If $y = f(x)$ is an even function, then the graph of $r = f(\theta)$ is symmetric with respect to the polar axis.

88. It is impossible to find a polar equation for a straight line.

89. The distance between any point on a parabola from the focus is at least p.

90. If the asymptotes of a hyperbola are perpendicular and the hyperbola is centered at the origin with horizontally aligned foci, then the equation of the hyperbola can be written as $x^2 - y^2 = a^2$.

Chapter 9
Technology Exercises

91. Find the equation of the graph of $r = 1 - 2\cos\theta$ after a clockwise rotation by $\pi/4$ radians. Name the resulting curve and use a graphing utility to sketch it. (See Exercise 73 in Section 9.3.)

92–93 Use a graphing utility to sketch the given curve for various values of the parameter(s) and explore the effects on the shape of your graph.

92. $r = \theta\cos k\theta$

93. $x = \pm a\cos^{2/n} t, \quad y = \pm b\sin^{2/n} t, \quad a, b, n > 0$
(Lamé curves)

94–95 Use a graphing utility to sketch the curve and then find all horizontal and vertical tangent lines. Confirm your results by paper and pencil calculations.

94. $x = t^3 - t, \quad y = t^2 + 1, \quad -2 \le t \le 2$

95. $r = 2\sin 2\theta, \quad 0 \le \theta \le \pi/2$

96–97 Use a graphing utility to sketch the region enclosed by the given curve and find its area.

96. $x = t\sin t, \quad y = \sin 2t, \quad 0 \le t \le \pi$

97. Inner loop of $r = 2 - 3\cos\theta$

98–99 Use a graphing utility to approximate the arc length of the curve with the given parametrization.

98. $x = \sin 2t, \quad y = \sin t, \quad 0 \le t \le \pi$

99. $r = \cos(2\sin\theta), \quad 0 \le \theta \le 2\pi$

100–101 Use a graphing utility to approximate the surface area of the solid obtained by rotating the given curve about the indicated axis.

100. $(t^3 - 3t, t^2 - 2), \quad 0 \le t \le \sqrt{3};$ about the y-axis

101. $r = 3\sin 2\theta, \quad \pi/4 \le \theta \le \pi/2;$ about the polar axis

Chapter 10
Review Exercises

1–2 Starting with $n = 0$, list the first five terms of the given sequence.

1. $a_n = \left(\dfrac{1-n}{n^2+1}\right)^n$ **2.** $a_n = \dfrac{2^{n+1}-n}{n!}$

3–4 Find the first five terms of the given recursively defined sequence.

3. $a_1 = 0, \quad a_n = 3a_{n-1} + 2$

4. $a_1 = 0, \quad a_2 = 1, \quad a_n = 2a_{n-2} - a_{n-1}$

5–6 Use the definition of the limit of a sequence to prove the limit statement.

5. $\lim\limits_{n\to\infty} \dfrac{2}{n^3+1} = 0$ **6.** $\lim\limits_{n\to\infty} \dfrac{2n+1}{n} = 2$

7–10 Find the limit of the sequence if it converges or prove that the sequence diverges.

7. $a_n = \dfrac{n^2-2}{3n+1}$ **8.** $a_n = \sqrt{n^2+1} - n$

9. $a_n = \left(1+\dfrac{2}{n}\right)^{3n}$ **10.** $a_n = 2\tan^{-1}\left(n^2\right)$

11–12 Use the Squeeze Theorem to prove that the given sequence converges.

11. $\left\{\dfrac{(-1)^n \cos n}{n^2}\right\}_{n=1}^{\infty}$ **12.** $\left\{\dfrac{\sin 2n}{2^n}\right\}_{n=1}^{\infty}$

13–14 Recognize the repeating decimal as a geometric series and write the decimal as a ratio of two integers.

13. $0.\overline{219}$ **14.** $0.2\overline{19}$

15. Use the Bounded Monotonic Sequence Theorem to prove that the recursive sequence $a_1 = \sqrt{3}, a_{n+1} = \sqrt{3a_n}$ converges, and find its limit.

16. Suppose that $\lim\limits_{n\to\infty} a_n = -\infty$ and $\{b_n\}$ is a sequence such that there is an N with $b_n \le a_n$ for all $n \ge N$. Prove that $\lim\limits_{n\to\infty} b_n = -\infty$.

17. Prove that if $\lim\limits_{n\to\infty} a_n = \infty$, then the sequence has a smallest term.

18. Let $\{a_n\}$ be a positive null sequence. Prove that $\lim\limits_{n\to\infty}(1/a_n) = \infty$. (See Exercises 92 and 95 in Section 10.1 for the definitions of a null sequence and a positive sequence, respectively.)

19. Suppose that the dosage of a certain medication is d milligrams once a day, to be taken at the same time of day.

a. Assuming that after 24 hours, half of the initial amount is still present in the bloodstream, find a recursive formula for the sequence $\{a_n\}$, the amount of medication present in the bloodstream right after the n^{th} dose is taken.

b. Find $\lim\limits_{n\to\infty} d_n$. (This is how much medication is in the patient's bloodstream, if it is taken over a long time period.)

20–21 Use partial sums to prove that the given series is convergent.

20. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$ **21.** $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-n}$

22–25 Decide whether the series converges. If so, find its sum.

22. $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{3}{2}\right)^{-n}$ **23.** $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{2+2n}{n^2+2n}$ **25.** $\displaystyle\sum_{n=2}^{\infty} \frac{3^n-1}{5^{n-2}}$

26. Referring back to Exercise 66 of Section 10.2, supposing that the positive series $\sum a_n$ is divergent, what can you say about $\sum(1/a_n)$? (See Exercise 70 in Section 10.2 for the definition of positive series.)

27–32 Use the Integral Test to determine whether the series converges or diverges.

27. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^2+4}$ **28.** $\displaystyle\sum_{n=0}^{\infty} \frac{4}{n^2+4}$

29. $\displaystyle\sum_{n=0}^{\infty} \frac{2n}{2^n}$ **30.** $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{2+e^{1/n}}{n^3}$ **32.** $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(\arctan n)}{n^2+1}$

33–34 Find the smallest possible n to approximate the sum of the series within the indicated error ε and provide the requested estimate.

33. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}; \quad \varepsilon = 0.04$ **34.** $\displaystyle\sum_{n=1}^{\infty} ne^{-n}; \quad \varepsilon = 0.03$

35–38 Use the Direct Comparison Test to determine whether the series converges or diverges.

35. $\sum_{n=2}^{\infty} \frac{1}{n \ln n - 1}$ **36.** $\sum_{n=2}^{\infty} \frac{n^2+1}{n^3-1}$

37. $\sum_{n=0}^{\infty} \frac{2}{2^n + \sqrt{n}}$ **38.** $\sum_{n=0}^{\infty} \frac{\sin^4 n}{n^2+1}$

39–42 Use the Limit Comparison Test to determine whether the series converges or diverges.

39. $\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^5+1}}$ **40.** $\sum_{n=1}^{\infty} \frac{\sqrt{1+\ln n}}{n^2}$

41. $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+n}}{n^{5/2}}$ **42.** $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^4+1}}$

43–50 Use the Ratio or Root Test, as appropriate, to determine whether the series converges or diverges.

43. $\sum_{n=1}^{\infty} \frac{n^7}{7^n}$ **44.** $\sum_{n=5}^{\infty} \frac{5^n}{(n-5)!}$

45. $\sum_{n=1}^{\infty} \frac{2^n}{n \cdot n!}$ **46.** $\sum_{n=1}^{\infty} \frac{2^n (n!)}{(2n-1)!}$

47. $\sum_{n=0}^{\infty} \frac{3^n}{4^n+1}$ **48.** $\sum_{n=1}^{\infty} \frac{n^n}{2^{n+1}}$

49. $\sum_{n=1}^{\infty} \frac{(\ln n)^{2n}}{n^{n+1}}$ **50.** $\sum_{n=1}^{\infty} \left(\frac{1}{2} + \frac{1}{2n}\right)^n$

51–54 Determine whether the alternating series converges and give a reason for your answer.

51. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{n^2}$ **52.** $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n \sqrt{n}}$

53. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n-1}{5n+1}$ **54.** $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n^5}$

55–56 Approximate the sum of the alternating series, accurate to at least the indicated number of decimal places. How many terms did you use? According to the n^{th}-remainder estimate for the error (see Section 10.6), how many terms guarantee the indicated accuracy? (Note that the n^{th}-remainder estimate is not necessarily "sharp".)

55. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$; accurate to 2 decimal places

56. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n 2^n}$; accurate to 3 decimal places

57–62 Determine whether the given series converges absolutely, converges conditionally, or diverges.

57. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$ **58.** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+1}$

59. $\sum_{n=1}^{\infty} \frac{\sin n}{n^{3/2}}$ **60.** $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^6}$

61. $\sum_{n=3}^{\infty} \left(\frac{2-n}{4n}\right)^n$ **62.** $\sum_{n=1}^{\infty} \frac{(-2)^{n+1} n}{4n-2}$

63–68 Determine the interval of convergence for the given power series.

63. $\sum_{n=0}^{\infty} 2^n x^n$ **64.** $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^3+2}$

65. $\sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$ **66.** $\sum_{n=1}^{\infty} \frac{n!(x-1)^n}{n^4}$

67. $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+1)3^n}$ **68.** $\sum_{n=2}^{\infty} \frac{(2x-6)^n}{\sqrt{n} \ln n}$

69–70 Determine the interval of convergence for the series, and the limiting function of the series on that interval.

69. $\sum_{n=0}^{\infty} (2x-3)^n$ **70.** $\sum_{n=0}^{\infty} \frac{(3x-1)^n}{2^n}$

71–72 Find the power series expansion for the given function about 0. What is the radius of convergence? (**Hint:** Use the same approach as in Exercises 41–48 of Section 10.7.)

71. $f(x) = \frac{1}{1+3x}$ **72.** $f(x) = \frac{1}{2-4x^2}$

73. Find the power series representation of $f(x) = 1/x$, centered at $a = 2$. Differentiating twice, find the series for $g(x) = 1/x^3$ around $a = 2$. What is the radius of convergence? (**Hint:** Start by rewriting $1/x$ as $\frac{1}{2-(2-x)}$.)

74–81 Determine the Taylor series (or Maclaurin series if the center is not specified) of the given function about the indicated point. Find the radius of convergence. (**Hint:** Do not use the definition.)

74. $f(x) = e^{x^4/2}$

75. $f(x) = \cos(2x^3)$

76. $f(x) = \frac{1}{x+3}$; $a = 2$

77. $f(x) = \ln x$; $a = 3$

78. $f(x) = \sin \frac{x}{2}$

79. $f(x) = \sin^2 x$
(**Hint:** Use a trigonometric indentity.)

80. $f(x) = x^2 e^{3x^2}$

81. $f(x) = \frac{x}{x+2}$; $a = 1$

82–83 Use the definition to find the first four nonzero terms of the Taylor series generated by the given function about the indicated point.

82. $f(x) = \cot x; \quad a = \pi/2$

83. $f(x) = \csc x; \quad a = \pi/4$

84–86 Find the first five nonzero terms of the Maclaurin series generated by the given function by using operations on familiar series (try not to use the definition).

84. $f(x) = xe^x - \sin x$

85. $f(x) = \dfrac{\cos x}{x+1}$

86. $f(x) = \cos(\sin x)$ (**Hint:** Substitute the series of $\sin x$ into that of $\cos x$.)

87–89 Use Taylor series to approximate the given function value or definite integral to within the indicated accuracy. How many (nonzero) terms did you need?

87. $\cos 1; \quad \text{error} \le 10^{-6}$

88. $\int_0^1 e^{-x^3}\, dx; \quad \text{error} \le 10^{-4}$

89. $\sqrt[4]{1.2}; \quad \text{error} \le 10^{-5}$ (**Hint:** Use a binomial series.)

90. Use series expansion to verify the trigonometric limit $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x} = 0.$

91–92 Use Taylor series to find the indicated limit.

91. $\lim\limits_{x\to 0} \dfrac{e^{-x}-1}{x}$

92. $\lim\limits_{x\to 0} \dfrac{x \sin 2x}{1-e^{x^2}}$

93. Using an appropriate power series, find the sum of the series $\sum\limits_{n=1}^{\infty} (n/2^n)$. (**Hint:** Start with the familiar series expansion of $1/(1-x)$, and use termwise differentiation in its interval of convergence; then make an appropriate substitution.)

94. Using an appropriate Taylor series, verify that $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n} = \ln 2.$

95–98 Find the first six nonzero terms of the binomial series expansion of the indicated function.

95. $f(x) = \dfrac{1}{\sqrt{x^2+1}}$

96. $f(x) = \dfrac{1}{\sqrt[3]{x+1}}$

97. $f(x) = \sqrt{2x^3+1}$

98. $f(x) = \left(1 - \dfrac{x}{2}\right)^{4/3}$

99–100 Find a power series solution of the equation. In each case, use the initial conditions $y(0) = a$ and $y'(0) = b$.

99. $y'' + 4y = 0$

100. $y'' - \dfrac{2}{(1-x)} y' = 0$

101. Find a power series solution of the differential equation $y' + y = 0$ satisfying the initial condition $y(0) = 2$. Then solve the equation by traditional means and conclude that the solutions are equal.

102. Use the Maclaurin series you obtained in Exercise 86 to find $f^{(6)}(0)$.

103.* Prove that if every bounded monotonic sequence converges, then the Completeness Property of real numbers holds. (**Hint:** Let S be a set that is bounded above by M. Let $a_0 = M$ and for each $n > 0$, let s_n be the least positive integer such that $s_n M/2^n$ is an upper bound for S. Letting $a_n = s_n M/2^n$, proceed to show that a_n converges and its limit is the least upper bound of S. The existence of the greatest lower bound can be shown analogously. Note that this statement is the converse of Exercise 74 in Section 10.1, showing that the Bounded Monotonic Sequence Theorem is equivalent to the Completeness Property of $\mathbb{R}$.)

104. Prove that the alternating series $\sum\limits_{n=2}^{\infty} (-1)^{n+1} \dfrac{\ln(n^p)}{n}$ converges for all values of p.

105. Let $\sum\limits_{n=1}^{\infty} a_n$ be a divergent series and assume that $\sum\limits_{n=1}^{\infty} b_n$ is convergent. Prove that $\sum\limits_{n=1}^{\infty} (a_n + b_n)$ is divergent.

106. Suppose that $\sum\limits_{n=1}^{\infty} a_n$ is divergent and $k \ne 0$. Prove that $\sum\limits_{n=1}^{\infty} k a_n$ is also divergent.

107. Decide whether the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{1+2+\cdots+n}$ converges or diverges. Prove your answer.

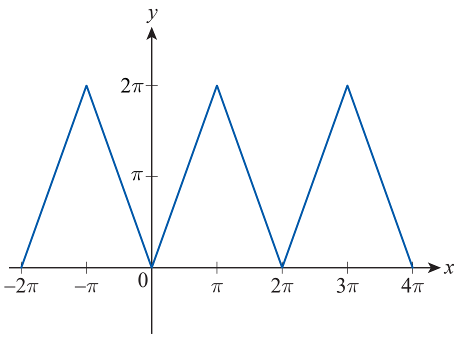
108. If a_n and b_n are positive sequences such that $\sum\limits_{n=1}^{\infty} a_n$ is convergent and b_n is a null sequence, prove that $\sum\limits_{n=1}^{\infty} a_n b_n$ is convergent.

109. Let $a_n = \begin{cases} 1/n & \text{if } n \text{ is even} \\ 1/n^2 & \text{if } n \text{ is odd} \end{cases}$. Show that $a_n \to 0$, yet $\sum\limits_{n=1}^{\infty} (-1)^{n+1} a_n$ diverges. Why does this not contradict Leibniz's Test?

110.* Use Taylor's formula to provide a proof of the Second Derivative Test as follows. Assuming that $f'(c)=0$, use Taylor's formula to conclude that $f(x)=f(c)+\frac{1}{2}f''(a)(x-c)^2$, for some a between x and c. Then examine the signs of $f(x)-f(c)$ and $f''(a)$. Next, assuming $f'(c)=f''(c)=0$ and $f'''(c)\neq 0$, argue that $f(c)$ is neither a relative maximum nor a minimum. (Assume initially that f is continuosly differentiable through at least the third order; then think about whether you can relax this condition.)

111. Find a second solution to Exercise 71 using long division.

112. Find the Fourier series expansion of the 2π-periodic extension of the function graphed below.



Concept Check

113–124 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

113. If a_n is monotonically decreasing, then $\lim_{n\to\infty} a_n = -\infty$.

114. If $\{a_n\}$ is convergent, then $\{a_n/n\}$ is a null sequence.

115. If $\{a_n/n\}$ is a null sequence, then $\{a_n\}$ is convergent.

116. If $\{a_n\}$ is convergent, then $\{a_{n+1}-a_n\}$ is a null sequence.

117. If $\{a_n\}$ is monotonically decreasing to zero, then $\sum_{n=1}^{\infty}(-1)^n a_n$ is absolutely convergent.

118. If $\{a_n\}$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

119. If $\sum_{n=1}^{\infty}|a_n|$ is divergent, then either $\sum_{n=1}^{\infty} a_n$ or $\sum_{n=1}^{\infty}(-a_n)$ is divergent.

120. If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty}|a_n|$ is divergent.

121. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

122. If the power series $\sum_{n=1}^{\infty} a_n x^n$ diverges at $x=c$, then it diverges at $x=-c$.

123. All power series converge at infinitely many x-values.

124. There is a power series whose convergence set is empty.

Chapter 10
Technology Exercises

125–127 Use a graphing utility to solve the problem.

125. We already know that the harmonic series diverges to infinity, and that it does so at a very slow pace. In this exercise, we will examine this series a bit further.

a. Find out how many terms are needed for the partial sum of the harmonic series to exceed 12.

b. What is the sum of the first 2 million terms? (Compare with Example 4 of Section 10.3. Notice that this calculation takes a bit of time even for today's powerful technology!)

126. The simple series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$ was shown by Gregory and Leibniz to converge to $\pi/4$ (hence its name, the *Gregory series*). However, it converges rather slowly. Find out how many terms of this series are necessary to approximate π accurate to two decimal places.

127. **a.** Graph $y=\sin x$ and its 11^{th}-order Maclaurin polynomial on the same screen, over the interval $[-4\pi,4\pi]$. Visually estimate the subinterval over which you find the approximation acceptable.

b. Repeat part a. with the 21^{st}-order Maclaurin polynomial.