Chapter 9
Review Exercises

1–4 Sketch the curve defined by the parametric equations by eliminating the parameter.

1. $x=\frac{1}{36t}, \quad y=t^2$

2. $x=t+5, \quad y=|t-2|$

3. $x=\frac{3}{4t-2}, \quad y=2t-2$

4. $x=4\sin\theta, \quad y=\cos\theta+1$

5–6 Construct parametric equations defining the graph of the given equation.

5. $y^2=x^2+4$

6. $6x=2-y$

7–8 Find parametric equations to represent the graph described. (Answers will vary.)

7. Line, passing through $(14,4)$ and $(-3,-8)$

8. Circle, center $(1,1)$, radius 1

9–10 Find the equations of any horizontal or vertical tangent lines to the given curve.

9. $x=2t+1, \quad y=t^2-4$

10. $x=-2t^2, \quad y=\frac{3}{t-2}, \quad t<2$

11–14 Find the values of dy/dx and d^2y/dx^2 for the given curve at the indicated point.

11. $x=2t^2+1, \quad y=\sqrt{t-1}; \quad (9,1)$

12. $x=e^t, \quad y=t^2e^{-t}; \quad (1,0)$

13. $x=\frac{1}{t}, \quad y=t^2+t; \quad (1,2)$

14. $x=\sin t, \quad y=\cos 2t; \quad \left(\frac{1}{2},\frac{1}{2}\right)$

15–16 Find the value(s) of the parameter for any inflection point(s) of the given curve.

15. $x=t-t^3, \quad y=3t+1$

16. $x=5t-1, \quad y=t^3\left(2-t^2\right)$

17–18 Find the area enclosed by the given curve.

17. $x=\sin 3t, \quad y=\cos 2t, \quad -\frac{\pi}{3}\le t\le\frac{\pi}{3}$

18. $x=\cos^3 t, \quad y=\sin^3 t, \quad 0\le t\le 2\pi$

19–20 Find the arc length of the given curve on the indicated interval.

19. $x=\frac{4}{3}t^{3/2}, \quad y=4t, \quad 0\le t\le 5$

20. $x=t\sin t+\cos t, \quad y=t\cos t-\sin t, \quad 0\le t\le\pi$

21–23 Find the area of the surface generated by revolving the parametric curve about the indicated axis.

21. $x=\frac{t^3}{2}, \quad y=2t+1, \quad 0\le t\le 1;$ about the y-axis

22. $x=\frac{1-t}{2}, \quad y=\sqrt{t}, \quad 0\le t\le 1;$ about the x-axis

23. $x=3t-\frac{t^3}{3}, \quad y=\sqrt{3}\left(9-t^2\right), \quad 0\le t\le 3;$ about the y-axis

24–25 Convert the point from polar to Cartesian coordinates.

24. $\left(-3.45,\frac{\pi}{3}\right)$

25. $\left(7,\frac{7\pi}{6}\right)$

26–27 Convert the point from Cartesian to polar coordinates.

26. $\left(-\sqrt{3},-1\right)$

27. $(10,12)$

28–29 Rewrite the rectangular equation in polar form.

28. $x^2+y^2=16a^2$

29. $x^2+y^2=9ax$

30–31 Rewrite the polar equation in rectangular form.

30. $r=4\cos\theta$

31. $r=\frac{16}{4\cos\theta+4\sin\theta}$

32–33 Sketch a graph of the given polar equation.

32. $r=4\sin 3\theta$

33. $r^2=25\cos 2\theta$

34–35 Find the slope of the line tangent to the given polar curve at the indicated point.

34. $r = 4\cos 3\theta; \quad \theta = \dfrac{\pi}{12}$

35. $r = \dfrac{1}{\theta^2 + 1}; \quad \theta = \dfrac{\pi}{2}$

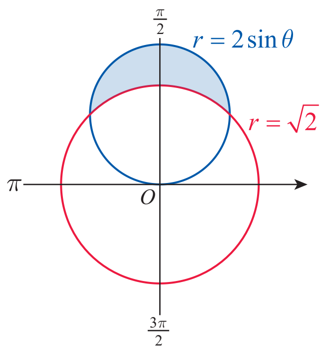
36–37 Find all points where the given polar curve has a horizontal or vertical tangent line.

36. $r = 2\sin\theta, \quad 0 \le \theta \le 2\pi$

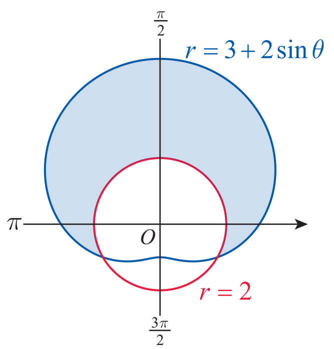
37. $r = 1 + \cos 2\theta, \quad 0 \le \theta \le \pi$

38–39 Find the area of the shaded region.

38.



39.



40–41 Find the area of the specified region.

40. A large loop of $r = 1 + 2\cos 2\theta$

41. The portion of the rose $r = 4\cos 2\theta$ outside the circle $r = 2$

42–45 Use polar coordinates to find the arc length of the curve.

42. The circle $r = 2\cos\theta$

43. The line segment $r = \sec\theta, -\pi/4 \le \theta \le \pi/4$

44. The spiral $r = 2\theta, 0 \le \theta \le 2\pi$

45. The cardioid $r = 2 + 2\cos\theta$

46. Find the area of the surface generated by revolving the curve $r = 2\cos\theta, 0 \le \theta \le \pi$ about $\theta = \pi/2$. (**Hint:** See Exercise 58 in Section 9.4)

47–52 Sketch the graph of the given conic section, and determine the coordinates of the foci and the equations of the directrix or asymptotes as appropriate.

47. $\dfrac{(x+1)^2}{9} + \dfrac{(y-2)^2}{16} = 1$

48. $x^2 + 9y^2 - 6x + 18y = -9$

49. $(y+1)^2 = -12(x+3)$

50. $x^2 - 8x + 2y + 14 = 0$

51. $\dfrac{(y+2)^2}{9} - \dfrac{(x-2)^2}{16} = 1$

52. $9x^2 - 4y^2 + 54x - 8y + 41 = 0$

53–63 Find the equation, in standard form, of the conic section with the given properties.

53. Ellipse, center at $(-1,4)$, major axis is vertical and of length 8, foci $\sqrt{7}$ units from the center

54. Ellipse, foci at $(1,2)$ and $(7,2)$, $e = \dfrac{1}{2}$

55. Ellipse, vertices at $\left(\dfrac{7}{2},-1\right)$ and $\left(\dfrac{1}{2},-1\right)$, $e = 0$

56. Ellipse, vertices at $(1,-8)$ and $(1,2)$, minor axis of length 6

57. Parabola, vertex at $(-2,3)$, directrix is the line $y = 2$

58. Parabola, vertex at $(5,-3)$, focus at $(5,1)$

59. Parabola, focus at $(3,-1)$, directrix is the line $x = 2$

60. Hyperbola, vertices at $(4,-1)$ and $(-2,-1)$, foci at $(5,-1)$ and $(-3,-1)$

61. Hyperbola, asymptotes of $y = \pm\frac{5}{2}(x+1) - 2$, vertices at $(-3,-2)$ and $(1,-2)$

62. Hyperbola, foci at $(-1,-2)$ and $(-1,8)$, asymptotes of $y = \pm\left(\frac{3}{4}x + \frac{3}{4}\right) + 3$

63. Hyperbola, asymptotes of $y = 3x - 4$, $y = 8 - 3x$, vertices at $(2,-1)$ and $(2,5)$

64–70 Identify the given conic section and find the equation for its directrix.

64. $r = \dfrac{8}{1+2\sin\theta}$

65. $r = \dfrac{5}{4-8\sin\theta}$

66. $r = \dfrac{3}{7+6\sin\theta}$

67. $r = \dfrac{6}{9-9\cos\theta}$

68. $r = \dfrac{7}{4+4\sin\theta}$

69. $r = \dfrac{4}{6-3\cos\theta}$

70. $r = \dfrac{7}{5+2\cos\theta}$

71–76 Construct a polar equation for the conic section with the focus at the origin and the given eccentricity and directrix.

71. Eccentricity: $e = 1$; directrix: $x = -3$

72. Eccentricity: $e = 4$; directrix: $y = 3$

73. Eccentricity: $e = \frac{1}{5}$; directrix: $y = -15$

74. Eccentricity: $e = \frac{1}{4}$; directrix: $x = 16$

75. Eccentricity: $e = 1$; directrix: $y = -7$

76. Eccentricity: $e = 9$; directrix: $x = \frac{1}{3}$

77. A motorcycle headlight is made by placing a strong light bulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 5y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

78.* Prove that the graph of $r = \cos\theta + \sin\theta$ is a circle.

79. Show that the two limaçons $r = 1 + \sin\theta$ and $r = 1 - \sin\theta$ are orthogonal, that is, they intersect at right angles. (Ignore their intersection at the pole.)

Concept Check

80–90 Determine whether each of the following statements is true or false. In case of a false statement, explain or provide a counterexample.

80. Two different sets of parametric equations may describe the same curve.

81. If $x = f(t)$ and $y = g(t)$ both pass through $(0,0)$, the parametric curve $x = f(t)$, $y = g(t)$ also passes through the origin.

82. The line $y = 2x$ can be parametrized as $x = t^2$, $y = 2t^2$.

83. If a parametric curve C is differentiable on (a,b) then its graph has a tangent line at any of its points.

84. Let C be the curve $x = t^5$, $y = t^4$. Since $\left.\dfrac{dy}{dt}\right|_{t=0} = 0$, the graph of C has a horizontal tangent at the origin.

85. The polar coordinates (r,θ) and $(-r,\theta-\pi)$ describe the same point.

86. If the polar coordinates (r_1,θ_1) and (r_2,θ_2) describe the same point, then $r_1 = \pm r_2$.

87. If $y = f(x)$ is an even function, then the graph of $r = f(\theta)$ is symmetric with respect to the polar axis.

88. It is impossible to find a polar equation for a straight line.

89. The distance between any point on a parabola from the focus is at least p.

90. If the asymptotes of a hyperbola are perpendicular and the hyperbola is centered at the origin with horizontally aligned foci, then the equation of the hyperbola can be written as $x^2 - y^2 = a^2$.

Chapter 9
Technology Exercises

91. Find the equation of the graph of $r = 1 - 2\cos\theta$ after a clockwise rotation by $\pi/4$ radians. Name the resulting curve and use a graphing utility to sketch it. (See Exercise 73 in Section 9.3.)

92–93 Use a graphing utility to sketch the given curve for various values of the parameter(s) and explore the effects on the shape of your graph.

92. $r = \theta\cos k\theta$

93. $x = \pm a\cos^{2/n} t, \quad y = \pm b\sin^{2/n} t, \quad a, b, n > 0$
(Lamé curves)

94–95 Use a graphing utility to sketch the curve and then find all horizontal and vertical tangent lines. Confirm your results by paper and pencil calculations.

94. $x = t^3 - t, \quad y = t^2 + 1, \quad -2 \le t \le 2$

95. $r = 2\sin 2\theta, \quad 0 \le \theta \le \pi/2$

96–97 Use a graphing utility to sketch the region enclosed by the given curve and find its area.

96. $x = t\sin t, \quad y = \sin 2t, \quad 0 \le t \le \pi$

97. Inner loop of $r = 2 - 3\cos\theta$

98–99 Use a graphing utility to approximate the arc length of the curve with the given parametrization.

98. $x = \sin 2t, \quad y = \sin t, \quad 0 \le t \le \pi$

99. $r = \cos(2\sin\theta), \quad 0 \le \theta \le 2\pi$

100–101 Use a graphing utility to approximate the surface area of the solid obtained by rotating the given curve about the indicated axis.

100. $(t^3 - 3t, t^2 - 2), \quad 0 \le t \le \sqrt{3};$ about the y-axis

101. $r = 3\sin 2\theta, \quad \pi/4 \le \theta \le \pi/2;$ about the polar axis