Chapter 4
Review Exercises

1–4 Sketch by hand the graph of a function f on the specified domain, with the specified properties. (Answers will vary.)

1. Defined on $(0,2)$, absolute minimum at 1, no absolute maximum
2. Defined on $[-2,2]$, absolute maximum occurs twice, no absolute minimum
3. Defined on $(-1,1)$, no absolute or relative extrema
4. Differentiable on $(0,1)$, no critical points, no extrema

5–14 Find all absolute extrema of the function on the indicated domain.

5. $f(x)=x^2-\frac{4}{3}x^3;\quad D=[-1,1]$
6. $f(x)=-x^4+8x^3-16x^2;\quad D=[-1,5]$
7. $f(x)=|x^2-2x-8|;\quad D=[-3,5]$
8. $f(x)=(x+2)|x|;\quad D=[-2,2]$
9. $f(x)=\sqrt{x}(1-x);\quad D=[0,2]$
10. $f(x)=\frac{x^2+1}{x+1};\quad D=[0,1]$
11. $f(x)=3x^5-2x^3+1;\quad D=\mathbb{R}$
12. $f(x)=\sqrt{1-x^4};\quad D=(-1,1)$
13. $f(x)=\csc\frac{x}{2};\quad D=(0,2\pi)$
14. $f(x)=x^2-x[\![x]\!];\quad D=[1,2]$

15–16 Prove that the equation has exactly one real solution on the given interval.

15. $3x^3-x^4=1$ on $(0,1)$
16. $x\arcsin x=e^{-x}$ on $(0,1)$

17–18 Determine whether Rolle's Theorem applies to the function on the given interval. If so, find all possible values of c as in the conclusion of the theorem. If the theorem does not apply, state the reason.

17. $f(x)=x^3+x^2-8x-12$ on $[-2,3]$
18. $g(x)=x^4+2x^2-2$ on $[0,1]$

19–20 Determine whether the Mean Value Theorem applies to the function on the given interval. If so, find all possible values of c as in the conclusion of the theorem. If the theorem does not apply, state the reason.

19. $f(x)=|x^4-3x|$ on $[2,3]$
20. $f(x)=|x^4-3x|$ on $[0,2]$
21. If $|f'(x)|\le 3$ for all x, prove that $|f(10)-f(2)|\le 24$.
22. If $g(-5)=-1$ and $g'(x)\le 4$ for all x, what is the largest possible value of $g(1)$?
23. Find the function f that passes through $(0,3)$ and whose derivative is $\cos x+e^x$.
24. An object is moving along the x-axis, starting at $x_0=-4$ with velocity function $v(t)=3t^2-2t$ $(0\le t\le 5)$. Find the time t when it reaches the origin.
25. A car driving at 70 mph passes a mile marker, and then exactly 48 seconds later, still driving at 70 mph, passes the next mile marker.
    a. Prove that there was at least one instant when the car traveled at 75 mph between the markers.
    b. Prove that there was at least one instant when the car's acceleration was zero.

26–29 Use the first derivative to determine where the function is increasing and decreasing.

26. $f(x)=|2x-2|$
27. $g(x)=-x^2+6x+7$
28. $h(x)=x^3-6x^2+9x+1$
29. $k(x)=x^4-4x^3-20x^2+96x$

30–33 Determine the intervals of concavity of the given function.

30. $f(x)=\frac{x^3}{6}-x^2-2$
31. $g(x)=\frac{e^x}{x}$
32. $h(x)=(x-2)\sqrt[3]{x}$
33. $k(x)=\frac{x+3}{x^2-1}$

34–35 Use the first and second derivatives to identify the intervals of monotonicity, extrema, intervals of concavity, and inflection points of the given function.

34. $f(x)=3x^5-20x^3$
35. $g(x)=\arctan(x^2)$

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36–37 The function $p(t)$ gives the position, relative to its starting point, of an object moving along a straight line. Identify the time intervals when the object is moving in the positive versus negative direction, as well as those intervals when it is accelerating or slowing down. Find the times when the object changes direction as well as when its acceleration is zero.

36. $p(t) = t^3 - 3t^2, \quad 0 \le t \le 5$

37. $p(t) = 3t^2 - \dfrac{t^4}{2}, \quad 0 \le t \le 3$

38–49 Check whether L'Hôpital's Rule applies to the given limit. If it does, use it to determine the value of the limit. If it does not, find the limit some other way. (When necessary, apply L'Hôpital's Rule several times.)

38. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

39. $\lim\limits_{x\to 0} \dfrac{x - \tan x}{\sec x - 1}$

40. $\lim\limits_{x\to \infty} \dfrac{e^{-x}}{\ln x}$

41. $\lim\limits_{x\to 0^+} \cot x \csc x$

42. $\lim\limits_{x\to 0} \left(1+4x^2\right)^{1/x^2}$

43. $\lim\limits_{x\to (1/2)^+} \left(\dfrac{1}{4x-2} - \dfrac{1}{\ln 2x}\right)$

44. $\lim\limits_{x\to 0^+} x^{\sqrt{x}}$

45. $\lim\limits_{x\to 0^+} \left(\sqrt{x}\right)^{\ln x}$

46. $\lim\limits_{x\to 0^-} x \cot x$

47. $\lim\limits_{x\to 0} \dfrac{\arctan x}{\arctan 2x}$

48. $\lim\limits_{x\to 0^+} \sin x \ln x$

49. $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \csc x\right)$

50. By examining the limits $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^a}$ and $\lim\limits_{x\to\infty} \dfrac{x^a}{b^x}$ using L'Hôpital's Rule, compare the relative growth rates of the functions $y = \ln x$, $y = x^a$, and $y = b^x$ $(a > 0,\ b > 1)$. (See Exercise 88 in the Chapter 3 Review.)

51–60 Use the curve-sketching strategy to construct a graph of the function.

51. $f(x) = \dfrac{x^3}{3} - x^2 - 15x$

52. $g(x) = -x^3 + 12x - 16$

53. $h(x) = x^4 - 2x^3$

54. $m(x) = 3x^3 - 4x^5$

55. $f(x) = \dfrac{2x}{x^2+1}$

56. $f(x) = \dfrac{2x}{x^2-1}$

57. $f(x) = |x|(x-1)$

58. $f(x) = x\sqrt{9-x^2}$

59. $f(x) = \dfrac{x^3}{x^2-4}$

60. $f(x) = \sin^2 x \cos x$

61–62 Use Newton's method to approximate the given number to five decimal places.

61. $\sqrt[5]{30}$

62. $\log 11$

63–64 Use Newton's method to solve the equation on the given interval. Approximate the root to six decimal places.

63. $2x^5 = 1 - x^2$ on $(0,1)$

64. $\ln x = \cos x$ on $(0,\infty)$

65. Use Newton's method to approximate to four decimal places the fixed point(s) of $f(x) = 1 - \tan x$ on $(0, \pi/2)$. (See Exercise 62 of Section 4.2.)

66. Find a positive number that is greater than its own cube by the greatest possible amount.

67. Generalize Exercise 66 to the n^{th} power of a number $(n \ge 2)$.

68. Find a number a so that for given $a_1, a_2, a_3 \in \mathbb{R}$, the quantity $S_3 = (a-a_1)^2 + (a-a_2)^2 + (a-a_3)^2$ is minimal.

69. Generalize Exercise 68 for n given numbers to minimize the following quantity: $S_n = (a-a_1)^2 + (a-a_2)^2 + \cdots + (a-a_n)^2$.

70. A wire of length l is bent into an L shape. Where should it be bent in order to minimize the distance between the two endpoints?

71. Find the length l and width w of the rectangle inscribed in the unit circle for which $l^2 w$ is maximal.

72. Find the dimensions of the rectangle whose diagonal is d units and whose area is maximum.

73. A book page of area 500 cm^2 is required to have 1 cm margins on the sides, while the margins on the top and bottom are to be 2 cm. Find the dimensions of the page that maximize the printed area.

74. Among all isosceles triangles whose legs are l units long, find the base angle that maximizes the area.

75. A vertex of a rectangle is at the origin and the opposite vertex sits in the first quadrant and on the graph of $y = \dfrac{2-x}{x+1}$. Find the maximum possible area for such a rectangle.

76. Find the point on the graph of $y = 1 - x^2$ that is closest to the point $(-3,1)$.

77. Among all isosceles triangles that can be inscribed in the circle of radius R, find the one with maximum area.

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78. A vending machine sells 500 bars of a certain type of candy when the price is \$1.50. It was discovered that 10 fewer customers will buy the candy bar for each 5¢ increase in price. What is the price that will bring maximum revenue from the sales of this type of candy bar?

79. Maximize the surface area of the can in Example 3 of Section 4.6. Explain your findings.

80. Minimize the cost of producing the can in Example 3 of Section 4.6 if the top and bottom are produced using a material that is 50% more expensive than the material used for the side.

81. Nate needs to reach a restaurant that is 600 ft upstream on the other side of a 150 ft wide river. Find the point where he has to reach the other side in order to make the best time if he can swim at 5 ft/s and walk at 9 ft/s. (Ignore the flow of the river.)

82–89 Find the general antiderivative of the given function, and check your answer by differentiation. (If necessary, rewrite the function before antidifferentiation.)

82. $f(x)=2x^3-6x^2+3x$

83. $f(x)=5x^4-4.8x^3+e^2$

84. $f(x)=x(x+2)(2x-3)$

85. $f(x)=0.4x\sqrt{x}-\dfrac{2}{\sqrt{x}}$

86. $f(x)=\dfrac{x^4-4x}{x^2}$

87. $f(x)=2(x+\sec^2 2x)$

88. $f(x)=6e^{3x}$

89. $f(x)=\dfrac{3}{4x^2+1}$

90–91 Find $f(x)$ that satisfies the specified conditions.

90. $f''(x)=x,\quad f'(1)=1,\quad f(1)=0$

91. $f'''(x)=2,\quad f''(2)=-1,\quad f'(2)=2,\quad f(2)=3$

92. A tennis ball is thrown upward from an initial height of 4 feet with an initial velocity of 56 feet per second. How high will it go and for how long is it rising? (Ignore air resistance.)

93. With what initial velocity do we need to throw a golf ball vertically upward in order for it to rise 100 feet high? (Ignore the initial height and air resistance.)

94. A pebble is shot horizontally using a slingshot at 10 meters per second from the top of a building that is 20 meters high. If the terrain around the building is nearly flat, approximately how far from the building will the pebble hit the ground? (Use the approximation $g\approx 10\text{ m/s}^2$ and ignore air resistance.)

Concept Check

95–101 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

95. A continuous function on a finite interval always attains its maximum and minimum.

96. If $f(x)$ has a relative maximum or minimum at $x=c$, then $f'(c)=0$.

97. If $f(x)$ has a relative maximum or minimum at $x=c$, then c is a critical point of f.

98. A cubic polynomial has exactly one inflection point.

99. If $f(x)$ is a polynomial, then between two consecutive local extrema there must be an $x=c$ so that $f''(c)=0$.

100. If $f(x)$ is a polynomial and c is a critical point, then there is a relative maximum or minimum at $x=c$.

101. If $f'''(c)=0$, then $f'(x)$ has a point of inflection at $x=c$.

Chapter 4 Technology Exercises

102–111. Use a graphing utility to verify the answers you obtained for Exercises 51–60.

112–113. Use a graphing utility to verify the conclusions of Exercises 15 and 16.