# Chapter 3
# Review Exercises

**1–2** Find the derivative of the given function at the specified point and express your answer using the differential notation due to Leibniz.

1. $f(x) = x^3 + x; \quad x = 1$
2. $g(x) = \frac{2}{x}; \quad x = 2$

**3–4** Find the derivative of the function and use the differentiation operator $D_x$ to express your answer.

3. $s(x) = \sqrt{x-2}$
4. $t(x) = \frac{1}{x^2+1}$

**5–6** Find the first, second, and third derivatives of the function.

5. $f(x) = x^2 - 1$
6. $g(x) = x^4$

**7–10** Find all points where the function is not differentiable. For each of those points, find the one-sided derivatives (if they exist).

7. $f(x) = \sqrt[3]{x}$
8. $g(x) = |x+1| + |x-3|$
9. $h(x) = [\![x]\!] + x$
10. $F(t) = \begin{cases} t^2 \sin\frac{1}{t} & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases}$

**11–20** Use differentiation rules to find the derivative of the function.

11. $f(x) = 0.2x^5 - 2x^4 + x^3 + 0.5x^2 + 2^{3/4}$
12. $g(x) = \sqrt{3x} + \sqrt{3x} + \frac{1}{\sqrt{3x}}$
13. $h(x) = (2x-1)(x^2+4)$
14. $k(x) = \frac{x-2}{x}e^{x-2}$
15. $F(t) = (t-1)\left(t^2 + \frac{1}{t}\right)\left(\sqrt{t} + t\right)$
16. $G(s) = \frac{1}{2s + s^2}$
17. $u(x) = \frac{2e^x + 1}{3e^x + 5}$
18. $t(x) = \dfrac{\frac{2}{x} + \frac{1}{x^2} + 4}{\frac{2}{x^2} - \frac{1}{x}}$
19. $v(x) = \ln(x^2 + 2)$
20. $w(x) = \sin(\sin(\sin x))$

**21–24** Find the first, second, and third derivatives of the function.

21. $f(x) = \frac{x}{x+1}$
22. $f(x) = 3\sqrt{x}$
23. $f(x) = \tan x$
24. $f(x) = \arctan x$

**25–26** Find a function $f$ that satisfies the given conditions. (**Hint:** A polynomial is the most natural choice. Answers will vary.)

25. $f(0) = 0$, $f'(1) = 1$, and $f''(2) = 4$.
26. $f(0) = 2$ and $y = 2x + 1$ is tangent to the graph at $x = -1$.

**27–28** Find the equation of the line tangent to the graph of the function at the given point.

27. $f(x) = \frac{1}{\sqrt{3x^2+1}}; \quad \left(1, \frac{1}{2}\right)$
28. $f(x) = \tan(\sin x); \quad (\pi, 0)$

29. Find the equation(s) of the line(s) tangent to the graph of $f(x) = x^2 + 3x + 1$ through the point $(2, 2)$, which is not on the graph of $f$.

30. Assuming $f$ is differentiable, find the derivative of $y = \ln\sqrt{[f(x)]^2 + 1}$.

**31–32** Find the indicated limit.

31. $\lim\limits_{x \to 0} \frac{-\sin 2x}{4x}$
32. $\lim\limits_{x \to 0^+} \frac{x\cos x}{1 - \cos x}$

33. The position function of a moving particle is given by $x(t) = \frac{50t}{t+1}$ feet at $t$ seconds. Find its velocity and acceleration at $t = 1$ second.

34. An object is moving along a straight line so that its distance from the start at $t$ seconds is given by $d(t) = 12t - t^3$ meters. Find its position and acceleration at the instant when its velocity changes directions.

35. The radius of a spherical balloon being inflated increases according to the function $r(t) = 3 + 4\sqrt[3]{t}$, where $r$ is measured in centimeters and $t$ in seconds. Find the rate of change of the balloon's volume and surface area with respect to time at $t = 1$ second.

**36–37** Find $dy/dx$ by implicit differentiation.

**36.** $x^3 + y^3 = 2$

**37.** $6\left(x^2 + y^2\right) = 15xy$

**38–39** Find $dx/dy$ by implicit differentiation.

**38.** $x\sin(x+y) = y^2 + 6$

**39.** $6\left(y^2 - x^2\right) = y^4$

**40–43** Use implicit differentiation to find the equation of the line tangent to the curve at the indicated point.

**40.** $\frac{1}{x^3} + \frac{1}{y^3} = 2;\quad (1,1)$

**41.** $x^3 + y^2 = 2x + 1;\quad (0,1)$

**42.** $\frac{3(x+y)}{xy} = 16\sqrt{x+y};\quad \left(\frac{3}{4}, \frac{1}{4}\right)$

**43.** $3\sqrt{x} + \frac{2}{\sqrt{y}} = xy;\quad (1,4)$

**44.** Find all points on the lemniscate

$$\left(x^2 + y^2\right)^2 = 2\left(x^2 - y^2\right)$$

where its graph has horizontal tangent lines.

**45.** Use implicit differentiation to find $d^2y/dx^2$ for $x^{2/3} + y^{2/3} = 1$.

**46–49** Use the Derivative Rule for Inverse Functions to determine $\left(f^{-1}\right)'(a)$ for the indicated value of *a*. (The domain of *f* is assumed to have been restricted so that the inverse exists and is differentiable, whenever appropriate.)

**46.** $f(x) = x^3 + x;\quad a = 10$

**47.** $f(x) = \sqrt[4]{x+1};\quad a = 2$

**48.** $f(x) = \frac{2}{x^2};\quad a = \frac{1}{2}$

**49.** $f(x) = e^x + x;\quad a = 1$

**50–53** Determine the derivative of the given function.

**50.** $f(x) = \log\sqrt{x^2+1}$

**51.** $f(x) = \tan^{-1}\sqrt{x}$

**52.** $f(x) = e^{\arcsin x}$

**53.** $f(x) = \sec^{-1}(\ln x)$

**54–55** Use logarithmic differentiation to find $y'$.

**54.** $y = \frac{\sqrt[3]{x^2+1}\,(x+3)}{x^{2/3}\sqrt{2x^2+3}}$

**55.** $y = \left(\sqrt{x}\right)^{\ln x}$

**56.** A fast-growing population of bacteria doubles every half hour. If the initial count is 1000, how many bacteria are there in 100 minutes?

**57.** A 350 °F pizza is left on the counter and cools to 250 °F in 4 minutes. If the room temperature is 70 °F, determine the total time it takes for the pizza to cool down from 350 °F to 185 °F. (**Hint:** See Exercise 9 of Section 3.7.)

**58.** Find the rate of change of the distance from the origin of a point moving on the graph of $f(x) = x^3$ when $x = 1$ and $dx/dt = 2$ units per second.

**59.** A spherical balloon is being filled with helium at a rate of $20\ \text{in.}^3/\text{s}$. How fast is the radius increasing at the instant when the radius is 4 in.?

**60.** A small plane, flying at an altitude of 0.1 miles at a ground speed of 85 miles per hour, passes directly over an observer. How fast is the distance between the observer and the plane increasing a minute later?

**61.** Radar is tracking a rocket that was launched vertically upward. It is found that the rocket's distance from the radar is increasing at a rate of $1200\ \text{km/h}$ at the instant when that distance is 5 km. If the radar station is 4 km from the launch site, find the speed of the rocket.

**62.** A ship sailing west at 9 miles per hour passes a buoy 20 minutes before another ship sailing due north at 12 miles per hour passes the same buoy. How fast will they be separating an hour later?

**63.** Tiffany walks toward a light source that is 8 feet above ground. If the speed of the tip of her shadow is three times that of her walking speed, how tall is Tiffany?

**64–65** Find the linearization of the function at the given value.

**64.** $f(x) = \frac{1}{(x-1)^2};\quad x = 2$

**65.** $f(x) = \sin x;\quad x = \frac{\pi}{4}$

**66–67** Use linear approximation to approximate the given number. Round your answer to four decimal places.

**66.** $\sqrt[3]{8.2}$

**67.** $\arctan 0.9$

**68.** The diameter of a large bouncy ball was measured to be 65 cm with a possible error of 1 mm. Approximate the propagated errors in the calculated volume and surface area of the ball, respectively. Express your answers as percentage errors.

69. The proper dosage $d$ of a certain over-the-counter medicine for children depends on body weight $w$ according to the function $d(w)=\frac{5}{4}w^{3/5}$, where $d$ is measured in milligrams and $w$ in pounds. Use differentials to estimate how accurately (in terms of percentage error) we need to know a 32-pound child's weight if we cannot stray from the proper dosage by more than 6 percent.

70. A manufacturing business found its daily revenue to be $R(x)=150x-\frac{1}{4}x^2$ dollars when $x$ units are produced and sold.

   a. Use linearization and marginal revenue to estimate the extra revenue when production is increased from 100 to 102 units.

   b. Use the revenue function to calculate the actual revenue increase. Compare your answers.

71. Use the concept of the derivative function to explain why the graph of $y=x^a$, $a>1$ curves upward, while the graph of $y=x^b$, $0<b<1$ curves downward.

## Concept Check

**72–82** Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

72. If both one-sided derivatives of $f(x)$ either exist or are equal to $\pm\infty$ at $c$, then $f$ is continuous at $c$.

73. If $p(x)$ is a polynomial of degree $n$, then all $k^{\text{th}}$-order derivatives of $p(x)$ for $k>n$ are 0.

74. If $y=\pi^n \sin x$, then $y'=n\pi^{n-1}\sin x+\pi^n \cos x$.

75. If $y=1/(x^2-3x+1)$, then $y'=1/(2x-3)$.

76. If $y=\ln(3x+1)$, then $y'=1/(3x+1)$.

77. If $y=x^x$, then $y'=x\cdot x^{x-1}$.

78. Since $(e^x)'=e^x$, therefore $\left(e^{e^x}\right)'=e^{e^x}$.

79. If $f(x)=x$, then $df=dx$.

80. If $f(x)$ is linear, then its linearization at any point is itself.

81. If $x\to 0$, then $\Delta x\to dx$ and $\Delta y\to dy$.

82. If $\Delta x\to 0$, then $\Delta y/\Delta x\to dy/dx$.

# Chapter 3 Technology Exercises

**83–85** Use a graphing utility to graph the function and identify all points where the function is not differentiable. Explain.

83. $f(x)=\left|x^2-x\right|$

84. $f(x)=|x|(x+2)$

85. $f(x)=\sqrt[4]{x^2-1}$

86. Use the differentiation capabilities of a graphing utility to find the derivative of $f(x)=2\cos^2 x-\cos 2x$. Then find the derivative by hand, applying a trigonometric identity before differentiating. Does your answer agree with that of your technology? If not, what do you think is the reason? Can you "force" your graphing utility to represent its answer in a simpler form?

87. Repeat Exercise 86 for the function $f(x)=2\sin(x/2)\cos(x/2)$.

88. Use a graphing utility to graph the functions $y=\ln x$, $y=a^x$, $a>1$ and $y=x^b$, $0<b<1$ for various values of the parameters $a$ and $b$. By zooming out appropriately, compare their relative growth rates; that is, conjecture "who wins the race toward infinity" in general among these three types of functions. Use the concept of the derivative to support your conjecture.

89. The displacement of a mass attached to a spring is given by the function $h(t)=e^{-t/6}\cos 2t$.

   a. Use a graphing utility to graph the function and explain why it is realistic.

   b. Use a graphing utility to graph the velocity and acceleration functions together with $h(t)$ on the same screen. What seems to be the position of the mass when velocity is maximum? When is velocity 0? When is acceleration maximum, and when is it 0?