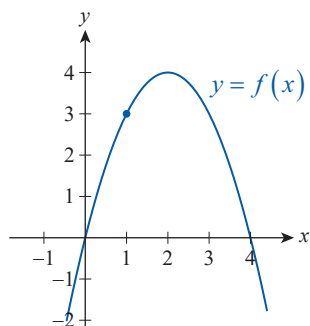


Chapter 2

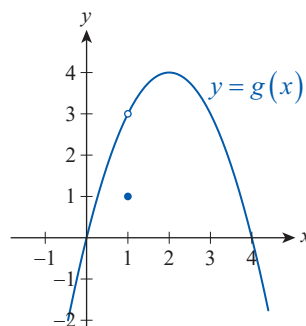
Review Exercises

1–4 Use the graph of the function to find the indicated (possibly one-sided) limit, if it exists. Also examine the continuity of the function at the indicated point and classify any discontinuities.

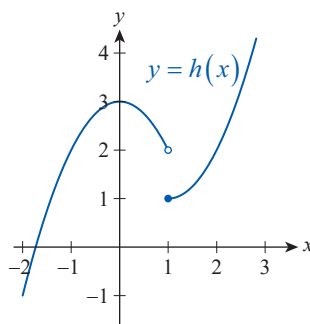
1. $\lim_{x \rightarrow 1} f(x)$



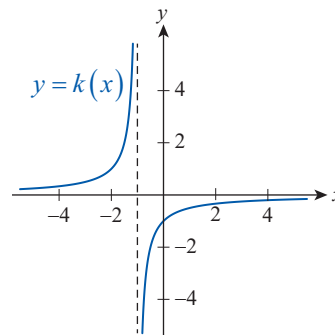
2. $\lim_{x \rightarrow 1} g(x)$



3. $\lim_{x \rightarrow 1^-} h(x)$



4. $\lim_{x \rightarrow -1^+} k(x)$



5–8 Use difference quotients to approximate the slope of the tangent to the graph of the function at the given point. Use at least five different h -values that are decreasing in magnitude. (Answers will vary.)

5. $f(x) = 3x + 2$; $(0, 2)$

6. $g(x) = 2 - x^2$; $(1, 1)$

7. $h(x) = \sqrt{x-1}$; $(2, 1)$

8. $k(x) = \sin x$; $(0, 0)$

9. A pellet is shot vertically upward from an initial height of 6 feet. Its height after t seconds is given by $h(t) = 6 + 608t - 16t^2$ feet. Use difference quotients to answer the questions below.

- What will be the pellet's height at the end of the first second?
- What is the average velocity of the pellet during the first two seconds?
- Estimate the instantaneous velocity at $t = 0$ seconds.
- Estimate the instantaneous velocity at $t = 2$ seconds.
- When will the velocity be 0?

10–11 Approximate the area of the region between the graph of $f(x)$ and the x -axis on the given interval. Use $n = 4$, as in Example 5a of Section 2.1. (Round your answer to four decimal places.)

10. $f(x) = x^3$ on $[0, 1]$ 11. $f(x) = \ln x$ on $[1, 3]$

12–13 Approximate the arc length of the graph of $g(x)$ on the given interval. Use $n = 5$, as in Example 5b of Section 2.1. (Round your answer to four decimal places.)

12. $g(x) = x^{2/3}$ on $[3, 8]$ 13. $g(x) = e^x$ on $[-2, 3]$

14–17 Create a table of values to estimate the value of the indicated limit without graphing the function. Choose the last x -value so that it is no more than 0.001 units from the given c -value.

14. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$

15. $\lim_{x \rightarrow 0} x^x$

16. $\lim_{x \rightarrow 0} \frac{\sin 2x}{4x}$

17. $\lim_{x \rightarrow 0} \left(2x \sin \frac{1}{4x} \right)$

18. Use one-sided limit notation to describe the behavior of $f(x) = \frac{1}{x-1}$ near $x = 1$.

19–20 Find a $\delta > 0$ that satisfies the limit claim corresponding to $\varepsilon = 0.01$, that is, such that $0 < |x - c| < \delta$ would imply $|f(x) - L| < 0.01$.

19. $\lim_{x \rightarrow 0} (3 - 2x) = 3$

20. $\lim_{x \rightarrow 4} \sqrt{x} = 2$

21–24 Give the formal definition of the limit claim. Then use the definition to prove the claim.

21. $\lim_{x \rightarrow 1} (3x + 1) = 4$

22. $\lim_{x \rightarrow 1} x^2 = 1$

23. $\lim_{x \rightarrow 1} \sqrt{x} = 1$

24. $\lim_{x \rightarrow 2} \frac{2}{x} = 1$

25–41 Use algebra and/or appropriate limit laws to evaluate the given limit (one-sided limit where indicated). If the limit is unbounded, use the symbol ∞ or $-\infty$ in your answer.

25. $\lim_{x \rightarrow 3} (2x^2 - 3x + 5)$

26. $\lim_{x \rightarrow -2} \left(\frac{x^3}{4} + 2x^2 - x + 1 \right)$

27. $\lim_{x \rightarrow 3} \sqrt{x^3 + 2x^2 + 4}$

28. $\lim_{x \rightarrow -2} \frac{2x + 1}{x^2 - x}$

29. $\lim_{t \rightarrow 1} \left(\frac{3t + 5t^3}{t^2 + 1} \right)^{3/2}$

30. $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$

31. $\lim_{x \rightarrow -5} \frac{x + 5}{x^2 - 25}$

32. $\lim_{x \rightarrow 5^-} \frac{x + 5}{x^2 - 25}$

33. $\lim_{x \rightarrow 1^+} \frac{x^2 + 1}{x^4 - 1}$

34. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^4 - 1}$

35. $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

36. $\lim_{x \rightarrow 0} \frac{1}{2+x} - \frac{1}{2}$

37. $\lim_{x \rightarrow 0^+} \frac{2|x|}{x}$

38. $\lim_{x \rightarrow -2^+} \sqrt{4 - x^2}$

39. $\lim_{x \rightarrow 2^-} (\lfloor x \rfloor + 2x)$

40. $\lim_{x \rightarrow 1^+} \lfloor x \rfloor x$

41. If $f(x) = x^2$, find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

42–43 Use the Squeeze Theorem to prove the limit claim.

42. $\lim_{x \rightarrow 0} x \cos \frac{1}{x} = 0$

43. $\lim_{x \rightarrow \infty} \frac{\sin x}{\ln x} = 0$

44. Sketch a graph of a function (a formula is not necessary) that is not continuous at $x = 0$ from either direction, but both of its one-sided limits exist at $x = 0$. (Answers will vary.)

45. Sketch a graph of a function that is left-continuous at $x = 0$, but its right-hand limit at $x = 0$ doesn't exist. (Answers will vary.)

46–51 Find and classify the discontinuities (if any) of the given function as removable or nonremovable.

46. $f(x) = \frac{x-9}{\sqrt{x}-3}, \quad x \geq 0$

47. $g(x) = \frac{\sqrt{x}+2}{x-4}, \quad x \geq 0$

48. $h(x) = \frac{1}{\sqrt{x^2+1}}$

49. $t(x) = 2 + 2\lfloor x \rfloor$

50. $G(x) = \frac{x}{\sqrt{x+1}-1}, \quad x \geq -1$

51. $k(x) = |x-3| + |x+1|$

52–53 Use the ε - δ definition to prove that the function is continuous.

52. $f(x) = 3x - 1$

53. $g(x) = 2x^2$

54. Find the values of a and b such that f is continuous on the entire real line.

$$f(x) = \begin{cases} -1 & \text{if } x \leq -3 \\ ax + b & \text{if } -3 < x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$$

55. Use the Intermediate Value Theorem to prove that the equation $2x^5 + x + 1 = 0$ has a solution on the interval $[-1, 1]$.

56. Use the Intermediate Value Theorem to show that the graphs of $f(x) = x^3$ and $g(x) = e^{-x}$ intersect.

57–58 Find the equation of the tangent line to the graph of $f(x)$ at the given point.

57. $f(x) = x^2 + x; (1, 2)$

58. $f(x) = \sqrt{x}; (4, 2)$

59–60 Use the definition (also called the limit process) to find the derivative function f' of the given function f . Find all x -values (if any) where the tangent line is horizontal.

59. $f(x) = 2x - x^2$ 60. $f(x) = \frac{3}{x-2}$

61–62 Sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

61. f is continuous at 0, $f(0) = 1$, $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$, and $f'(0)$ does not exist
62. $g(1) < 0$, $g'(1) > 0$, and $g(2) > 0$, but $g'(2) < 0$
63. Prove that if $f(x)$ is a quadratic function, then $f'(x)$ is linear.
64. A small object is thrown upward with an initial velocity of 12 m/s from the top of a 15 m high building.
- How high does it go and when does it reach the ground?
 - What is the speed of impact?

(Hint: Use $h(t) = -5t^2 + 12t + 15$ as the position function, where h is in meters, t in seconds.)

65. The owner of a small toy manufacturer has determined that he can sell x toys if the price is $p = D(x) = 0.2x + 30$ dollars. The total cost as a function of x is given by $C(x) = 0.1x^2 + 15x + 247.5$ dollars.
- Find the profit function $P(x)$.
 - Find any break-even points.
 - Find the marginal profit function.

Concept Check

66–73 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

66. Instantaneous velocity can be interpreted as the slope of a tangent line.

67. If $\lim_{x \rightarrow c} f(x)$ doesn't exist, then $f(x)$ has a vertical asymptote at $x = c$.

68. Any rational function has at least one vertical asymptote.

69. If $\lim_{x \rightarrow c} f(x) = A$ and $\lim_{x \rightarrow c} g(x) = B$, then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{A}{B}$.

70. If f is defined on $[a, b]$, L is a real number between $f(a)$ and $f(b)$, and $\lim_{x \rightarrow c} f(x)$ exists for all $x \in (a, b)$, then there is a c in the interval (a, b) such that $f(c) = L$.

71. If f is continuous at c , then $f(c)$ is equal to both one-sided limits at c .

72. If both one-sided limits of f exist at c , and if f is defined at c , then f is continuous at c .

73. If $g(x) \leq f(x) \leq h(x)$ for all x in some open interval containing c , and if $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} h(x) = L$, then by the Squeeze Theorem $f(c) = L$ as well.

Chapter 2 Technology Exercises

74. Use a computer algebra system to find approximations for the areas in Exercises 10 and 11 by using $n = 100$. (Round your answers to four decimal places.)
75. Use a computer algebra system to find approximations for the arc lengths in Exercises 12 and 13 by using $n = 100$. (Round your answers to four decimal places.)
76. Use a graphing utility to verify your answers given for Exercises 14–17.
77. Use a graphing utility to approximate the solutions for Exercises 55 and 56. Round your answers to four decimal places.

78–81 Use a graphing utility to graph the function, and estimate from the graph the value of the given limit.

78. $\lim_{x \rightarrow \infty} x^{1/x}$

79. $\lim_{x \rightarrow 0} \frac{\arcsin x}{x}$

80. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{2x}$

81. $\lim_{x \rightarrow 1} \frac{\ln(x^3)}{x-1}$