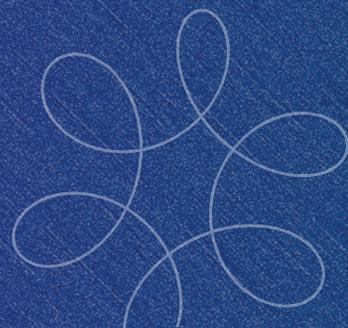


Chapter 5 Project



The topic of this project is the so-called *sine integral function*, which is important for its applications, most notably in electrical engineering and signal processing.

1. Consider the following piecewise defined function:

$$f(t) = \begin{cases} \frac{\sin t}{t} & \text{if } t > 0 \\ 1 & \text{if } t = 0 \end{cases}$$

Prove that for any $x \geq 0$, $f(t)$ is integrable on $[0, x]$.

2. The **sine integral function** is defined as follows:

$$\text{Si}(x) = \int_0^x f(t) dt, \text{ for } x \geq 0$$

Prove that $\text{Si}(x)$ is continuous.

3. Find the derivative $\frac{d}{dx} \text{Si}(x)$.

4. Without graphing first, write a short paragraph on why you would expect the graph of $\text{Si}(x)$ to be oscillating. Explain why its amplitude is expected to decrease as $x \rightarrow \infty$.
5. Find the x -values where the relative maxima and minima of $\text{Si}(x)$ occur.
6. Extend the definition of $\text{Si}(x)$ to negative x -values and prove that for any $a > 0$,
$$\int_{-a}^a \text{Si}(x) dx = 0.$$
7. Use a computer algebra system to plot the graph of $\text{Si}(x)$ on the interval $[-8\pi, 8\pi]$.
8. Use a computer algebra system to approximate the range of $y = \text{Si}(x)$ to four decimal places.