74–77 Use the Trapezoidal Rule with $n = 6$ to approximate the integral, and compare the result to the exact value of the integral by determining the absolute value of the error E_T.

74. $\int_1^4 x^{3/2}\,dx$

75. $\int_1^2 \frac{1}{x^2}\,dx$

76. $\int_0^4 \sqrt{x^2+1}\,dx$

77. $\int_0^{\pi/3} \tan x\,dx$

78–81. Use the error estimate for the Trapezoidal Rule to estimate $|E_T|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 74–77.

82–85. Use Simpson's Rule to approximate the integrals from Exercises 74–77 with $n = 6$. Determine the absolute value of the error E_S.

86–89. Use the error estimate for Simpson's Rule to estimate $|E_S|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 82–85.

90. Prove that if $f(x) = ax + b$ is a linear function on a closed interval $[a,b]$, then for any n, $T_n = \int_a^b f(x)\,dx$.

91–96 Identify the type of the improper integral and determine whether it is convergent or divergent. If it is convergent, find its value.

91. $\int_2^4 \frac{dx}{(x-2)^4}$

92. $\int_{-\infty}^0 x^2 e^x\,dx$

93. $\int_2^6 \frac{dx}{\sqrt{x-2}}$

94. $\int_0^\infty \frac{2}{(x+3)^{2/3}}\,dx$

95. $\int_{-\infty}^\infty \frac{2e^x}{e^{2x}+4}\,dx$

96. $\int_e^\infty \frac{dx}{x\ln x}$

97–98 Use the Direct Comparison Test to determine whether the integral converges.

97. $\int_0^\infty \frac{dx}{\sqrt{x^3+1}}$

98. $\int_1^\infty \frac{\ln x}{\sqrt{x}}\,dx$

99. Use substitution to turn the improper integral $\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ into a proper one and evaluate it.

100. Rotate about the x-axis the region bounded by the graph of $y = \sqrt{\ln x}/x^2$ and the x-axis over the interval $[1,\infty)$. Use the disk method to determine if the resulting unbounded solid has finite volume. If so, find the volume.

101.* Rotate about the x-axis the region bounded by the x-axis and the graph of $y = e^{-x}$ over the infinite interval $[0,\infty)$. Determine if the resulting infinite solid has finite volume or surface area. If so, find their values.

102. Prove that the improper integral $\int_2^\infty \frac{dx}{x(\ln x)^a}$ converges if and only if $a > 1$.

103–106 Find the Laplace transform. (See Exercises 88–93 in Section 7.7.)

103. $L\{te^{at}\}$

104.* $L\{t^2e^{at}\}$

105.* $L\{t\sin kt\}$

106.* $L\{t\cos kt\}$

Concept Check

107–113 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

107. If $\lim_{x\to\infty} f(x) = L$ and $L \neq 0$, then $\int_0^\infty f(x)\,dx$ diverges.

108. If $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

109. Let $f(x)$ be defined on $[a,\infty)$ and S be the solid generated by rotating the graph of $f(x)$ about the x-axis. If the surface area of S is infinite, then the volume of S is also infinite.

110. If $f(x)$ is an odd function, then $\int_{-\infty}^\infty f(x)\,dx = 0$.

111. If $f(x)$ has a vertical asymptote at $x = 0$ and $a > 0$, then $\int_{-a}^a f(x)\,dx$ diverges.

112. Any rational function is integrable on any finite interval that doesn't include a zero of the denominator.

113. If an integrand contains the expression $\sqrt{a^2 \pm x^2}$, a trigonometric substitution must be used to evaluate the integral.

Chapter 7
Technology Exercises

114. Use a graphing utility to find the integral from Exercise 33. If the answer appears different from what you obtained by hand, prove that the answers are equivalent.

115. Write a program for a computer algebra system or programmable calculator that evaluates the trapezoidal approximation T_n for a given input function on a specified interval and positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal.

116. Use the program you wrote for Exercise 115 for the integral from Exercise 75.

117. Write a program for a computer algebra system or programmable calculator that evaluates the Simpson approximation S_n for a given integral and an even positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal. Compare this with your answer for Exercise 115. What n ensures that the answer is correct to at least five decimal places?

118. Use the program you wrote for Exercise 117 for the integral from Exercise 75.

119–120 Use the programs you wrote for Exercises 115 and 117 to approximate the given nonelementary integral with $n = 50$. Which method (the Trapezoidal Rule or Simpson's Rule) do you expect to be more accurate? Use the built-in numerical integration command of your technology to verify your conjecture.

119. $\int_0^1 \sqrt{1+x^4}\, dx$

120. $\int_0^1 e^{e^x}\, dx$