56. Use the Intermediate Value Theorem to show that the graphs of $f(x) = x^3$ and $g(x) = e^{-x}$ intersect.

57–58 Find the equation of the tangent line to the graph of $f(x)$ at the given point.

57. $f(x) = x^2 + x; \quad (1,2)$

58. $f(x) = \sqrt{x}; \quad (4,2)$

59–60 Use the definition (also called the limit process) to find the derivative function f' of the given function f. Find all x-values (if any) where the tangent line is horizontal.

59. $f(x) = 2x - x^2$ **60.** $f(x) = \dfrac{3}{x-2}$

61–62 Sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

61. f is continuous at 0, $f(0) = 1$, $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$, and $f'(0)$ does not exist

62. $g(1) < 0$, $g'(1) > 0$, and $g(2) > 0$, but $g'(2) < 0$

63. Prove that if $f(x)$ is a quadratic function, then $f'(x)$ is linear.

64. A small object is thrown upward with an initial velocity of 12 m/s from the top of a 15 m high building.

a. How high does it go and when does it reach the ground?

b. What is the speed of impact?

(**Hint:** Use $h(t) = -5t^2 + 12t + 15$ as the position function, where h is in meters, t in seconds.)

65. The owner of a small toy manufacturer has determined that he can sell x toys if the price is $p = D(x) = 0.2x + 30$ dollars. The total cost as a function of x is given by $C(x) = 0.1x^2 + 15x + 247.5$ dollars.

a. Find the profit function $P(x)$.

b. Find any break-even points.

c. Find the marginal profit function.

Concept Check

66–73 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

66. Instantaneous velocity can be interpreted as the slope of a tangent line.

67. If $\lim_{x \to c} f(x)$ doesn't exist, then $f(x)$ has a vertical asymptote at $x = c$.

68. Any rational function has at least one vertical asymptote.

69. If $\lim_{x \to c} f(x) = A$ and $\lim_{x \to c} g(x) = B$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{A}{B}$.

70. If f is defined on $[a,b]$, L is a real number between $f(a)$ and $f(b)$, and $\lim_{x \to c} f(x)$ exists for all $x \in (a,b)$, then there is a c in the interval (a,b) such that $f(c) = L$.

71. If f is continuous at c, then $f(c)$ is equal to both one-sided limits at c.

72. If both one-sided limits of f exist at c, and if f is defined at c, then f is continuous at c.

73. If $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, and if $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$, then by the Squeeze Theorem $f(c) = L$ as well.

Chapter 2
Technology Exercises

74. Use a computer algebra system to find approximations for the areas in Exercises 10 and 11 by using $n = 100$. (Round your answers to four decimal places.)

75. Use a computer algebra system to find approximations for the arc lengths in Exercises 12 and 13 by using $n = 100$. (Round your answers to four decimal places.)

76. Use a graphing utility to verify your answers given for Exercises 14–17.

77. Use a graphing utility to approximate the solutions for Exercises 55 and 56. Round your answers to four decimal places.

78–81 Use a graphing utility to graph the function, and estimate from the graph the value of the given limit.

78. $\lim_{x \to \infty} x^{1/x}$ **79.** $\lim_{x \to 0} \dfrac{\arcsin x}{x}$

80. $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^{2x}$ **81.** $\lim_{x \to 1} \dfrac{\ln(x^3)}{x-1}$