95. $\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma$, where $\mathbf{F}=\left\langle x-\frac{yz^2}{4},3x^2z,z-y^2\right\rangle$ and S is the cone frustum $z=2\sqrt{x^2+y^2}$ with $2\le z\le 6$, oriented with an inward-pointing unit normal vector field

96–97 Verify the Divergence Theorem by showing the equality of the integrals $\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma$ and $\iiint_D \nabla\cdot\mathbf{F}\,dV$ for the given vector field **F** on the solid *D*.

96. $\mathbf{F}(x,y,z)=\left\langle 3z,\frac{y}{4},-2x\right\rangle$, where D is the ball of radius 3 centered at the origin

97. $\mathbf{F}(x,y,z)=\langle 3xy,-x,2z\rangle$, where D is the solid bounded by the paraboloid $z=1-x^2-y^2$ and the xy-plane

98–103 Use the Divergence Theorem to find the flux of the vector field **F** over the surface of the given solid *D*. Consider cylindrical or spherical coordinates where appropriate.

98. $\mathbf{F}(x,y,z)=\langle -x^2y,yz,5z\rangle$, where D is the solid bounded by the parabolic cylinder $z=4-y^2$, the xy-plane, and the planes $x=0$ and $x=2$

99. $\mathbf{F}(x,y,z)=\langle 5x(z-y),z^2+\cos x,y\sin x\rangle$, where D is the solid cylinder bounded by $x^2+y^2=9$, the xy-plane, and the plane $z=3$

100. $\mathbf{F}(x,y,z)=\left\langle 2xy,y-z,\frac{x^2y}{3}\right\rangle$, where D is the tetrahedron with vertices at the origin, $(6,0,0)$, $(0,3,0)$, and $(0,0,4)$

101. $\mathbf{F}(x,y,z)=\langle 2x+z,x-y^3,2z\rangle$, where D is the solid inside the cylinder $x^2+y^2=4$, bounded by the xy-plane and $z=3-y$

102. $\mathbf{F}(x,y,z)=\langle -xy^2,3yz^2,y^3\rangle$, where D is the solid cylindrical shell $2\le x^2+y^2\le 4$ between the planes $z=1$ and $z=6$

103. $\mathbf{F}(x,y,z)=\left\langle \frac{yz^2}{2},\frac{y^3}{8},x(z-x)\right\rangle$, where D is the portion of the solid cone $z=\sqrt{x^2+y^2}$ between the planes $z=2$ and $z=4$

104. Write a paragraph discussing the relationships and analogies between the Fundamental Theorem of Calculus, the Fundamental Theorem for Line Integrals, Green's and Stokes' Theorems, and the Divergence Theorem.

Concept Check

105–112 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

105. If a vector field **F** is constant, then $\nabla\times\mathbf{F}=\mathbf{0}$.

106. If a vector field **F** is constant, then $\nabla\cdot\mathbf{F}=0$.

107. A vector field is conservative if and only if its curl is zero.

108. The Fundamental Theorem for Line Integrals can only be used if the underlying vector field is conservative.

109. If a force field **F** is conservative, then the work done by **F** on a particle moving along a smooth path is zero.

110. Green's Theorem and Stokes' Theorem are unrelated.

111. When a charged particle moves along a piecewise smooth closed curve in an electric force field, the total work done by the force field is zero.

112. If the vector field **F** has continuous partials in an open neighborhood of the closed, piecewise smooth surface S, then $\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma=0$.

Chapter 15 Technology Exercises

113–116 Use a graphing utility to find the mass and center of mass of the solid with the given density.

113. The solid bounded by the cone $z=\sqrt{4-x^2-y^2}$ and the xy-plane, with constant density ρ

114. The solid bounded by the paraboloid $z=4-x^2-y^2$ and the xy-plane, with constant density ρ

115. The solid of Exercise 114, with its density at any point being proportional to the distance from the xy-plane

116. The solid of Exercise 114, with its density at any point being proportional to the distance from the z-axis