

122. If $f(x, y) = \frac{g(x)}{h(y)}$, then

$$f_x(x, y) + f_y(x, y) = \frac{g'(x)h(y) - g(x)h'(y)}{h^2(y)}.$$

123. If $f_x(a, b)$ and $f_y(a, b)$ both exist, then $f(x, y)$ is differentiable at (a, b) .

124. If $f_x(x, y)$ and $f_y(x, y)$ are continuous at (a, b) , then $f(x, y)$ is differentiable at (a, b) .

125. If f has a relative maximum or minimum at (a, b) , then all directional derivatives $D_u f(a, b)$ are 0.

126. If f has a relative maximum or minimum at (a, b) , then the tangent plane to its graph at (a, b) is horizontal.

Chapter 13

Technology Exercises

127–130. Write a program on your computer algebra system or programmable calculator that accepts the equation of a surface and a point on the surface, and generates the corresponding tangent plane and normal line to the surface. Test your program by checking the answers you obtained for Exercises 82–85.

131–134. Write a program on your computer algebra system or programmable calculator that accepts a two-variable function $f(x, y)$ and then locates and classifies its critical points by utilizing the Second Partial Derivative Test. Use your program to check the answers you obtained for Exercises 93–96.

135. Recall the Ideal Gas Law, $pV = nRT$ from Section 3.4, Exercise 97. Suppose $\frac{1}{2}$ mole of an ideal gas is present in an expandable container. Assuming the volume of the container can expand up to 1 m^3 , and that it can withstand a maximum pressure of 120 kPa along with temperatures of up to 500 K, use a computer algebra system to generate a contour map that illustrates how the volume depends on pressure when temperature is kept at constant levels. (Note that the resulting contour lines are called *isotherms*. **Hint:** Start by expressing T as a function of p and V from the Ideal Gas Law.)

136. Mimic Exercise 135 to generate a few contour lines of the pressure function under the same conditions (these curves are called *isobars*).