We will conclude this section with an example of rotation of conics in polar form. In general, the graph of an equation $r = f(\theta - \varphi)$ is the rotation of the graph of $r = f(\theta)$ by the angle $\theta$ counterclockwise. This makes rotation in polar coordinates particularly easy to handle.

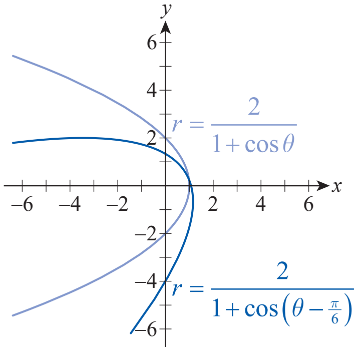



Figure 6

### Example 5 Graphing a Rotated Conic Section in Polar Form

Sketch the graph of the conic section $r = \dfrac{2}{1+\cos\left(\theta - \dfrac{\pi}{6}\right)}$.

**Solution**

We constructed the equation $r = 2/(1+\cos\theta)$ in Example 2, so we know its graph is a parabola opening to the left with directrix $x = 2$.

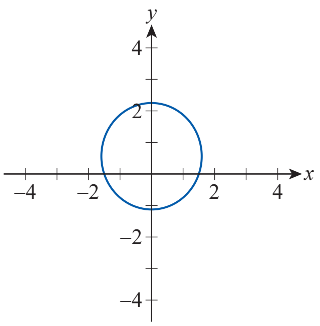
The graph of $r = \dfrac{2}{1+\cos\left(\theta - \dfrac{\pi}{6}\right)}$ is the same shape rotated $\pi/6$ radians counterclockwise, as shown in Figure 6.

## 9.6 Exercises

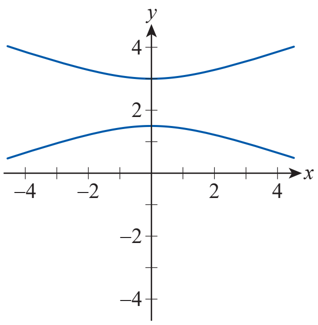
**1-6** Match the given polar equation with its graph (labeled A–F).

1. $r = \dfrac{3}{4-\cos\theta}$
2. $r = \dfrac{9}{6-2\sin\theta}$
3. $r = \dfrac{3}{3+4\sin\theta}$
4. $r = \dfrac{1}{2+2\cos\theta}$
5. $r = \dfrac{6}{1+3\sin\theta}$
6. $r = \dfrac{6}{1+3\cos\theta}$

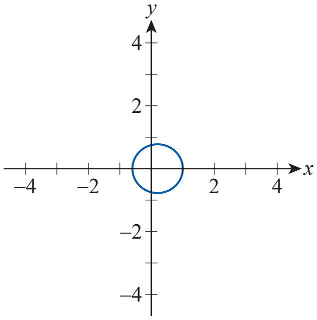
**A.**



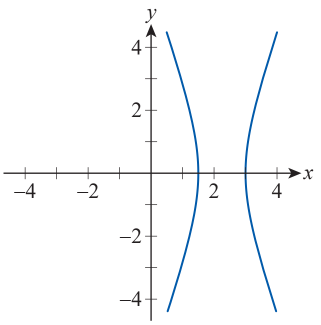
**B.**



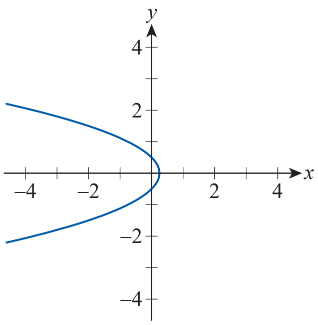
**C.**



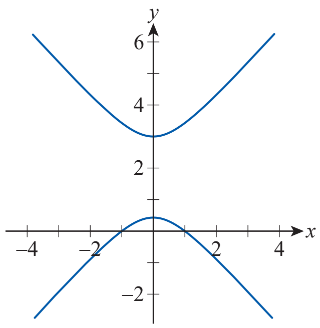
**D.**



**E.**



**F.**



**7–20** Identify the given conic section as an ellipse, parabola, or hyperbola and find the equation for its directrix.

**7.** $r = \dfrac{7}{1+6\sin\theta}$

**8.** $r = \dfrac{2}{1-\sin\theta}$

**9.** $r = \dfrac{3}{4-\cos\theta}$

**10.** $r = \dfrac{4}{2-2\cos\theta}$

**11.** $r = \dfrac{1}{1+3\cos\theta}$

**12.** $r = \dfrac{7}{3+2\sin\theta}$

**13.** $r = \dfrac{5}{2+\cos\theta}$

**14.** $r = \dfrac{3}{4-3\sin\theta}$

**15.** $r = \dfrac{6}{3-5\cos\theta}$

**16.** $r = \dfrac{8}{5-6\sin\theta}$

**17.** $r = \dfrac{3}{2+2\sin\theta}$

**18.** $r = \dfrac{-1}{3+4\cos\theta}$

**19.** $r = \dfrac{4}{6-7\cos\theta}$

**20.** $r = \dfrac{9}{5-4\sin\theta}$

**21–26** Construct a polar equation for the conic section with the focus at the origin and the given eccentricity and directrix.

**21.** Parabola; eccentricity: $e = 1$; directrix: $x = -2$

**22.** Hyperbola; eccentricity: $e = 2$; directrix: $x = -3$

**23.** Hyperbola; eccentricity: $e = 4$; directrix: $y = -\frac{3}{4}$

**24.** Parabola; eccentricity: $e = 1$; directrix: $x = 2$

**25.** Ellipse; eccentricity: $e = \frac{1}{4}$; directrix: $x = 12$

**26.** Ellipse; eccentricity: $e = \frac{1}{2}$; directrix: $y = 8$

**27–36** Sketch the graph of the conic section.

**27.** $r = \dfrac{5}{1+3\cos\theta}$

**28.** $r = \dfrac{3}{2+\sin\theta}$

**29.** $r = \dfrac{4}{1-2\sin\theta}$

**30.** $r = \dfrac{6}{2-4\cos\theta}$

**31.** $r = \dfrac{9}{3-2\cos\theta}$

**32.** $r = \dfrac{5}{3+\sin\theta}$

**33.** $r = \dfrac{4}{1+2\cos\theta}$

**34.** $r = \dfrac{4}{2+2\sin\theta}$

**35.** $r = \dfrac{2}{1+\cos\left(\theta-\dfrac{\pi}{4}\right)}$

**36.** $r = \dfrac{4}{2+2\sin\left(\theta-\dfrac{\pi}{3}\right)}$

**37.** The planets of our solar system follow elliptical orbits with the sun located at one of the foci. If we assume that the sun is located at the pole and the major axes of these elliptical orbits lie along the polar axis, the orbits of the planets can be expressed by the polar equation

$$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$$

where $e$ is the eccentricity. Verify the above equation.

**38.** Using the equation from Exercise 37, answer the following exercises.

**a.** Show that the shortest distance from the sun to a planet, called the *perihelion* distance, is $r = a(1-e)$.

**b.** Show that the longest distance from the sun to a planet, called the *aphelion* distance, is $r = a(1+e)$.

**c.** The distance from Uranus to the sun is approximately $2.74 \times 10^9$ km at perihelion and $3.00 \times 10^9$ km at aphelion. Find the eccentricity of Uranus' orbit.

**d.** The eccentricity of Neptune's orbit is 0.0113 and $a = 4.495 \times 10^9$ km. Determine the perihelion and aphelion distances for Neptune.

**39.** Derive the polar form of the equation of a conic with vertical directrix $x = -d$ and focus at the origin.

**40.** Derive the polar form of the equation of a conic with horizontal directrix **a.** $y = d$, **b.** $y = -d$, and focus at the origin.

**41.** A chord through a focus of a conic section that is parallel to the directrix is called its *latus rectum* (from the Latin words "latus," meaning "side," and "rectum," meaning "straight"). Find the length of the latus rectum for the conic $r = ed/(1+e\cos\theta)$.

**42.** Find the polar coordinates of the vertices for the ellipse with polar equation $r = ed/(1+e\cos\theta)$, $0 < e < 1$.

**43.** Find the polar coordinates of the vertices for the hyperbola with polar equation $r = ed/(1+e\cos\theta)$, $e > 1$.

**44.** Use Exercise 42 to find the rectangular equation of the ellipse $r = 12/(5+\cos\theta)$.

**45.** Use Exercise 43 to find the rectangular equation of the hyperbola $r = 12/(5+7\cos\theta)$.

# 9.6 Technology Exercises

**46–55** Use a graphing utility to graph the conic section.

**46.** $r = \dfrac{-3}{4 - 9\cos\theta}$

**47.** $r = \dfrac{9}{-4 + \frac{3}{2}\sin\theta}$

**48.** $r = \dfrac{-11}{3 - \cos\theta}$

**49.** $r = \dfrac{2}{10 + 4\sin\theta}$

**50.** $r = \dfrac{3}{7 + 3\cos\theta}$

**51.** $r = \dfrac{2}{2 + 3\cos\left(\theta - \frac{\pi}{4}\right)}$

**52.** $r = \dfrac{-7}{5 + 3\sin\left(\theta - \frac{\pi}{6}\right)}$

**53.** $r = \dfrac{5}{-2 - 4\sin\left(\theta + \frac{2\pi}{3}\right)}$

**54.** $r = \dfrac{4}{-3 - 2\cos\left(\theta + \frac{\pi}{3}\right)}$

**55.** $r = \dfrac{1}{1 + 4\sin\left(\theta + \frac{\pi}{6}\right)}$

**56.** Use technology to sketch the conic section $r = ed/(1 + e\cos\theta)$ for various values of $d$ and $e$, $e > 0$, and examine how these values affect the shape of the graph.