9.5 Exercises

1–6 Use the discriminant to determine whether the given equation represents an ellipse, a parabola, or a hyperbola.

1. $y^2 + 2y + 12x + 13 = 0$

2. $2x^2 + 12x - y^2 - 2y + 9 = 0$

3. $4x^2 + 3y^2 + 18y + 19 = 8x$

4. $-2x^2 - 8xy + 2y^2 + 2y + 5 = 0$

5. $3x^2 - 6xy + 3y^2 + 3x - 9 = 0$

6. $x^2 - xy + 4y^2 + 2x - 3y + 1 = 0$

7–21 Identify the type of conic section defined by the equation and match the equation with its graph (labeled A–O).

7. $\dfrac{(x-1)^2}{4} + \dfrac{y^2}{16} = 1$

8. $(x-2)^2 = 4y$

9. $x^2 - y^2 = 1$

10. $x^2 + \dfrac{(y-3)^2}{4} = 1$

11. $\dfrac{y^2}{4} - (x-1)^2 = 1$

12. $\dfrac{(x-3)^2}{9} + \dfrac{(y-2)^2}{25} = 1$

13. $\dfrac{x^2}{9} - \dfrac{(y+2)^2}{4} = 1$

14. $(x+2)^2 = 3(y-1)$

15. $\dfrac{(x-1)^2}{4} + y^2 = 1$

16. $\dfrac{(x-3)^2}{4} - \dfrac{(y+1)^2}{9} = 1$

17. $y^2 = 4(x+1)$

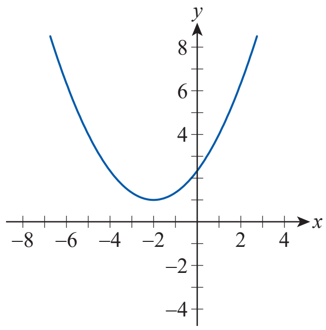
18. $(x-1)^2 = -(y-2)$

19. $(y-1)^2 = -2(x-2)$

20. $\dfrac{(x+2)^2}{9} + \dfrac{(y-1)^2}{4} = 1$

21. $(y+2)^2 - \dfrac{(x-2)^2}{4} = 1$

A.



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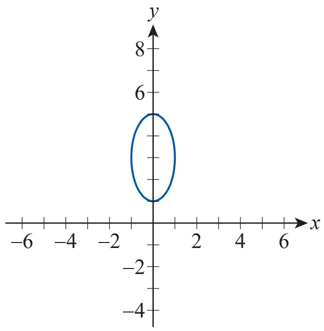

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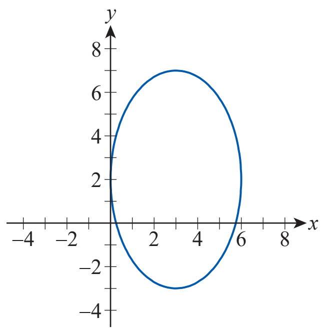

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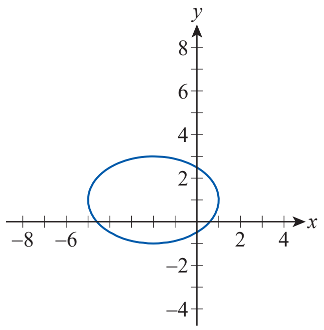

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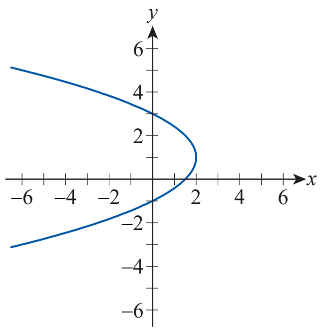

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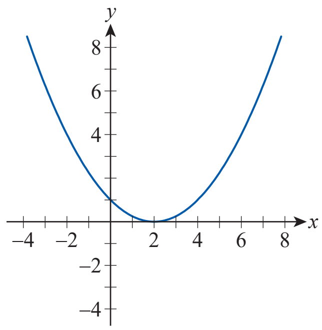

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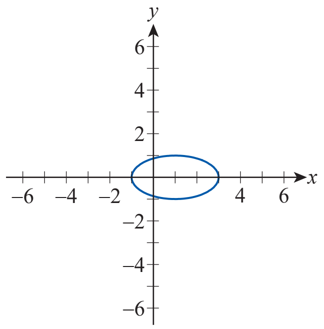

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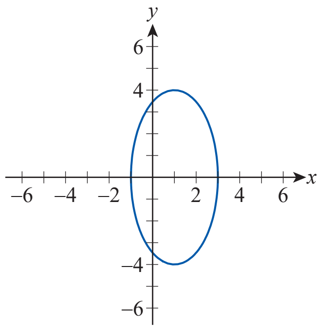
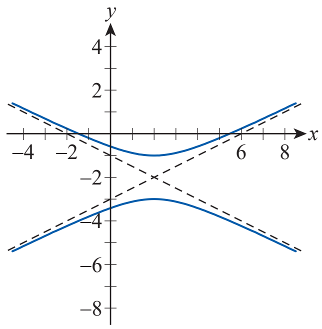


I.



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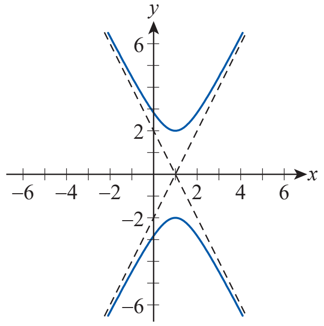

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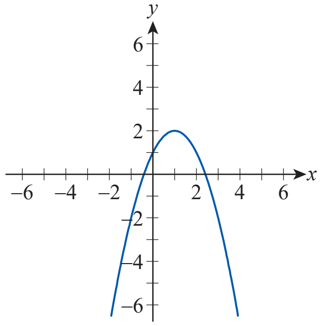

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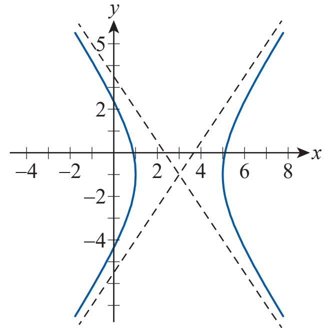

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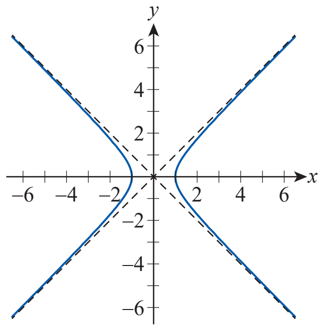

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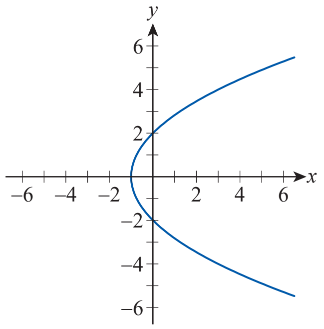

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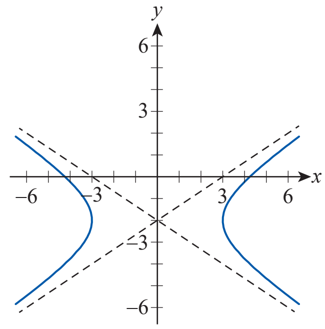


22–25 Find the eccentricity and the lengths of the major and minor axes of the ellipse.

22. $x^2 + 9y^2 = 36$

23. $5x^2 + 8y^2 = 40$

24. $20x^2 + 10y^2 = 40$

25. $\frac{1}{4}x^2 + \frac{1}{12}y^2 = \frac{1}{2}$

26–35 Graph the ellipse and determine the coordinates of the foci.

26. $\frac{(x-4)^2}{16} + \frac{(y-4)^2}{4} = 1$

27. $\frac{(x+2)^2}{16} + \frac{(y+1)^2}{9} = 1$

28. $9x^2 + 16y^2 + 18x - 64y = 71$

29. $9x^2 + 4y^2 - 36x - 24y + 36 = 0$

30. $16x^2 + y^2 + 160x - 6y = -393$

31. $25x^2 + 4y^2 - 100x + 8y + 4 = 0$

32. $4x^2 + 9y^2 + 40x + 90y + 289 = 0$

33. $16x^2 + y^2 - 64x + 6y + 57 = 0$

34. $4x^2 + y^2 + 4y = 0$

35. $9x^2 + 4y^2 + 108x - 32y = -352$

36–45 Graph the parabola and determine its focus and directrix.

36. $(x+1)^2 = 4(y-3)$

37. $(y-4)^2 = -2(x-1)$

38. $y^2 + 2y + 12x + 37 = 0$

39. $x^2 - 8y = 6x - 1$

40. $x^2 + 6x + 8y = -17$

41. $x^2 + 2x + 8y = 31$

42. $y^2 + 6y - 2x + 13 = 0$

43. $x^2 - 2x - 4y + 13 = 0$

44. $4y + 2x^2 = 4$

45. $2y^2 - 10x = 10$

46–55 Graph the hyperbola, using asymptotes as guides, and determine the coordinates of the foci.

46. $\frac{(x+3)^2}{4} - \frac{(y+1)^2}{9} = 1$

47. $4y^2 - x^2 - 24y + 2x = -19$

48. $x^2 - 9y^2 + 4x + 18y - 14 = 0$

49. $9x^2 - 25y^2 = 18x - 50y + 241$

50. $9x^2 - 16y^2 + 116 = 36x + 64y$

51. $\frac{(y-1)^2}{9} - (x+3)^2 = 1$

52. $9y^2 - 25x^2 - 36y - 100x = 289$

53. $9x^2 + 18x = 4y^2 + 27$

54. $9x^2 - 16y^2 - 36x + 32y - 124 = 0$

55. $x^2 - y^2 + 6x - 6y = 4$

56–73 Find the equation, in standard form, for the conic with the given properties or with the given graph.

56. Ellipse, center at $(-2,3)$, horizontal major axis of length 8, minor axis of length 4

57. Parabola, focus at $(-2,1)$, directrix is the x-axis

58. Ellipse, vertices at $(5,-1)$ and $(1,-1)$, minor axis of length 2

59. Hyperbola, foci at $(1,5)$ and $(1,-1)$, vertices at $(1,3)$ and $(1,1)$

60. Parabola, focus at $\left(-3,-\frac{3}{2}\right)$, directrix is the line $y=-\frac{1}{2}$

61. Hyperbola, foci at $(-1,3)$ and $(-1,-1)$, asymptotes given by $y=\pm(x+1)+1$

62. Parabola, vertex at $(-4,3)$, focus at $\left(-\frac{3}{2},3\right)$

63. Ellipse, foci at $(0,0)$ and $(6,0)$, $e=\frac{1}{2}$

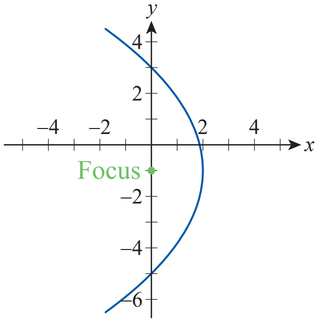
64. Hyperbola, asymptotes given by $y=\pm(2x+8)+3$, vertices at $(-6,3)$ and $(-2,3)$

65. Ellipse, vertices at $(-4,6)$ and $(-14,6)$, $e=\frac{2}{5}$

66. Hyperbola, foci at $(2,4)$ and $(-2,4)$, asymptotes given by $y=\pm 3x+4$

67. Parabola, symmetric with respect to the line $y=1$, directrix is the line $x=2$, and $p=-3$

68.



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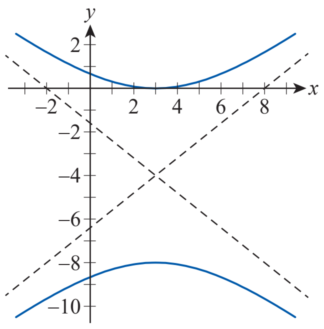

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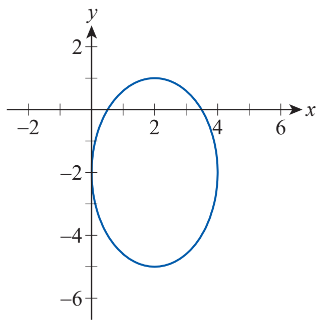

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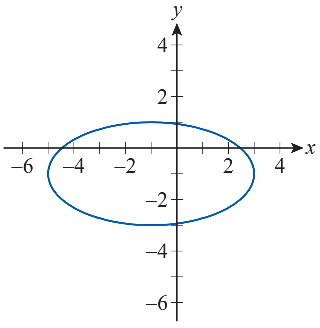

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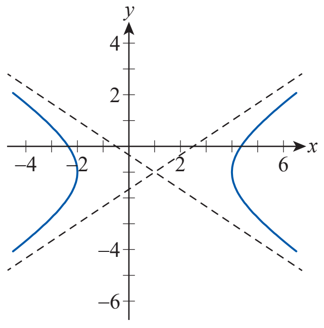

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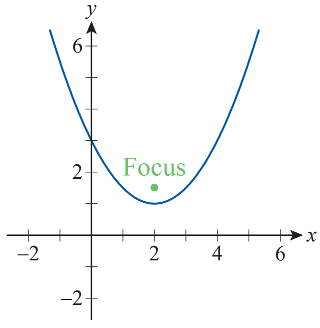
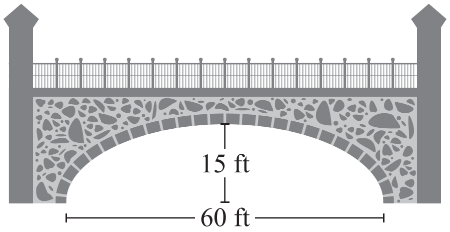


74. The orbit of Halley's Comet is an ellipse with the sun at one focus and an eccentricity of 0.967. Its closest approach to the sun is approximately 54,591,000 miles. What is the furthest Halley's Comet ever gets from the sun?

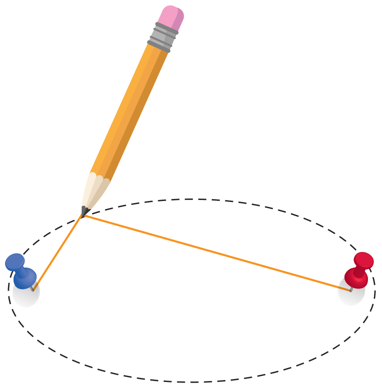
75. Pluto's closest approach to the sun is approximately 4.43×10^9 kilometers, and its maximum distance from the sun is approximately 7.37×10^9 kilometers. What is the eccentricity of Pluto's orbit?

76. The archway supporting a bridge over a river is in the shape of half an ellipse. The archway is 60 feet wide and is 15 feet tall at the middle. A large boat is 10 feet wide and 14 feet 9 inches tall. Is the boat capable of passing under the archway?



77. Use the information given in Example 2 to determine the length of the minor axis of the ellipse formed by Earth's orbit around the sun.

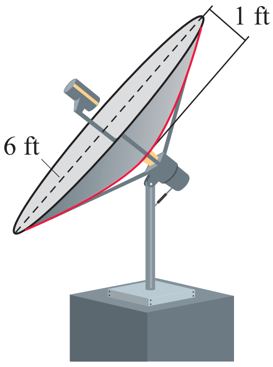
78. Since the sum of the distances from each of the two foci to any point on an ellipse is constant, we can draw an ellipse using the following method. Tack the ends of a length of string at two points (the foci) and, keeping the string taut by pulling outward with the tip of a pencil, trace around the foci to form an ellipse (the total length of the string remains constant). If you want to create an ellipse with a major axis of length 5 cm and a minor axis of length 3 cm, how long should your string be and how far apart should you place the tacks?



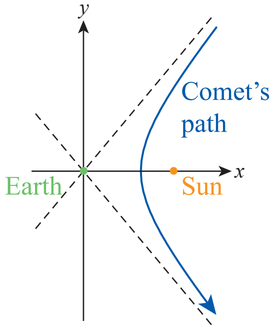
79. One design for a solar furnace is based on the paraboloid formed by rotating the parabola $x^2 = 8y$ around its axis of symmetry. The object to be heated in the furnace is then placed at the focus of the paraboloid (assume that x and y are in units of feet). How far from the vertex of the paraboloid is the hottest part of the furnace?

80. A certain satellite dish antenna is a paraboloid with a diameter of 6 feet and a depth of 1 foot. How far from the vertex of the dish should the receiver of the antenna be placed, given that the receiver should be located at the focus of the paraboloid?

81. A spotlight is made by placing a strong lightbulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 6y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?



82. As mentioned in this section, some comets trace one branch of a hyperbola through the solar system, with the sun at one focus. Suppose a comet is spotted that appears to be headed straight for Earth as shown in the figure. As the comet gets closer, however, it becomes apparent that it will pass between Earth, which lies at the center of the hyperbolic path of the comet, and the sun. In the end, the closest the comet comes to Earth is 60,000,000 miles. Using an estimate of 94,000,000 miles for the distance from Earth to the sun, and positioning Earth at the origin of a coordinate system, find the equation for the path of the comet.



83. Placing the foci at $(-c, 0)$ and $(c, 0)$ and introducing $d_1 + d_2 = 2a$, derive the standard form of the equation of an ellipse.

84. Denoting $|d_1 - d_2|$ by $2a$, use the approach suggested by Exercise 83 to derive the standard equation of a hyperbola.

85. Suppose two LORAN (LOng RAnge Navigation) radio transmitters are 26 miles apart. A ship at sea receives signals sent simultaneously from the two transmitters and is able to determine that the difference between the distances from the ship to each of the transmitters is 24 miles. By positioning the two transmitters on the y-axis, each 13 miles from the origin, find the equation of the hyperbola that describes the set of possible locations for the ship. (**Hint:** See Exercise 84.)

86.* Three high-sensitivity microphones are located in a forest preserve, with microphone A two miles due north of microphone B and microphone C two miles due east of microphone B. During an early morning thunderstorm, microphone A detects a thunderclap (and possible lightning strike) at 3:28:15 a.m. The same thunderclap is detected by microphone B at 3:28:19 a.m. and by microphone C at 3:28:25 a.m. Assuming that sound travels at 1100 feet per second, graphically approximate the source of the thunderclap. (**Hint:** Place microphone B at the origin, with A and C on the y- and x-axes, respectively. Then, by a repeated application of the method used in Exercise 85, construct two intersecting hyperbolas to locate the thunderclap.)

87–90 Find the $x'y'$-coordinates of the point for the given rotation angle θ.

87. $(8,6);\quad \theta = 30°$

88. $(-5,1);\quad \theta = \frac{\pi}{3}$

89. $\left(-\frac{1}{2},-\frac{1}{8}\right);\quad \theta = \frac{\pi}{4}$

90. $(-1,1);\quad \theta = \frac{\pi}{2}$

91–96 Use the discriminant to classify the conic section as an ellipse, parabola, or hyperbola. Then determine the appropriate angle θ by which to rotate the coordinate axes, and use that angle to convert the equation by eliminating the xy-term. Finally, sketch the graph of the conic section.

91. $xy - 4 = 0$

92. $x^2 + 2xy + y^2 - x + y = 0$

93. $7x^2 + 5\sqrt{3}xy + 2y^2 = 14$

94. $22x^2 + 6\sqrt{3}xy + 16y^2 - 49 = 276$

95. $2\sqrt{3}x^2 - 6xy + \sqrt{3}x - 9y = 0$

96. $34x^2 + 8\sqrt{3}xy + 42y^2 = 1380$

97–100 The given equation is that of a rotated conic. Match the equation with its graph (labeled A–D).

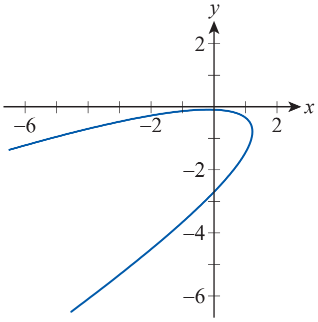
97. $3x^2 + 2xy + y^2 - 10 = 0$

98. $x^2 - 4xy + 4y^2 + 5\sqrt{5}y + 1 = 0$

99. $3x^2 + 8xy + 4y^2 - 7 = 0$

100. $x^2 - 6xy + 9y^2 - 2y + 1 = 0$

A.



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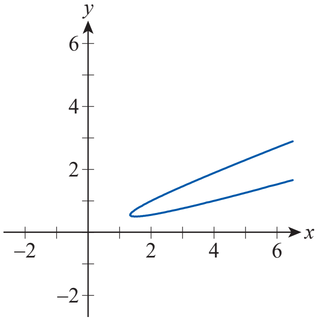

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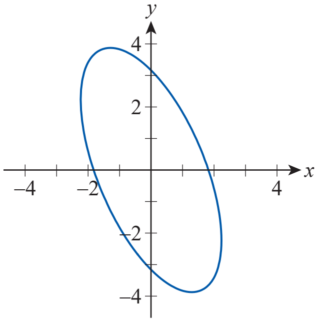

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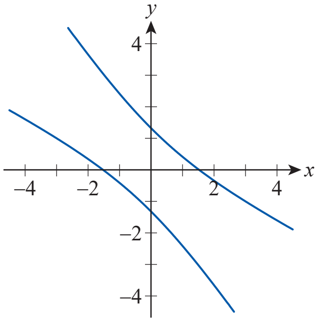

101. You have just used the rotation of axes to rotate the x- and y-axes until they were parallel to the axes of the conic. The resulting equation in the $x'y'$-plane is of the form

$$A'x'^2 + B'x'y' + E'y' + F' = 0,$$

where A', B', E', and F' are all nonzero. What is wrong with the resulting equation?

102. What must the angle of rotation θ be if the coefficients of x^2 and y^2 are equal and $B \neq 0$? Support your answer.

103. An expression involving the coefficients of the general form of a conic section is said to be *rotation invariant* if it has the same value under every possible rotation. Using the equation $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$,

a. show that the relationship $F = F'$ is true (so F is rotation invariant in this equation);

b. show that the relationship $A + C = A' + C'$ is true (so $A + C$ is rotation invariant in this equation);

c. show that the relationship $B^2 - 4AC = B'^2 - 4A'C'$ is true (so $B^2 - 4AC$ is rotation invariant in this equation).

104.* Show in general that the quantity $A + C$ and the discriminant $B^2 - 4AC$ are rotation invariant. (See Exercise 103.)

Concept Check

105–108 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

105. It is possible for a parabola to be tangent to its directrix.

106. The graph of $Ax^2 + Cy^2 + Dx + Ey = 0$ is a hyperbola if A and C have different signs and $D, E \neq 0$.

107. It is not possible for a line tangent to a hyperbola to have more than one point in common with the graph.

108. If the eccentricity of an ellipse is greater than 1, the ellipse is extremely narrow.

9.5 Technology Exercises

109–118 Use a graphing utility to sketch the given curve.

109. $15x^2 + 9y^2 + 150x - 36y = -276$

110. $5x^2 + 12y^2 - 20x + 144y + 392 = 0$

111. $x^2 - 6x + 12y + 21 = 0$

112. $x^2 - 5y^2 = 14x + 20y - 4$

113. $x^2 + 6xy + y^2 = 18$

114. $x^2 - 4xy + 3y^2 = 12$

115. $36x^2 - 19xy + 8y^2 = 72$

116. $72x^2 + 19xy + 4y^2 = 20$

117. $40x^2 + 20xy + 10y^2 + \left(2\sqrt{2} - 6\right)x - \left(4\sqrt{2} + 8\right)y = 90$

118. $72x^2 + 18xy - 9y^2 = 14$

119–120 Use a graphing utility to sketch the given curve. Explore how different values of the parameters k_1 and k_2 affect the graph. Experiment with both nonnegative and negative values. (Answers will vary.)

119. $k_1x^2 + k_2xy + 5y^2 - 6x + 7y + 15 = 0$

120. $k_1x^2 - 4xy + k_2y^2 + 2x + 3y - 1 = 0$