# 7.5 Exercises

**1–20** Use the integration guidelines listed in this section to evaluate the given indefinite or definite integral. (**Hint:** Whenever possible, try to simplify before integrating.)

1. $\displaystyle\int \frac{dx}{5x(x+5)}$

2. $\displaystyle\int \frac{2x}{(2x+3)^2}\,dx$

3. $\displaystyle\int \frac{2\,dx}{x\sqrt{x^2+4}}$

4. $\displaystyle\int \sin^2 x \sec^2 x\,dx$

5. $\displaystyle\int \frac{\tan^2 2x+1}{\csc^2 2x}\,dx$

6. $\displaystyle\int \frac{\cos^2\theta\csc^2\theta}{\tan\theta}\,d\theta$

7. $\displaystyle\int \frac{dx}{x^2+4x+8}$

8. $\displaystyle\int \frac{3x^3-12x^2+15x+2}{x^2-4x+5}\,dx$

9. $\displaystyle\int \frac{x\left(\sqrt{x}+2\right)}{\sqrt{x}}\,dx$

10. $\displaystyle\int \frac{1-\cos 2x}{\sin x}\,dx$

11. $\displaystyle\int \frac{x}{\sqrt{4-x^4}}\,dx$

12. $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

13. $\displaystyle\int \sqrt{x^2+2x+5}\,dx$

14. $\displaystyle\int \frac{\sqrt{4t+1}}{4t+5}\,dt$

15. $\displaystyle\int_{\sqrt{3}/3}^{\sqrt{3}} \frac{\arctan u}{u^2}\,du$

16. $\displaystyle\int \sqrt{e^2+e^x}\,dx$

17. $\displaystyle\int_{\pi/4}^{\pi/2} \frac{1+\cos z}{1-\cos z}\,dz$

18. $\displaystyle\int \frac{t^2}{t^6-9}\,dt$

19. $\displaystyle\int \frac{2}{\sqrt{z+2}+\sqrt{z}}\,dz$

20. $\displaystyle\int \sqrt{x}\cos\sqrt{x}\,dx$

**21–40** Transform the integral into a form that you can integrate by using a table of integrals or any of the techniques discussed in the previous sections. Then evaluate the integral.

21. $\displaystyle\int \sqrt{\frac{x+2}{x-2}}\,dx$

22. $\displaystyle\int \frac{\cos x}{\sin x(2\sin x+7)}\,dx$

23. $\displaystyle\int \frac{dx}{x\sqrt{2x^2+9}}$

24. $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-9}}$

25. $\displaystyle\int \frac{3x^2}{x^6(2x^3+1)}\,dx$

26. $\displaystyle\int \frac{4x}{(x^4+4)^2}\,dx$

27. $\displaystyle\int \frac{\tan u}{\cos^2 u\sqrt{5+2\tan u}}\,du$

28. $\displaystyle\int \frac{dx}{\sqrt{3e^x-16}}$

29. $\displaystyle\int \ln\sqrt{x}\,dx$

30. $\displaystyle\int \sin 2x(2\sin x+3)^3\,dx$

31. $\displaystyle\int \frac{\cot x}{\sqrt{2\sin x-\sin^2 x}}\,dx$

32. $\displaystyle\int t^2\sqrt{t^6-4}\,dt$

33. $\displaystyle\int \frac{\sqrt{3x-2}}{x}\,dx$

34. $\displaystyle\int \frac{\sqrt{2x+1}}{x}\,dx$

35. $\displaystyle\int \frac{x^2-2x+1}{\sqrt{1+6x-3x^2}}\,dx$

36. $\displaystyle\int x\sqrt{x^2+2x+5}\,dx$

37. $\displaystyle\int_{\pi/3}^{2\pi/3} \csc^5 t\,dt$

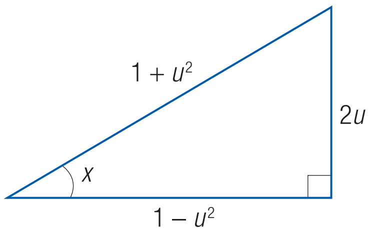
38. $\displaystyle\int \frac{\sin 2x}{\sqrt{2\sin x+9}}\,dx$

39. $\displaystyle\int_0^1 \arccos\sqrt{z}\,dz$

40. $\displaystyle\int e^{2t}\tan^{-1}e^t\,dt$

**41–45** If the integrand is a rational function of $\sin x$ and $\cos x$, we can turn it into a rational function of $u$ by the substitution $x = 2\arctan u$. The figure reflects the aforementioned substitution (as you will determine in Exercise 41). Using the figure along with differentiation, we obtain the following.

$$dx = \frac{2\,du}{1+u^2},\quad \sin x = \frac{2u}{1+u^2},\quad \text{and}\quad \cos x = \frac{1-u^2}{1+u^2}$$




After integrating the resulting rational function, we express our answer in terms of the original variable $x$ by using $u = \tan(x/2)$.

41. By using the identity $\tan\dfrac{x}{2} = \dfrac{\sin x}{1+\cos x}$, show that the figure indeed reflects the substitution $x = 2\arctan u$.

Use the above substitution technique to evaluate the given integral.

42. $\displaystyle\int \frac{dx}{3-\sin x}$

43. $\displaystyle\int \frac{dx}{\cos x-\sin x+2}$

44. $\displaystyle\int \frac{\sin x}{1+\cos x-\sin x}\,dx$

45. $\displaystyle\int \frac{du}{2\cos u+3}$

**46–55** The given integration formula can be found in most tables of integrals. Verify it by an appropriate integration technique learned from this chapter.

46. $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

47. $\displaystyle\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right) + C$

48. $\displaystyle\int \sin^{-1}u\,du = u\sin^{-1}u + \sqrt{1-u^2} + C$

**49.** $\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

**50.** $\int \frac{u}{a+bu}\,du = \frac{1}{b^2}\left(a+bu-a\ln|a+bu|\right)+C$

**51.** $\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$

**52.** $\int \frac{u^2}{\sqrt{a^2-u^2}}\,du = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin\frac{u}{a} + C$

**53.** $\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

**54.** $\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$

**55.** $\int \frac{du}{\left(u^2-a^2\right)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$

## 7.5 **Technology Exercises**

**56–59** Use a computer algebra system to solve the given exercise. If the answer looks different from what you obtained by hand, prove that the answers are equivalent.

**56.** Exercise 6 **57.** Exercise 13

**58.** Exercise 27 **59.** Exercise 34

**60–65** Compare a computer algebra system's answer to the given exercise with those obtained by substitution and/or integration tables. Use the differentiation feature of your technology to prove that both answers are correct.

**60.** Exercise 5 **61.** Exercise 16

**62.** Exercise 21 **63.** Exercise 24

**64.** Exercise 30 **65.** Exercise 36

**66–68** Use a computer algebra system to find $F(x)$ that satisfies the given condition. (This problem type is called an initial value problem. We have already seen similar problems in Section 4.7, but you will learn more about them in Chapter 8.)

**66.** $F(x) = \int \frac{dx}{x^2-2x+4};\quad F(0)=0$

**67.** $F(x) = \int 3x\sqrt{x^2-4x+5}\,dx;\quad F(1)=0$

**68.** $F(x) = \int x^2 \arccos x\,dx;\quad F(0)=1$

**69–71** Use a computer algebra system to give an approximate solution to the given equation. (These are examples of *integral equations*.)

**69.** $\int_0^x \frac{dt}{\sqrt{t^2+2}} = 2$ **70.** $\int_1^x \frac{\tan^{-1} t}{t^2}\,dt = 1$

**71.** $\int_x^{\pi/3} \frac{\sqrt{1-\sin^2 t}}{\tan t}\,dt = 2$

**72.** Paper-and-pencil skills are important, even when powerful software is at your disposal. Use an appropriate substitution to evaluate

$$\int \frac{(1+x)e^x}{\sqrt{x^2e^{2x}+1}}\,dx,$$

and then use a computer algebra system to check your answer. What do you find?

**73–78** Even with all of our integration techniques, tables, and computer algebra systems, we are far from being able to find antiderivatives for all elementary functions (roughly speaking, these are finite combinations of the types of functions you have studied so far). Even more surprisingly, many elementary functions do not even have elementary antiderivatives! Some of these appear relatively "easy," so that you might even be tempted to try and integrate them. For example, $f(x) = e^{x^2}$ does not have an elementary antiderivative. Integrals such as $\int e^{x^2}\,dx$ are called *nonelementary integrals*. Working with them requires infinite series (see Chapter 10) or the numerical methods that we shall learn in Section 7.6.

Try to evaluate the nonelementary integral using a computer algebra system. What answer do you get? (Answers will vary.)

**73.** $\int e^{x^2}\,dx$ **74.** $\int \frac{e^x}{x}\,dx$

**75.** $\int \cos\left(x^2\right)dx$ **76.** $\int \frac{\sin x}{x}\,dx$

**77.** $\int \ln(\ln x)\,dx$ **78.** $\int \frac{1}{\ln x}\,dx$