6.3 Exercises

1–4 Use integration to determine the length of the given line segment. Then use the distance formula to check your answer.

1. $y = 3x - 5; \quad 1 \le x \le 7$
2. $x = 2\sqrt{2}y + 1; \quad \sqrt{2} \le y \le \sqrt{18}$
3. $y = \frac{1}{2}x + 1; \quad 2 \le x \le 8$
4. $x = 3 - \frac{1}{3}y; \quad 0 \le y \le 6$

5–16 Determine the arc length L of the curve defined by the equation over the given interval.

5. $y = \frac{5 + 2x^{3/2}}{3}; \quad 1 \le x \le 8$
6. $y = x^{3/2} - \frac{1}{3}\sqrt{x}; \quad 1 \le x \le 4$
7. $y = \frac{x^2}{8} - \ln x; \quad 1 \le x \le e$
8. $y = \frac{x^3}{3} + \frac{1}{4x}; \quad 1 \le x \le 3$
9. $yx^3 - x^8 = \frac{1}{60}; \quad \frac{1}{2} \le x \le 1$
10. $8x^2y = x^6 + 2; \quad 1 \le x \le 3$
11. $y = \sqrt{1 - x^2}; \quad 0 \le x \le 1$
12. $y = \sqrt[3]{x}\left(x^{4/3} - \frac{9}{20}\right); \quad 1 \le x \le 8$
13. $y = \frac{e^x}{4} + e^{-x}; \quad 0 \le x \le 1$
14. $y = \int_{1/2}^{x} \sqrt{\cos 2t}\, dt; \quad 0 \le x \le \frac{\pi}{4}$
15. $x = \int_{1/3}^{y} \sqrt{\frac{1}{t^2} - 1}\, dt; \quad \frac{1}{e} \le y \le 1$
16. $x = \ln(\cos y); \quad 0 \le y \le \frac{\pi}{4}$

17–25 Set up, but do not evaluate, an integral defining the arc length of the graph of the equation over the given interval. Then find the corresponding arc length function $s(x)$ or $s(y)$ as appropriate.

17. $y = x^2 + 1; \quad 0 \le x \le 2$
18. $3x - y^2 = y + 2; \quad 1 \le y \le 4$
19. $x + y = y^2; \quad 0 \le y \le 1$
20. $y = \frac{1}{x - 2}; \quad 3 \le x \le 4$
21. $y = \frac{1}{x^2}; \quad 1 \le x \le 2$
22. $y = 2\ln x; \quad 1 \le x \le e$
23. $y = e^{2-x}; \quad 0 \le x \le 2$
24. $y = \cos x; \quad \frac{\pi}{2} \le x \le \frac{3\pi}{2}$
25. $y = \sin^{-1} x; \quad -\frac{1}{2} \le x \le \frac{1}{2}$

26–35 Find the surface area of the solid obtained by revolving the indicated curve about the x-axis.

26. $y = \frac{1}{2}x - 1; \quad 2 \le x \le 5$
27. $x = 5 - y; \quad 1 \le x \le 3$
28. $x = \sqrt[3]{y}; \quad 0 \le y \le 1$
29. $y = 2\sqrt{x}; \quad 0 \le x \le 4$
30. $x = y^2 + 3; \quad 1 \le y \le \sqrt{2}$
31. $y = x^3 + \frac{1}{12x}; \quad \frac{1}{2} \le x \le 1$
32. $6y\sqrt{x} = 12x^2 - x; \quad 1 \le x \le 2$
33. $y = e^x + \frac{e^{-x}}{4}; \quad 0 \le x \le \frac{1}{2}$
34. $y = \sqrt{4x - x^2}; \quad 0 \le x \le 4$
35. $x^2 + y^2 = 2x; \quad 1 \le x \le 2, \quad 0 \le y \le 1$

36–41 Find the surface area of the solid obtained by revolving the indicated curve about the y-axis.

36. $3y = x + 1; \quad 1 \le y \le 3$
37. $4y = x^2; \quad 0 \le y \le 2$
38. $y = \sqrt[3]{4x}; \quad 0 \le y \le 1$
39. $2y = x^2 - 1; \quad 0 \le y \le 1$
40. $20xy^{-1/3} = 20y^{4/3} - 9; \quad 0 \le y \le 1$
41. $12xy = 4y^4 + 3; \quad 1 \le y \le 2$

42–45 Find the surface area of the solid obtained by revolving the given curve about the indicated line. (**Hint:** Remember that $A = \int 2\pi r\, ds$, where r denotes the radius from the axis of revolution to the surface and ds is the arc length differential.)

42. $y = x - 1;\quad 1 \le x \le 3;\quad$ about $y = -2$

43. $y = \frac{1}{2}x - 1;\quad 0 \le y \le 2;\quad$ about $x = 1$

44. $12xy = 6y^4 + 2;\quad 1 \le y \le 2;\quad$ about $x = -\frac{1}{4}$

45. $2y = e^x + e^{-x};\quad 0 \le x \le 1;\quad$ about $y = -1$

46–53 Set up, but do not evaluate, an integral defining the surface area of the solid obtained by revolving the given curve about the indicated axis.

46. $y = \cos x;\quad \frac{\pi}{2} \le x \le \frac{3\pi}{2};\quad$ about the x-axis

47. $y = \sin^{-1} x;\quad 0 \le x \le \frac{\sqrt{3}}{2};\quad$ about the x-axis

48. $y = \ln x;\quad 1 \le x \le e;\quad$ about the y-axis

49. $y = \frac{1}{x^2};\quad 2 \le x \le 4;\quad$ about the x-axis

50. $x^2 - 7 = y^2 + 4y;\quad 0 \le y \le 2;\quad$ about the y-axis

51. $y = \sqrt[3]{x};\quad 1 \le x \le 8;\quad$ about $y = -3$

52. $x(y-5) = 1;\quad 6 \le y \le 8;\quad$ about the y-axis

53. $y = x^4 - 1;\quad 1 \le x \le \sqrt[4]{2};\quad$ about $x = -2$

54. By generalizing Exercise 11, prove that the circumference of a circle of radius R is $C = 2\pi R$.

55. Recall that the equation of the astroid of Exercise 28 of Section 3.5 is $x^{2/3} + y^{2/3} = 10$. Rotate the graph about the x-axis and find the surface area of the resulting solid. Do you get the same answer if you rotate about the y-axis? Why?

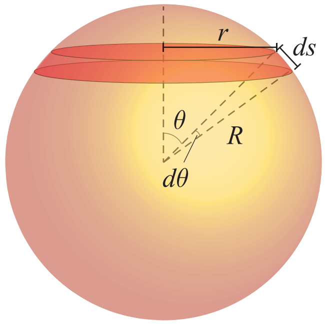
56. The shape of a clothesline stretched between two trees at a campsite can be approximated by the equation $y = \frac{1}{10}x^2 + 1,\ -1.5 \le x \le 1.5$ (distance is measured in yards). Find a formula for the length of the clothesline.

57. A particle is moving in the two-dimensional coordinate system so that its position (the x- and y-coordinates) as a function of time is given by $x = t/2$ and $y = t^{3/2},\ 0 \le t \le 4$.

a. Sketch the path of the particle. (**Hint:** Paying attention to the domain of the variable t, use t to express y in terms of x.)

b. Find the distance traveled by the particle. (**Hint:** Use the formula you found in part a. Alternatively, you may observe that $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, and use the notation of Example 4.)

58. Show that the surface area of a sphere of radius R is $4\pi R^2$ by the following steps.



a. Show that the distance r in the diagram above is given by $r = R\sin\theta$ and note that the arc length differential is given by $ds = R\,d\theta$.

b. Determine the appropriate limits of integration and apply the surface area formula $A = \int 2\pi r\, ds$.

59.* Suppose that a pair of parallel planes intersect a sphere, as in the illustration provided for Exercise 58. However this time assume that the fixed distance between the planes is D units. Modify your argument in Exercise 58 to prove that the surface area of the zone of the sphere that falls between the planes is $A = 2\pi RD$. (Notice that this area depends only on the distance between the two parallel planes, not their actual location!)

60. Recall that the lateral surface area of a circular cone of slant height s and base radius r is $A = \pi rs$ (circumference of base $\cdot$ slant height$/2$). Use the surface area integral of this section to verify this formula. (**Hint:** Rotate the line segment $y = rx/\sqrt{s^2 - r^2},\ 0 \le x \le \sqrt{s^2 - r^2}$ around the y-axis.)

61. Find the surface area of the solid generated by revolving the region bounded by the graphs of $x^2 = 5y$ and $5y + 36 = 5x^2$ about the y-axis.

62. Use the methods of this section to find the surface area of the torus that is obtained by revolving the circle $x^2 + (y-2)^2 = 1$ about the x-axis.

63.* Generalize your solution to Exercise 62 to obtain the surface area of the torus of Exercise 84 in Section 6.1.

64.* If we rotate the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ about the x-axis, the resulting solid is called an ellipsoid. Find the surface area of this solid. (**Hint:** Handle the integral of type $\int\sqrt{k^2-u^2}\,du$ by substituting $u=k\sin\theta$. Next, for the integral of type $\int\cos^2 t\,dt$ use the formula $\cos^2 t=(1+\cos 2t)/2$.)

65.* Generalize Exercise 64 for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$ to finds the surface area of the resulting ellipsoid.

6.3 Technology Exercises

66–74. Use the formulas you found and the integration capabilities of a graphing utility to evaluate the arc length integrals in Exercises 17–25.

75–82. Use the formulas you found and the integration capabilities of a graphing utility to evaluate the surface area integrals in Exercises 46–53.

83. Use a graphing utility to find the length of the clothesline in Exercise 56.

84. Rotate the parabola of Exercise 56 about the vertical axis. Find the surface area of the resulting paraboloid. (For example, a large satellite dish might have this shape.)