

- c. This integral is difficult to get a handle on in its original form, but it is more tractable if we multiply the numerator and denominator by 3^x .

$$\begin{aligned} \int \frac{dx}{3^x + 3^{-x}} &= \int \frac{1}{3^x + 3^{-x}} \left(\frac{3^x}{3^x} \right) dx && \text{Multiply the integrand by 1.} \\ &= \int \frac{3^x}{3^{2x} + 1} dx && \text{Set } u = 3^x; \text{ so } du = 3^x (\ln 3) dx. \\ &= \frac{1}{\ln 3} \int \frac{1}{u^2 + 1} du && 3^{2x} = (3^x)^2 = u^2 \\ &= \frac{1}{\ln 3} \tan^{-1} u + C \\ &= \frac{1}{\ln 3} \tan^{-1}(3^x) + C \end{aligned}$$

5.4 Exercises

1–15 Evaluate the integral, definite or indefinite, as indicated. (**Hint:** See Examples 1 and 2 and the subsequent table of integrals.)

1. $\int (12x^5 + 7.5x^4 - x^3 + 2) dx$

2. $\int (-3x^4 + 0.8x^3 - 6x^2 + 4x - \pi) dx$

3. $\int 2(x+1)(5x-2) dx$

4. $\int_{-1}^1 -x(x+4)(2x-1) dx$

5. $\int_0^1 x^3 \sqrt{x} dx$

6. $\int \frac{x^4 - 3\sqrt{x}}{x^2} dx$

7. $\int \frac{x^2 - 2x}{\sqrt{x} + \sqrt{2}} dx$

8. $\int \left(\sqrt[3]{x^2} + \frac{1}{\sqrt{x}} \right)^2 dx$

9. $\int (\pi \sec x - \tan x) \sec x dx$

10. $\int (e^{3x} + 2^{2x/3}) dx$

11. $\int \frac{\cot 2x}{2 \sin x \cos x} dx$

12. $\int_0^{\sqrt{3}/2} \frac{2}{1+4x^2} dx$

13. $\int_0^{1/2} \frac{2}{\sqrt{1-4x^2}} dx$

14. $\int \frac{3}{|4x| \sqrt{16x^2 - 1}} dx$

15. $\int \frac{-7}{\sqrt{1-25x^2}} dx$

16–36 Perform the suggested substitution to evaluate the given indefinite integral.

16. $\int 6x(3x^2 + 5)^7 dx; u = 3x^2 + 5$

17. $\int x^3 \sqrt{x^4 + 2} dx; u = x^4 + 2$

18. $\int \frac{2x}{4x^2 + 1} dx; u = 4x^2 + 1$

19. $\int \frac{2}{4x^2 + 1} dx; u = 2x$

20. $\int 4e^{2t+3} dt; u = 2t + 3$

21. $\int 4e^{2t+3} dt; u = e^{2t+3}$

22. $\int \cos 5\theta d\theta; u = 5\theta$

23. $\int \frac{5^{\arctan x}}{1+x^2} dx; u = \arctan x$

24. $\int \frac{\sin 2x}{\sqrt{\sin^2 x + 1}} dx; u = \sin^2 x + 1$

25. $\int \frac{\sec^2(1+\sqrt{s})}{\sqrt{s}} ds; u = 1 + \sqrt{s}$

26. $\int \frac{1}{3s^{2/3} \sqrt{1-s^{2/3}}} ds; u = \sqrt[3]{s}$

27. $\int \frac{1}{x^2} \sec^2 \frac{1}{x} dx; u = \frac{1}{x}$

28. $\int \sqrt{x} \cos^3(x^{3/2}) dx$; $u = x^{3/2}$ (**Hint:** Use $u = x^{3/2}$ and $\cos^3 u = \cos^2 u \cos u = (1 - \sin^2 u) \cos u$, and then perform another substitution $w = \sin u$.)

29. $\int \sqrt{1 + \cot^2 x} \cot x \csc^2 x dx$; $u = 1 + \cot^2 x$

30. $\int (x+1)(x-7)^8 dx$; $u = x-7$

31. $\int \frac{z}{2z-1} dz$; $u = 2z-1$

32. $\int z\sqrt{z^2-5} dz$; $u = z^2-5$

33. $\int x\sqrt{x-5} dx$; $u = x-5$

34. $\int \frac{(\sqrt{x}-2)\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$; $u = 1+\sqrt{x}$

35.* $\int \sqrt{1-x^2} dx$; $x = \sin u$

36. $\int \sin^4 x \cos^3 x dx$; $u = \sin x$

59. $\int xe^{x^2-3} dx$

61. $\int \cos \pi x dx$

63. $\int \sin^3 2t \cos 2t dt$

65. $\int \frac{\csc^2 v}{e^{\cot v - 1}} dv$

67. $\int \frac{\sqrt{x}}{(30-x^{3/2})^2} dx$

69. $\int \frac{\sin 2x}{\sin^2 x + 2} dx$

71. $\int \cos x \cos(\sin x) dx$

73. $\int \frac{\sin 2x}{1 - \cos 2x} dx$

75. $\int \tan^3 2x \sec 2x dx$

77. $\int \frac{\sqrt{1+\sqrt{w}}}{\sqrt{w}} dw$

79. $\int (x-5)(x+1)^{11} dx$

81. $\int \frac{x+2}{4x+1} dx$

83. $\int \frac{(2 \ln x + 5)(1 - \ln x)^3}{2x} dx$

84. $\int \sqrt{2+\sqrt{x}} dx$

60. $\int x \sin(x^2) dx$

62. $\int \frac{x}{\cos^2(x^2)} dx$

64. $\int 3x \sec^2(x^2+1) dx$

66. $\int \frac{5^{\sqrt{3v}}}{\sqrt{3v}} dv$

68. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

70. $\int \frac{\tan \sqrt{x} \sec \sqrt{x}}{\sqrt{x}} dx$

72. $\int \frac{\ln(x^3)}{x} dx$

74. $\int \frac{e^{2t}}{e^t - 2} dt$

76. $\int \frac{z^3}{1-z^2} dz$

78. $\int \sqrt{1+\sqrt{w}} dw$

80. $\int 2x\sqrt{x+2} dx$

82. $\int \frac{x^2+x+1}{x-2} dx$

37–84 Use an appropriate substitution to evaluate the indefinite integral.

37. $\int 3(3x-2)^7 dx$ 38. $\int -2x\sqrt{9-x^2} dx$

39. $\int (4x+3)(2x^2+3x)^{20} dx$

40. $\int -2x^3\sqrt{9-x^2} dx$

41. $\int \cot z dz$

42. $\int (z-2)(3z^2-12z)^{99} dz$

43. $\int x^2(x^3-5)^{19} dx$ 44. $\int 6x^3(x^4+2)^{14} dx$

45. $\int e^{\sin x} \cos x dx$ 46. $\int e^x \csc^2(e^x) dx$

47. $\int (5-s)^{37} ds$ 48. $\int s\sqrt{s^2+1} ds$

49. $\int x^3\sqrt[3]{x^4+11} dx$ 50. $\int \sqrt[3]{5x+9} dx$

51. $\int \frac{2x+1}{(x^2+x-7)^2} dx$ 52. $\int \frac{x^2}{x^3-1} dx$

53. $\int \frac{-x^3}{\sqrt{2+x^4}} dx$ 54. $\int \frac{4x^3+10x}{x^4+5x^2+6} dx$

55. $\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2} dt$ 56. $\int \frac{3t+9}{\sqrt{2t^2+12t}} dt$

57. $\int \frac{2}{t \ln 2t} dt$ 58. $\int \frac{\ln x}{x} dx$

85–90 Find the function that satisfies the given conditions.

85. $\frac{df}{dx} = 4x\sqrt{4x^2+4}$; $f(0) = 1$

86. $\frac{dg}{ds} = \frac{2 \ln s}{s}$; $g(1) = 5$

87. $\frac{dy}{dt} = \frac{\sin 2t}{\sin^2 t + e}$; $y(0) = 0$

88. $y'(x) = \frac{3\sqrt{x}}{(1-x^{3/2})^2}$; $y(0) = 0$

89. $y''(x) = \cos 4x$; $y'(0) = 1$; $y(0) = 0$

90. $\frac{d^2y}{dt^2} = 2 \cot t \csc^2 t$; $y'\left(\frac{\pi}{2}\right) = 0$; $y\left(\frac{\pi}{2}\right) = 0$

91. A particle that started at the origin and is moving along the x -axis has a velocity function given by

$$v(t) = \frac{1 + \sqrt{t+1}}{\sqrt{t+1}} \text{ units/s.}$$

What is the particle's position at $t = 3$ seconds?

92. A particle is undergoing simple harmonic motion along the y -axis around the equilibrium $y = 0$, while its acceleration is given by

$$a(t) = 4\pi^2 \sin \frac{\pi(1+8t)}{4} \text{ units/s}^2.$$

Find the particle's position at $t = 1.5$ seconds and the total distance covered by the particle from $t = 0$ seconds to $t = 1.5$ seconds.

Concept Check

- 93–98 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

93. If f is defined on the interval $[a, b]$ and has an antiderivative, then the indefinite integral of f on $[a, b]$ is a number.
94. If f is defined on the interval $[a, b]$ and has an antiderivative, then the indefinite integral of f on $[a, b]$ is a function.
95. Two different elements of $\int f(x) dx$ can only differ by a constant; that is, if both $F(x)$ and $G(x)$ are elements of the set, then there is a constant C such that $F(x) = G(x) + C$.
96. The Substitution Rule can be interpreted as the Chain Rule in reverse.

97. $\int (\cos x + 1)^2 dx = \frac{(\cos x + 1)^3}{3} + C$

98. $\int \frac{1}{x^2 + x + 1} dx = \ln|x^2 + x + 1| + C$