3.4 Exercises

1–9 Identify $f(x)$ and $u=g(x)$ such that $F(x)=f(u)=f(g(x))$. Also find $h(x)$ wherever $F(x)=f(g(h(x)))$. (Answers will vary.)

1. $F(x)=(3x-2.5)^6$

2. $F(x)=2(x^3-5x^2+\pi)^{-4}$

3. $F(x)=2\sqrt[3]{x^2-9}$

4. $F(x)=\dfrac{-3}{5+\sqrt{x^3+x}}$

5. $F(x)=\sin\dfrac{1}{x^2+1}$

6. $F(x)=3\cos\left(\dfrac{\tan x}{2}\right)$

7. $F(x)=\csc(3e^x)$

8. $F(x)=\sec\left(e^{2+\sqrt{x}}\right)$

9. $F(x)=\dfrac{3}{\sqrt{\ln(x^2+1)}}$

10–60 Find the derivative of the given function.

10. $f(x)=(2x^2+x)^7$

11. $g(x)=3(x^5-\pi x^2+7.5)^{11}$

12. $h(x)=\dfrac{1}{2}(x^8+5x^3-ex)^{100}$

13. $F(x)=-3(5+2\sqrt{x})^{-5}$

14. $G(x)=(2x^2-3x+1)^{2/3}$

15. $k(x)=-5(x^5-2x^3+10.5x)^{-2/5}$

16. $f(x)=\sqrt{2-4x}$

17. $g(x)=\sqrt{x^2-5x+2}$

18. $h(x)=(4x+5)^{21}(3x-7)^{13}$

19. $q(x)=2(x^3-5x)^{2/3}(x+3)^{5/4}$

20. $r(t)=\dfrac{1}{3t+1}$

21. $k(z)=\dfrac{1}{1+5z-2z^2}$

22. $F(x)=\left(\dfrac{2x-3}{1-7x}\right)^{10}$

23. $S(v)=\left(\dfrac{2v+1}{v^2-5}\right)^{-3}$

24. $G(y)=\left(\dfrac{3y^2-1}{2+4y}\right)^{7/5}$

25. $T(s)=\left(\dfrac{s^2-1}{s^2+1}\right)^{-2/3}$

26. $G(x)=\dfrac{(5-\pi x^2)^2}{(1+2x)^3}$

27. $H(x)=\dfrac{\sqrt{x^2-2}}{(x^2+2)^2}$

28. $R(x)=\sqrt{\dfrac{1}{x^2-1}}$

29. $B(t)=\sqrt[3]{\dfrac{t}{2t^2+1}}$

30. $K(s)=\sqrt{\dfrac{2s-5}{3s+1}}$

31. $t(x)=\sin(\cos x)$

32. $Q(x)=2\tan(\sin x)$

33. $P(x)=x\tan^2 x$

34. $w(x)=\cot(x^2)$

35. $U(z)=5\sec^2 z$

36. $R(x)=x\sqrt{\sin x}$

37. $C(x)=\sin^2(\tan x)$

38. $U(v)=\csc\left(\dfrac{v}{\cos v}\right)$

39. $V(x)=e^{\cos x}$

40. $R(\theta)=e^{\theta\tan\theta}$

41. $w(x)=\sin\sqrt{2x+1}+e^{\tan\sqrt{2x+1}}$

42. $t(x)=10^{\sqrt{x}}$

43. $f(x)=\pi 2^{\sin(\pi x)}$

44. $u(x)=2^{x^2}-4^{\sqrt{x}}$

45. $t(s)=\tan(2^s)$

46. $u(x)=\cot^2(2^{\sin x})$

47. $E(x)=5^{5^x}$

48. $K(x)=\sqrt[3]{3^x}+3^{\sqrt[3]{x}}$

49. $N(x)=\cos^2\left(e^{\cos(x^2)}\right)$

50. $u(t)=\tan^3(t^3+3^t)$

51. $C(x)=\cos^2(x^2)$

52. $F(x)=5^{x^5}$

53. $t(s)=\sqrt{\cos(10^s)}$

54. $G(t)=\sec^{-3}(5^t)$

55. $H(s)=\sin(2^s)\tan(2^s)$

56. $w(s)=\sin(\tan(2^s))$

57. $T(z)=\sin(e^z)+e^{\sin z}$

58.* $q(x)=\sin(\cos(\tan(\cot x)))$

59.* $U(\theta)=\theta+\tan(\theta+\tan(\theta+\tan\theta))$

60.* $v(x)=\left(1+\left(2+(3+4x)^5\right)^6\right)^7$

61–68 Find an equation for the tangent line to the graph of the given function at the specified point.

61. $f(x)=\sqrt{2x^2+1};\quad (2,3)$

62. $g(x)=(x^2+3x+4)^{2/3};\quad (1,4)$

63. $q(x)=\cos(\tan x);\quad (0,1)$

64. $S(x)=\sin(x^2)+\sin^2 x;\quad (0,0)$

65. $M(x)=\dfrac{e^{\cos x}}{x};\quad \left(\pi,\dfrac{1}{e\pi}\right)$

66. $a(x)=10^{\sqrt{x}};\quad (1,10)$

67. $h(x)=\dfrac{3x+1}{\sqrt{x^2+3}};\quad (1,2)$

68. $u(x)=\pi^{\pi^{\sin x}};\quad (0,\pi)$

69–76 Find all x-values where the line tangent to the given curve is horizontal.

69. $f(x)=(x^2-8x+15)^{100}$

70. $g(x)=\dfrac{2x+3}{x^2-2}$

71. $h(x)=\sqrt{x^2+1}$

72. $T(x)=\tan^{10}x$

73. $w(x)=\sec(x^2+2)$

74. $t(x)=\cos(\cos x)$

75. $k(x)=e^{x/(x^2+1)}$

76. $q(x)=\pi^{\cos^2 x}$

77–84 Determine the second derivative of the function.

77. $p(x)=(x^2+5)^{20}$

78. $r(t)=\sqrt{t^2+5}$

79. $g(x)=5\cos^2 x$

80. $c(x)=e^{\tan x}$

81. $F(t)=t\sin(t^2)$

82. $d(x)=5^{5^x}$

83. $G(x)=\sin^2 x+\cos^2 x$

84. $U(s)=\sec\sqrt{s}$

85. Suppose that $f(1)=1,\ f'(1)=-2,\ g(1)=1,$ and $g'(1)=5$. If $F(x)=(f\circ g)(x)$ and $G(x)=(g\circ f)(x)$, find $F'(1)+G'(1)$.

86. Let $P(x)=x(x+1)(x+2)\cdots(x+10)$. If $F(x)=(P\circ P)(x)$, find the value of $F'(0)$.

87. Find a formula for the n^{th} derivative of $f(x)=\cos(kx),\ k\in\mathbb{R}$. (**Hint:** Use the Chain Rule and recognize a pattern.)

88. Repeat Exercise 87 for the function $g(x)=2^{kx}$.

89. Use the Chain Rule to prove that the function $f(x)=\sin(1/x^2)$ is differentiable for $x\neq 0$.

90. Use the Chain Rule to construct a second proof of the Quotient Rule. (**Hint:** Rewrite $f(x)/g(x)$ as $f(x)\cdot[g(x)]^{-1}$.)

91. Use the Chain Rule to prove that the derivative of an even function is odd and vice versa.

92. Find all points where the line tangent to the graph of $y=\sqrt[3]{\cos x}$ is horizontal, as well as those where it is vertical.

93.* A spherical balloon is being inflated so that its radius is increasing at a rate of $dr/dt=0.1$ in./s. Find the rate at which the volume of the balloon is increasing when its radius is $r=4$ in. (**Hint:** Notice that $V(t)=V(r(t))$ and use the Chain Rule.)

94.* Pouring sand is forming a conical shape so that the radius of the bottom of the cone is always twice its height throughout the process. If the height of the cone is increasing at a rate of $dh/dt=0.5$ mm/s, find the rate at which the volume of the cone is increasing when its height is $h=50$ mm. (See the hint given in Exercise 93.)

95. The position function of a vibrating loudspeaker cone is given by $x(t)=10^{-3}\cos 1500t$, where distance is measured in meters, time in seconds. As indicated by the position function, the cone is at one of its extreme positions at $t=0$. Use the above information to find **a.** the maximum velocity of the cone and **b.** the maximum acceleration of the cone.

96. The position function for damped harmonic motion of an object of mass m is

$$x(t)=Ae^{-\frac{k}{2m}t}\cos(\omega t),$$

where A is the amplitude and k and ω are constants specific to the motion. Find the velocity and acceleration functions for this motion.

97. Unless conditions are extreme, most gases obey the so-called *Ideal Gas Law*, which says $PV=nRT$, where P stands for pressure measured in pascals (Pa), V for volume, n for the number of moles (mol) of gas in the container, T denotes temperature measured in kelvins (K), and R is the *universal gas constant*, which is the same for all gases. Suppose 5 moles of gas are being slowly compressed by a piston in a container so that $dV/dt=-2\cdot 10^{-8}\ \text{m}^3/\text{s}$. Assuming that temperature is being kept constant at $T=293$ K throughout the process, find the rate of change of pressure with respect to time when $V=10^{-3}\ \text{m}^3$. (Use $R\approx 8.315\,\text{J}/(\text{mol}\cdot\text{K})$.)

3.4 Technology Exercises

98–99 The Maclaurin polynomial of order 2 of the function $f(x)$ is used to approximate $f(x)$ near $x=0$. It is defined as

$$P_2(x)=f(0)+f'(0)x+\frac{1}{2}f''(0)x^2.$$

Find the Maclaurin polynomial of order 2 for $f(x)$. Then use a graphing utility to graph f along with its Maclaurin polynomial. (We will learn more about Maclaurin polynomials in Section 10.8.)

98. $f(x)=\cos(\sin x)$

99. $f(x)=\dfrac{1}{x^2+1}$