3.2 Exercises

1–12 Use the appropriate rules from this section to find the derivative of the given function.

1. $f(x)=5-2x$
2. $g(x)=\frac{4}{5}x+2$
3. $h(x)=\frac{1}{2}+2x-3x^2$
4. $F(x)=x^3-2x^2+\frac{1}{2}x-77$
5. $G(x)=\frac{1}{2}x^4+2x^3-x^2+3.2x+\sqrt{2}$
6. $k(x)=x^{11}-0.2x^{10}+\frac{\pi}{3}x^3+\pi$
7. $H(x)=x^8+\sqrt{2}x^5-2x^4$
8. $R(t)=t^{100}-2t^{59}+\pi t^{38}+et$
9. $S(z)=4z^3-3\sqrt{z}+11.2$
10. $Q(s)=\frac{1}{3s}-\sqrt{2}s+\sqrt{2s}$
11. $T(r)=\pi r^2+2e^r+\pi^2$
12. $N(t)=e^{2+t}+\frac{t+1}{t}+pt$

13–20 Use the Product Rule to find the indicated derivative. Then find the answer without the use of the Product Rule, by multiplying first, and compare your answers.

13. $\frac{d}{dx}\left[(x+2)(3x+5)\right]$
14. $\frac{d}{dx}\left[(3x+7)(x^2+2x)\right]$
15. $\frac{d}{dx}\left[(x^2-6)(2x^2+5x)\right]$
16. $\frac{d}{dx}\left[(2x^3+3x^2)(4x^2-2x+5)\right]$
17. $\frac{d}{dx}\left[\left(\frac{1}{3}x^3+\frac{7}{5}x^5\right)\left(\frac{2}{x}-4x^2\right)\right]$
18. $\frac{d}{dx}\left[(e^x+3)(e^2-5)\right]$
19. $\frac{d}{dt}\left[(3+2\sqrt{t})(4\sqrt{t}-5)\right]$
20. $\frac{d}{ds}\left[s\left(-3-\frac{1}{3}s^3\right)(s^4+2s)\right]$

21–32 Use the Reciprocal Rule or Quotient Rule to determine the derivative of the function.

21. $f(x)=\frac{1}{1-2x}$
22. $g(x)=\frac{1}{4x-2x^2}$
23. $h(x)=\frac{2}{2x^3-5x^2+3x+1}$
24. $F(x)=\frac{e}{e^x-\sqrt{x}}$
25. $G(x)=\frac{2x+1}{x-5}$
26. $k(x)=\frac{3x-4}{2x^2+5}$
27. $H(x)=\frac{x^3-3x^2}{2x^3+5x^2+1}$
28. $A(x)=\frac{6\sqrt{x}}{3x-4}$
29. $B(u)=\frac{u^2}{\sqrt{u}+1}$
30. $f(t)=\frac{4-\sqrt{t}}{t^2+3}$
31. $g(t)=\frac{3-t}{4-5\sqrt{t}}$
32. $w(s)=\frac{1+2e^s}{3e^s+5}$

33–38 Differentiate the quotient by simplifying it algebraically first.

33. $f(x)=\frac{30x^2-10x^6}{5x}$
34. $g(x)=\frac{1+5x+x^2}{x}$
35. $h(x)=\frac{3x^{1/2}-5x^{3/2}+7x^{5/2}-9x^{7/2}}{x^{1/2}}$
36. $F(x)=\frac{2\left(\sqrt{x}\right)^3+3\sqrt{x}}{\sqrt{x}+\left(\sqrt{x}\right)^3}$
37. $G(x)=\frac{\frac{6}{x^2}-\frac{5}{x}+1}{\frac{1}{x}-\frac{3}{x^2}}$
38. $H(x)=\frac{2-\frac{1}{e^x}}{2e^{-x}}$

39–61 Using the rules of this section, differentiate the given function.

39. $f(t)=t^{1/2}(4t+3)$
40. $g(x)=x^2\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$
41. $h(s)=\left(5+\frac{1}{s}\right)\left(s^2+\frac{1}{5}\right)$
42. $F(x)=\frac{1}{x}+\frac{2}{x^2}$
43. $G(x)=3x^{-5}+2x^{-3}$
44. $k(s)=s^2\left(\frac{3}{s}+\frac{1}{s-1}\right)$
45. $H(t)=\sqrt{t}(9-t^2)$
46. $K(x)=\frac{\frac{1}{x^2}-3}{x+2}$
47. $w(z)=z\left(2+\frac{4}{4-\sqrt{z}}\right)$

48. $L(T)=T^{-3}\left(2-4T^{-2}\right)$

49. $r(x)=\dfrac{x-a^2}{x+a^2}$

50. $Q(t)=\dfrac{at+b}{ct+d}$

51. $F(x)=e^x\left(2+\sqrt{x}\right)$

52. $E(s)=\dfrac{2+se^s}{e^s-s}$

53. $C(x)=\dfrac{a}{a+\dfrac{a}{x}}$

54. $D(x)=\dfrac{x}{x+\dfrac{x}{a}}$

55. $G(s)=\dfrac{3s^2}{2e^s+s}$

56. $S(t)=\left(4-\sqrt{t}\right)\left(2-e^t\right)$

57. $L(y)=\left(y^4-3y^3\right)\left(y^2-2y^5\right)$

58. $h(z)=\dfrac{1}{ae^z+z}$

59. $H(s)=\dfrac{a}{b+ce^s}$

60. $T(x)=(x+2)\left(2x^2-x\right)\left(x^3+5\right)$

61. $L(t)=t\left(2e^t+\sqrt{t}\right)\left(\dfrac{1}{t}-1\right)$

62–67 Find the first, second, and third derivatives of the function.

62. $f(x)=2x+5$

63. $g(x)=\dfrac{x}{x+1}$

64. $h(x)=3\sqrt{x}$

65. $F(x)=2-x+5x^2-\pi x^3$

66. $V(z)=2z^2+\dfrac{2}{z^2}$

67. $W(t)=3t^2+3e^t$

68–71 Find a function f that satisfies the given conditions. (**Hint:** A polynomial is the most natural choice. Answers will vary.)

68. $f(0)=2,\ f'(0)=1,$ and $f''(0)=-1.$

69. $f(0)=0$ and f has horizontal tangent lines at $x=2$ and $x=-2$.

70. $f(0)=1,$ $y=x+1.5$ is tangent to the graph at $x=1$, and $y=5.5-x$ is tangent to the graph at $x=3$.

71. $f(1)=5,\ f'(1)=8,$ f has a horizontal tangent line at $x=-1$, and $f'''(x)=6.$

72–75 Find a formula for the k^{th} derivative of the function. (**Hint:** Calculate the first few derivatives and try to recognize an emerging pattern.)

72. $f(x)=x^n$

73. $g(x)=\dfrac{1}{x}$

74. $h(x)=xe^x$

75.* $q(x)=x^ne^x$

76. If $f(1)=2,\ f'(1)=1,\ g(1)=-1,$ and $g'(1)=3,$ find the following function values.

a. $(f-g)'(1)$

b. $(fg)'(1)$

c. $\left(\dfrac{f}{g}\right)'(1)$

77. If $f(3)=-1,\ f'(3)=5,\ g(3)=\frac{1}{2},$ and $g'(3)=-2,$ find the following function values.

a. $(2f+5g)'(3)$

b. $(4fg)'(3)$

c. $\left(\dfrac{f}{2g}\right)'(3)$

78–82 Find the equation of the line tangent to the graph of the function at the given point.

78. $f(x)=\dfrac{x^2+1}{x};\quad (1,2)$

79. $w(x)=\dfrac{8}{x^2+4};\quad (2,1)$

(This curve is called the **witch of Agnesi**.)

80. $g(x)=\dfrac{2}{\sqrt{x}+1};\quad (1,1)$

81. $h(x)=\dfrac{2e^x}{x^2};\quad (1,2e)$

82. $k(x)=\dfrac{2x}{2+x^2};\quad \left(1,\dfrac{2}{3}\right)$

(This curve is called a **serpentine**.)

83. Find the equation of the **normal line** to the graph of the function $s(x)=\dfrac{9x}{x^2+9}$ at the point $(0,0)$. (We call a line **normal** to the graph at a point if it is perpendicular to the line tangent to the graph at the same point.)

84. Repeat Exercise 83 for the graph of the function $h(x)=\dfrac{e^x}{x^4+2}$ at $\left(0,\frac{1}{2}\right)$.

85–96 Find all x-values where the graph of the function has a horizontal tangent line, or prove that the graph has no horizontal tangent line.

85. $f(x)=x^2-2x$

86. $g(x)=2x^3-3x^2-12x+1$

87. $h(x)=\dfrac{2}{x^2}$

88. $F(x)=\dfrac{2}{x^2+1}$

89. $G(x)=\dfrac{1}{2}e^x-2$

90. $k(x)=x^2-a$

91. $H(x)=\dfrac{1}{x^2-a}$

92. $f(x)=\sqrt{x+5}$

93. $g(x)=\dfrac{x^2}{x^2+5}$

94. $F(x)=\dfrac{2x-1}{x^2}$

95. $P(x)=2x^3+3x^2+3x-5$

96. $Q(x)=e^x-x$

97. The line $y=8x+b$ is tangent to the graph of $f(x)=ax^2$ at $x=2$. Find the values of a and b.

98. Repeat Exercise 97 with the line $y=-2x+b$ that is tangent to the graph of $g(x)=-x^2+ax$ at $x=2$.

99. Show that the graphs of $y=e^{x/2}$ and $y=\dfrac{1}{(x+1)^2}$ intersect at $x=0$ in such a way that their respective tangent lines are perpendicular at their point of intersection.

100–103 Find the equation(s) of the line(s) tangent to the graph of f and passing through the indicated point, which does not lie on the graph of f. (**Hint:** If the point of tangency is $(x,f(x))$, then the slope of the tangent line going through (a,b) is $f'(x)=\dfrac{f(x)-b}{x-a}$.)

100. $f(x)=x^2$; $(0,-1)$

101. $f(x)=e^x$; $(0,0)$

102. $f(x)=\sqrt{x}$; $(-2,0)$

103. $f(x)=\dfrac{1}{x}$; $(-1,0)$

104–107 Assuming that f and g are differentiable functions, differentiate the given expression.

104. $\dfrac{f(x)}{x}$

105. $\dfrac{xf(x)}{g(x)}$

106. $e^x g(x)$

107. $\dfrac{f(x)e^x}{g(x)+2}$

108. Use the definition of the derivative to prove the Constant Multiple Rule. (**Hint:** For a given constant k, start out by using the definition $\dfrac{d}{dx}[kf(x)]=\lim\limits_{h\to 0}\dfrac{kf(x+h)-kf(x)}{h}$, and let k "pass through" the limit sign.)

109. Use the Product Rule and mathematical induction to provide a third proof of the Positive Integer Power Rule. (**Hint:** The base case of $n=1$ should be obvious. After setting up the induction hypothesis of $(x^n)'=nx^{n-1}$, find the derivative of x^{n+1} by treating it as $x^{n+1}=x\cdot x^n$ and use the Product Rule along with the induction hypothesis.)

110. Use the Product Rule to arrive at a rule for $\dfrac{d}{dx}[f(x)g(x)h(x)]$.

111. Use the Product Rule to arrive at a formula for $\dfrac{d}{dx}[f(x)]^2$, and then use Exercise 110 to find the formula for $\dfrac{d}{dx}[f(x)]^3$. Do you recognize a pattern?

112. Use mathematical induction to prove the **Generalized Positive Integer Power Rule**:

$$\frac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}f'(x).$$

(The hint provided in Exercise 109 might prove helpful, but you will need to appropriately modify your induction hypothesis.)

113. Use the Sum Rule and mathematical induction to prove that any finite sum of functions $f_1(x)+f_2(x)+\cdots+f_n(x)$ can be differentiated termwise, that is,

$$[f_1(x)+f_2(x)+\cdots+f_n(x)]'=f_1'(x)+f_2'(x)+\cdots+f_n'(x).$$

In particular, polynomials can be differentiated termwise. (Look at the hint provided in Exercise 109.)

114. Use Exercise 113 and the results of this section to prove that the derivative of a polynomial is always another polynomial.

115. A Formula One race car was moving in a parabolic curve with equation $y=\sqrt{x}$ when it hit an oil patch and the driver lost control at the point $(4,2)$. The car left the track along the tangent line at the same point. Where did he hit the tire wall if the equation of the tire wall was $y=4$? (Fortunately, there were no injuries.)

116. The position function of a golf ball rolling on an incline is given by $d(t)=2t^2+3t$, where d is measured in meters, t in seconds. Find the ball's velocity and acceleration at $t = 4$ seconds.

117. The velocity function of a moving particle is given by $v(t)=\frac{50t}{t+10}$ ft/s. Find its acceleration at **a.** $t = 2$ seconds and **b.** $t = 10$ seconds.

118. The position function of a moving object is given by $p(t)=2t^3-6t$ ft. Find its position and acceleration at the instant when its velocity changes directions.

119. The position function of an object dropped by an astronaut on the moon is $h(t)=-0.81t^2+1.5$, where h is measured in meters, t in seconds. What is the acceleration due to gravity on the moon? How long does it take for the above object to reach the ground and what is the speed of impact?

120. The radius of a spherical balloon being inflated increases according to the function $r(t)=2+5\sqrt[3]{t}$, where r is measured in centimeters and t in seconds. Find the rate of change of the balloon's volume and surface area with respect to time at $t = 8$ seconds.

Concept Check

121–129 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

121. If $y=\pi x^n$, then $y'=n\pi x^{n-1}$.

122. If $y=\pi^n$, then $y'=n\pi^{n-1}$.

123. If $y=\pi/x^n$, then $y'=\pi/\left(nx^{n-1}\right)$.

124. If $y=e^x$, then $y'=xe^{x-1}$.

125. If $y=\pi e^x$, then $y'=\pi e^x$.

126. If p is a fifth-degree polynomial, then its sixth derivative is 0.

127. If $F(x)=\frac{f(x)}{g(x)}$, then $\frac{d}{dx}F(x)=\frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$.

128. The jerk of a free-falling object is 0.

129. If a polynomial $p(x)$ has degree n, then its derivative has degree $n-1$.

3.2 Technology Exercises

130–135. Use the differentiation capabilities of a graphing utility to check your answers for Exercises 62–67, and then graph each function along with its derivatives in the same viewing window.

136–140. Referring back to Exercises 78–82, verify your answers by using a graphing utility to graph each function and its indicated tangent line in the same viewing window.