3.1 Exercises

1–12 Find the derivative of the given function at the specified point and express your answer using the differential notation due to Leibniz.

1. $f(x)=7;\quad x=1$

2. $g(x)=\frac{1}{2}x-5;\quad x=-1$

3. $h(x)=\frac{1}{2}x^2-5;\quad x=0$

4. $F(t)=\frac{1}{5}t+2t^2;\quad t=\frac{1}{5}$

5. $G(s)=\frac{1}{3}s^3-s;\quad s=-3$

6. $H(t)=\frac{1}{2}t^4+t^2;\quad t=-2$

7. $K(z)=\frac{5}{3z+1};\quad z=0$

8. $T(t)=\frac{2t-3}{t+1};\quad t=\sqrt{5}-1$

9. $w(z)=\frac{1}{z^2+2};\quad z=\sqrt{2}$

10. $A(t)=\sqrt{2t};\quad t=0$

11. $Q(y)=\frac{1}{\sqrt{3y}};\quad y=1$

12. $B(u)=\sqrt{u^2+1};\quad u=-2\sqrt{2}$

13–24 Find the derivative of the function and use the differentiation operator D_x to express your answer.

13. $f(x)=\pi^2$

14. $g(x)=1-\frac{2}{3}x$

15. $h(t)=\frac{3}{2}t^2+10t-1$

16. $F(s)=4-s+\frac{1}{2}s^2+s^3$

17. $G(y)=\frac{1}{3y}$

18. $H(t)=\frac{t-3}{t+3}$

19. $S(z)=\frac{-3}{z^2}$

20. $T(u)=\frac{4}{u^2-3u}$

21. $R(v)=\sqrt{2v-3}$

22. $f(y)=\frac{2}{\sqrt{y+2}}$

23. $X(y)=2y^4$

24. $u(x)=\frac{2}{\sqrt{2x^2+1}}$

25–33 Find the first, second, and third derivatives of the function. Then graph the function along with its derivatives in the same coordinate system and compare the graphs. (**Hint:** See Example 4.)

25. $f(x)=\frac{5}{2}x-1$

26. $g(x)=x^2+5$

27. $h(x)=-\frac{1}{2}x^2+x-\frac{3}{2}$

28. $U(x)=-x^3$

29. $V(x)=\frac{1}{3}(x-2)^3$

30. $F(t)=t^4-1$

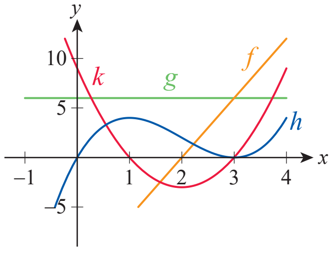
31. $G(x)=2(x-1)^4$

32. $H(x)=\frac{1}{x}$

33. $K(s)=\frac{-2}{s-1}$

34–37 The graphs of the position, velocity, acceleration, and jerk of a moving particle are given. Decide which one is which, label them accordingly, and explain.

34.



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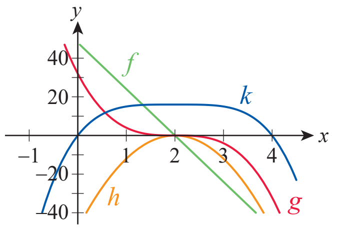

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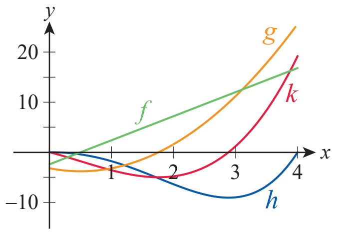

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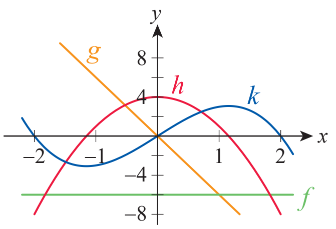
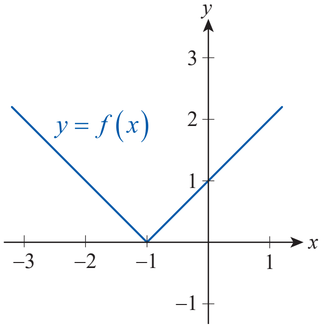


38–43 Use the given graph of the function to find all x-values where the function is differentiable.

38.



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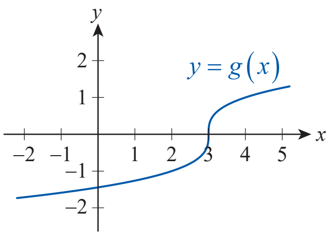

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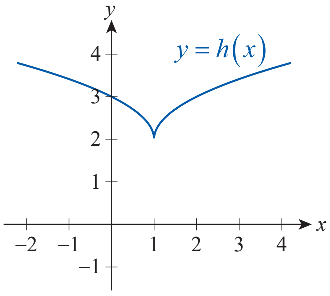

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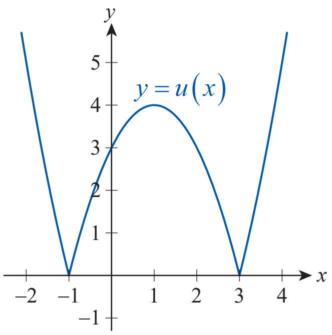

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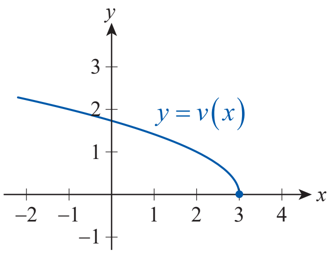

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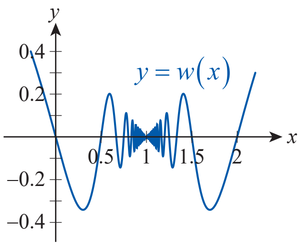

44–58 Find all points where the function is not differentiable. For each of those points, find the one-sided derivatives (if they exist).

44. $f(x)=|x+5|$

45. $g(x)=|x+2|-|x-4|$

46. $h(x)=(x-1)^{2/3}$

47. $F(x)=\sqrt[3]{x-1.5}+2$

48. $H(x)=\sqrt{1.8-x}$

49. $k(x)=\sqrt{3-x^2}$

50. $G(x)=\dfrac{x^2}{x^2-9}$

51. $m(x)=|x^2-6x+5|$

52. $A(t)=[\![t-4]\!]$

53. $B(x)=x-[\![x]\!]$

54. $F(t)=\begin{cases}\frac{1}{2}t\cos\frac{1}{t} & \text{if } t\neq 0\\ 0 & \text{if } t=0\end{cases}$

55. $H(z)=\begin{cases}\sqrt{z}\sin\frac{\pi}{z} & \text{if } z>0\\ 0 & \text{if } z=0\end{cases}$

56. $P(x)=\begin{cases}\sqrt[3]{x-1} & \text{if } x<1\\ (x-1)^2 & \text{if } x\geq 1\end{cases}$

57. $G(x)=\begin{cases}2x+2 & \text{if } x\leq -2\\ -\frac{1}{2}x^2 & \text{if } x>-2\end{cases}$

58. $S(t)=\begin{cases}\frac{1}{t} & \text{if } t\leq 1\\ t & \text{if } t>1\end{cases}$

59. Prove that the function

$$f(x)=\begin{cases}x^2\sin\frac{1}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$

is differentiable at 0. Contrast this result with Example 5.

60. The position function of a car crashing head-on at 62 mph during a crash test is $x(t)=-196t^2+27.78t$, where x is measured in meters and t in seconds. Find the deceleration of the dummy inside the car. What multiple of g is this (where g is the gravity constant)?

61. The position from its starting point of a small plane preparing for takeoff is given by $x(t)=1.1t^2$ meters (t is measured in seconds).

a. What is the acceleration of the plane?

b. How long does it take for the plane to reach the minimum takeoff speed of 33 m/s?

c. What is the minimum required runway length for this type of plane?

62. The position function of a theme park thrill ride moving along a straight line is $x(t)=\frac{14}{3}t^3+10t$ ft ($0\leq t\leq 3$, t is measured in seconds). Find the velocity, acceleration, and jerk. How far from starting position are the cars at the end of the 3-second time interval?

63. The **symmetric derivative** of a function f at a point c is defined as

$$f'_{sym}(c) = \lim_{h\to 0} \frac{f(c+h)-f(c-h)}{2h}.$$

a. Prove that if f is differentiable at c, then its symmetric derivative exists and $f'_{sym}(c) = f'(c)$.

b. Give an example of a function g and a point c such that $g'_{sym}(c)$ exists, but g is not differentiable at c.

64. Sketch the graph of $f(x) = -x^2 + 2x$ and its derivative on the interval $[0,2]$ in the same coordinate system. Where (on which interval) is f' positive? Where is f' negative? Identify those intervals where f is increasing versus decreasing. Do you see a connection? Can you give an intuitive reason for your findings?

65. Repeat Exercise 64 for the function $g(x) = 1/x^2$. Sketch both g and g' on their entire domains and summarize your observations. Can you formulate a general conjecture?

Concept Check

66–69 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

66. If f is continuous at c, then f is differentiable at c.

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68. If f is differentiable at c, then

$$f'(c) = \lim_{\Delta x\to 0} \frac{f(c-\Delta x)-f(c)}{-\Delta x}.$$

69. If both one-sided derivatives of f at c exist, then f is differentiable at c.

3.1 Technology Exercises

70–75 Use a graphing utility to graph the given function along with its derivative in the same viewing window and answer the questions of Exercise 64. (**Hint:** Use the differentiation capabilities of your technology to find f' first.)

70. $f(x) = \frac{1}{3}x^3 - 5x^2 - 1$

71. $f(x) = \frac{x^2}{x^2-4}$

72. $f(x) = \frac{x}{x^2+1}$

73. $f(x) = -\cos x$

74. $f(x) = \sin^2 x$

75. $f(x) = e^{1/(x^2+1)}$

76–79 Use a graphing utility to find the first four derivatives of f; then graph them along with f in the same viewing window and compare the graphs.

76. $f(x) = \frac{1}{2}x^5 - 2x^4 - 5x^3 + 7$

77. $f(x) = \arctan x$

78. $f(x) = \frac{2x}{x-3}$

79. $f(x) = x\sin\left(\frac{1}{10}x - 1\right)$

80–83 Use a graphing utility to graph the function and identify all points where the function is not differentiable. Explain.

80. $G(x) = (x^2-4)^{2/5}$

81. $H(t) = |2t-1|^{2/3}$

82. $P(x) = \begin{cases} \arctan x & \text{if } x < 0 \\ x^{3/2} & \text{if } x \ge 0 \end{cases}$

83. $L(t) = \sqrt{|t|}\sin\frac{1}{|t|}$