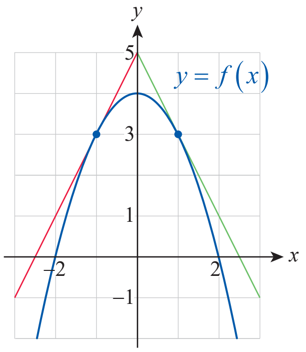
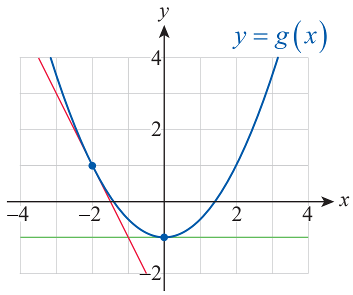
2.6 Exercises

1–2 Use the graph to estimate the derivative at the given points.

1. **a.** $x_1 = -1$ **b.** $x_2 = 1$



2. **a.** $x_1 = -2$ **b.** $x_2 = 0$



3–14 Find the equation of the tangent line to the graph of $f(x)$ at the given point.

3. $f(x) = x^2 - 2;\quad (2,2)$

4. $f(x) = 3x - 2x^2;\quad (-1,-5)$

5. $f(x) = \frac{1}{2}x + 4;\quad (2,5)$

6. $f(x) = 1 - 5x;\quad (0,1)$

7. $f(x) = x^3;\quad (2,8)$

8. $f(x) = 5x - 2x^3;\quad (-1,-3)$

9. $f(x) = \sqrt{x+1};\quad (0,1)$

10. $f(x) = 2\sqrt{1-3x};\quad (-1,4)$

11. $f(x) = \frac{1}{x};\quad \left(\frac{1}{2},2\right)$

12. $f(x) = \frac{5}{1-2x};\quad \left(-1,\frac{5}{3}\right)$

13. $f(x) = \frac{1}{\sqrt{x}};\quad \left(4,\frac{1}{2}\right)$

14. $f(x) = \frac{2}{\sqrt{x+1}};\quad (3,1)$

15–38 Use the definition (also called the *limit process*) to find the derivative function f' of the given function f. Find all x-values (if any) where the tangent line is horizontal.

15. $f(x) = 2$

16. $f(x) = 2x$

17. $f(x) = 4x + 5$

18. $f(x) = 3 - \frac{2}{5}x$

19. $f(x) = 3x^2$

20. $f(x) = 4 - 2x^2$

21. $f(x) = \frac{1}{2}x^2 + 5x - 7$

22. $f(x) = x - \frac{1}{3}x^2$

23. $f(x) = x^3 + x$

24. $f(x) = 7 + x - 3x^2 + x^3$

25. $f(x) = x^4$

26. $f(x) = \frac{1}{2x}$

27. $f(x) = \frac{5}{2x-4}$

28. $f(x) = \frac{x-2}{x+2}$

29. $f(x) = \frac{2x+1}{x-3}$

30. $f(x) = \frac{2}{x^2}$

31. $f(x) = \frac{1}{x^2+1}$

32. $f(x) = \frac{2}{x^2-2x}$

33. $f(x) = \sqrt{5x}$

34. $f(x) = \frac{1}{\sqrt{5x}}$

35. $f(x) = \sqrt{2x+1}$

36. $f(x) = \frac{1}{\sqrt{x-2}}$

37. $f(x) = \sqrt{x^2+1}$

38. $f(x) = \frac{1}{\sqrt{x^2+1}}$

39–44 Find the equation of a tangent line to the graph of the function that is parallel to the given line.

39. $f(x) = x^2 + 3;\quad y - 6x + 1 = 0$

40. $g(x) = 2x - x^2;\quad y - 5 = 4x$

41. $h(x) = \frac{1}{2x};\quad x + 2y = 3$

42. $F(x) = \frac{1}{x-3};\quad y + 4x + 7 = 0$

43. $G(x) = \frac{1}{\sqrt{x}};\quad 54y + x = 1$

44. $H(x) = \frac{1}{\sqrt{x^2-7}};\quad 27y + 4x - 2 = 0$

45–56 Use the alternate form of the definition of the derivative $f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ to evaluate the given slope.

45. $f(x)=5-\frac{1}{4}x;\quad f'(3.6)$

46. $g(x)=x^2+1;\quad g'(-1)$

47. $h(x)=(x+2)^2;\quad h'(3)$

48. $F(t)=\frac{1}{t-3};\quad F'(2)$

49. $G(x)=\frac{2}{5-x};\quad G'(7)$

50. $k(t)=\sqrt{t+5};\quad k'(11)$

51. $u(x)=2\sqrt{1-x};\quad u'(-3)$

52. $v(x)=\frac{1}{x^2+1};\quad v'(0)$

53. $w(s)=\frac{1}{\sqrt{s+4}};\quad w'(5)$

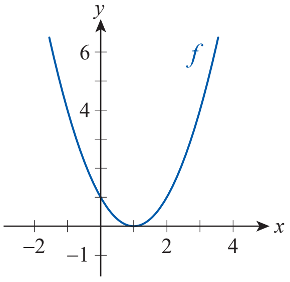
54. $F(t)=t^3-t;\quad F'(1)$

55. $G(s)=s^4;\quad G'(-2)$

56. $H(x)=\frac{2}{\sqrt{x^2+1}};\quad H'(0)$

57–60 Match the graph of f with the graph of its derivative f' (labeled A–D).

57.



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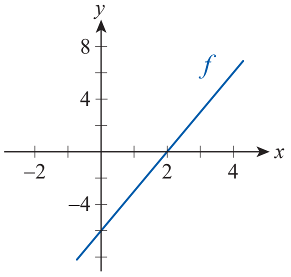

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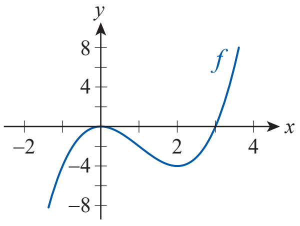

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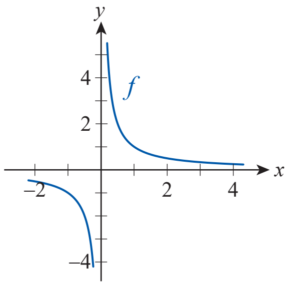
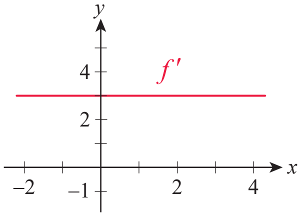
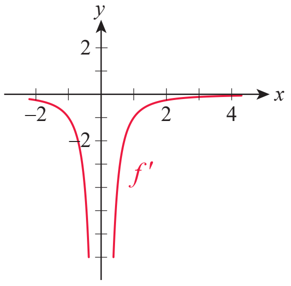


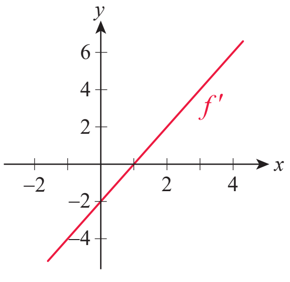
A.



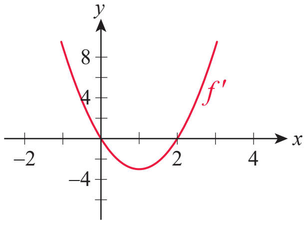
B.



C.



D.



61–65 Sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

61. $f(0)=1,\ f'(0)=0,\ f'(x)<0$ for $x<0$, $f'(x)>0$ for $x>0$

62. $f(1)=0,\ f'(1)=0,\ f'(x)\geq 0$ on the entire real line

63. $f(x)>0$ on the entire real line, $f'(x)<0$ on the entire real line

64. $f(1)=1,\ f'(1)=-1,\ f'$ is nonzero on the entire real line

65. $f(1)=5,\ f'(x)=5$ on the entire real line

66. Prove that if $f(x)=c$ (a constant function), then $f'(x)=0$.

67. Use the definition of the derivative to prove that if $f(x)=x$, then $f'(x)=1$.

68. Generalize Exercise 67 to prove that if $f(x)$ is a linear function, then $f'(x)$ is constant.

69.* Use the definition of the derivative to prove that if $f(x)=x^n$ for a positive integer n, then $f'(x)=nx^{n-1}$.

70.* Recall from Section 1.1 that a function f is even if $f(-x)=f(x)$ and odd if $f(-x)=-f(x)$ throughout its domain. Prove that the derivative of an even function is odd and, vice versa, an odd function has an even derivative.

71.* Find the equation of the line tangent to the graph of $f(x)=1/x$ at the point $(c, f(c))$. Prove that the area of the triangle bounded by the tangent line and the coordinate axes is the same for all $c\neq 0$.

72. The position function of a moving particle is given by $p(t)=t^2-3t+1$ feet at t seconds. Find all points in time where the particle's speed is $1\ \text{ft/s}$. When does it come to a momentary stop?

73. Repeat Exercise 72 with the position function $p(t)=\frac{1}{9}t^3-t^2+\frac{8}{3}t$.

74. A baseball is hit vertically upward with an initial speed of $80\ \text{ft/s}$. When does it slow down to $32\ \text{ft/s}$? How high does it go and how long is it aloft? (**Hint:** Use the position function $h(t)=-16t^2+80t$. Ignore air resistance.)

75. A rock is thrown upward from the edge of a 150 ft high cliff with an initial velocity of $48\ \text{ft/s}$.

a. Calculate the velocity and speed of the rock when it is exactly 32 ft above the person's hand.

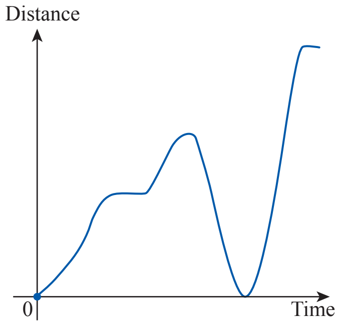
b. How high does it go and when does it reach the bottom of the cliff?

c. What is the velocity of impact?

(**Hint:** Use $h(t)=-16t^2+48t+150$ as the position function, where h is in feet, t in seconds. Ignore air resistance.)

76. A package is dropped from a small airplane 122.5 meters above Earth. If we ignore air resistance, how much time does the package need to reach the ground and what is the speed of impact? (**Hint:** The position function is $h(t)=-4.9t^2+122.5$ meters, where t is measured in seconds.)

77. The following graph is a position function of a student's car relative to her home as she drove to class one morning. From the graph, recreate a possible story of her trip, mentioning distance, velocity, speed, and so forth.



78. A manufacturer has determined that the revenue from the sale of x cell phones is given by $R(x)=94x-0.03x^2$ dollars. The cost of producing x telephones is $C(x)=10{,}800+34x$ dollars.

a. Find the profit function $P(x)$ and any break-even points.

b. Find $P(200)$, $P(400)$, and $P(600)$.

c. Find the marginal profit function $P'(x)$.

d. Find $P'(200)$, $P'(400)$, and $P'(600)$.

79. The owner of a leather retailer has determined that he can sell x attaché cases if the price is $p = D(x) = 46 + 0.25x$ dollars ($D(x)$ is often called the demand function). The total cost for these cases is $C(x) = 0.15x^2 + 6x + 190$ dollars.

a. Find the profit function $P(x)$. (**Hint:** Find the revenue function $R(x)$ first.)

b. Find any break-even points.

c. Find $P(25)$, $P(30)$, and $P(40)$.

d. Find the marginal profit function $P'(x)$.

e. Find $P'(25)$, $P'(30)$, and $P'(40)$.

80. The average cost $\overline{C}(x)$ of manufacturing x units of a certain product is $C(x)/x$, where $C(x)$ is the total cost function.

a. Find the average cost function if

$$C(x) = 30 + 2x + 0.003x^2.$$

b. What is the rate of change of average cost?

c. What value of x results in a minimum average cost? (**Hint:** Use the fact that when average cost is a minimum, its rate of change is 0. Alternatively, use technology to graph $C(x)$ for $x \geq 0$ and zoom in on the lowest point.)

81. The average manufacturing cost function for a product is given by $\overline{C}(x) = 20x^{-1} + 3$. Determine the cost function and the marginal cost function for the product. (**Hint:** See Exercise 80.)

2.6 Technology Exercises

82–105. Referring back to the functions given in Exercises 15–38, use a graphing utility to sketch the graph of f along with that of f' in the same viewing window. Compare the graphs and describe their relationship.