Again we see that, because of rounding errors and other reasons, our technology can sometimes mislead us by giving seemingly conflicting or inaccurate feedback, and we must be aware of this when using it.

We will stick with our guess that

$$\lim_{x\to 0} g(x) = \lim_{x\to 0}\frac{1-\cos x}{x} = 0,$$

but realize that we haven't actually proved this at all. We will learn how to do that after discussing limit theorems in upcoming sections.

Technology Note Finding a Limit

Computer algebra systems such as *Mathematica* provide additional tools for determining limits, but it should always be remembered that software has limitations and can be fooled. *Mathematica* contains the built-in command **`Limit`** that uses the same mathematical facts we will learn in the next two sections to correctly evaluate many types of limits. Its use is illustrated below. (For additional information on *Mathematica* and the use of the **`Limit`** command, see Appendix A.)

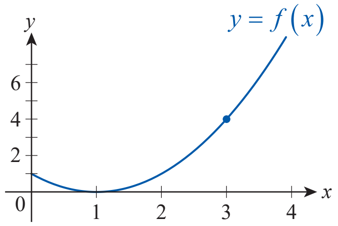
```
In[1]:= Limit[(Sqrt[x^2 + 4] - 2) / x^2, x → 0]

         1
Out[1]=  —
         4
```

Figure 17

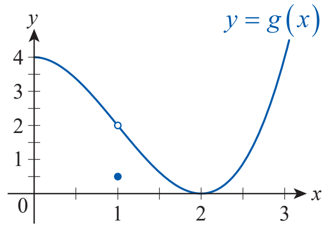
2.2 Exercises

1–4 Use the graph of the function to find the indicated limit (if it exists).

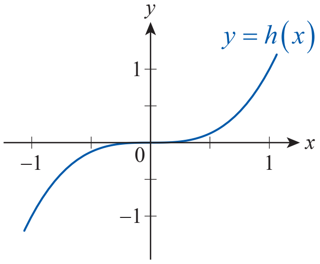
1. $\lim_{x\to 3} f(x)$



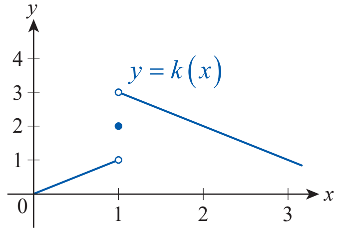
2. $\lim_{x\to 1} g(x)$



3. $\lim_{x\to 0} h(x)$



4. $\lim_{x\to 1} k(x)$



5–12 Create a table of values to estimate the value of the indicated limit without graphing the function. Choose the last x-value so that it is no more than 0.001 units from the given c-value.

5. $\lim_{x\to\sqrt{2}} \frac{x^2-2}{x-\sqrt{2}}$

6. $\lim_{x\to 3} \frac{x^3-9x^2+27x-27}{x-3}$

7. $\lim_{x\to 1} \frac{x^{10}-1}{x-1}$

8. $\lim_{x\to 0} \frac{4\sin x}{3x}$

9. $\lim_{x\to \pi} \frac{2\cos x-1}{1-\sin x}$

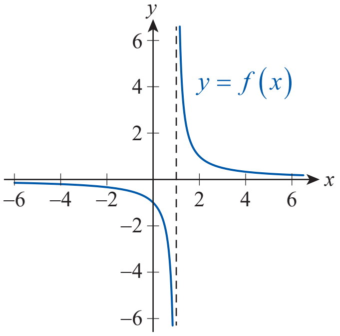
10. $\lim_{x\to 7^-} \frac{x^2-49}{x-7}$

11. $\lim_{x\to 7^+} \frac{x^2+49}{x-7}$

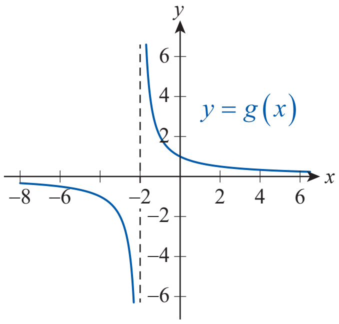
12. $\lim_{x\to 0^+} \frac{\sqrt{4+x}}{x}$

13–24 Use one-sided limit notation to describe the behavior of the function near its vertical asymptote(s).

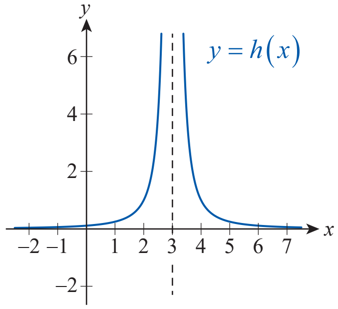
13.



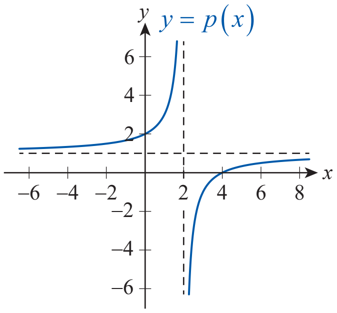
14.



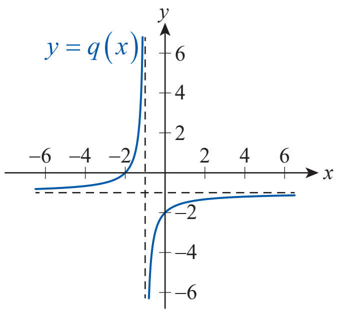
15.



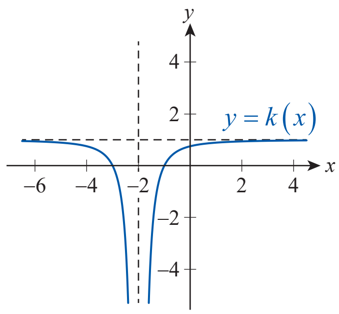
16.



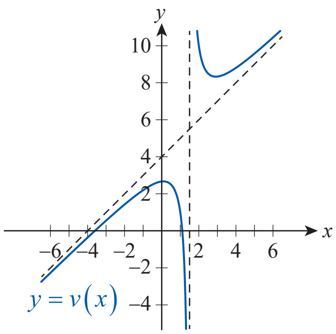
17.



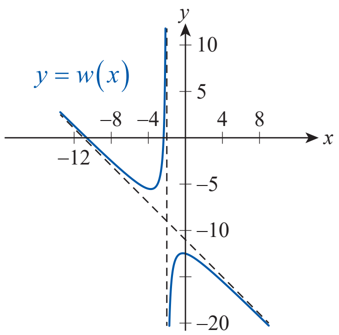
18.



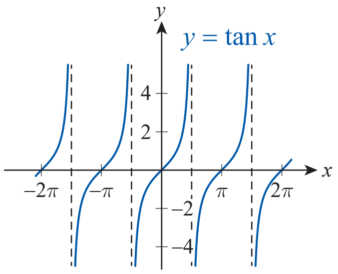
19.



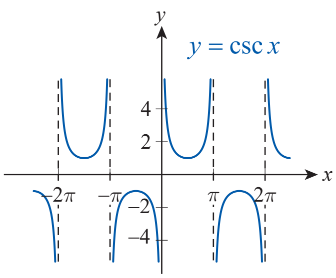
20.



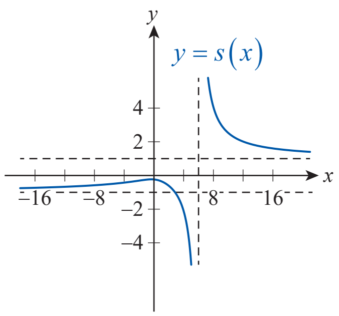
21.



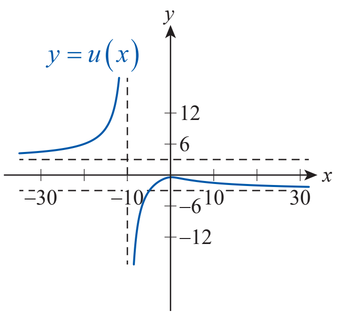
22.



23.



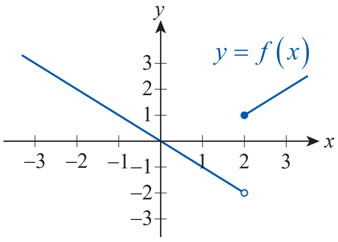
24.



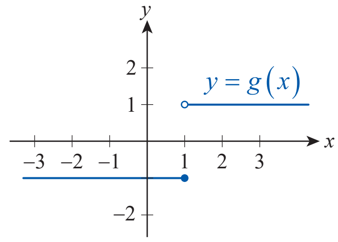
25–36. Consider the functions given in Exercises 13–24. Find their limits at ∞ and $-\infty$ (if they exist). When applicable, use the horizontal asymptote(s) as a guide.

37–46 Use the graph to find the indicated one-sided limits, if they exist.

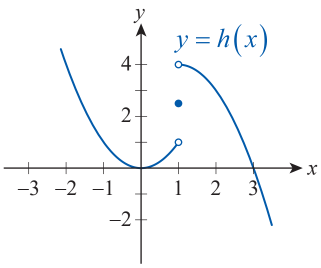
37. **a.** $\lim_{x\to 2^-} f(x)$ **b.** $\lim_{x\to 2^+} f(x)$



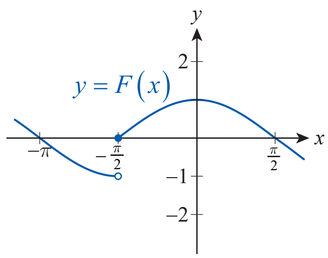
38. **a.** $\lim_{x\to 1^-} g(x)$ **b.** $\lim_{x\to 1^+} g(x)$



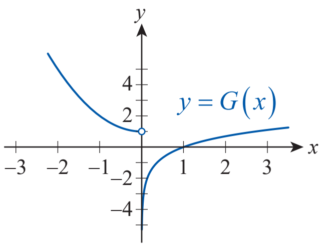
39. **a.** $\lim_{x\to 1^-} h(x)$ **b.** $\lim_{x\to 1^+} h(x)$



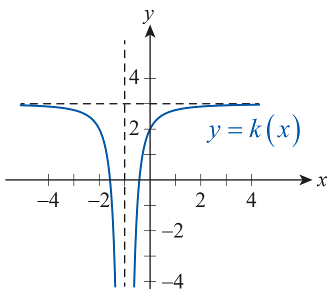
40. **a.** $\lim_{x\to(-\pi/2)^-} F(x)$ **b.** $\lim_{x\to(-\pi/2)^+} F(x)$



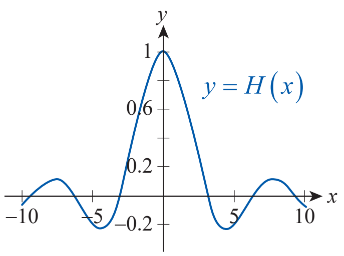
41. **a.** $\lim_{x\to 0^-} G(x)$ **b.** $\lim_{x\to 0^+} G(x)$



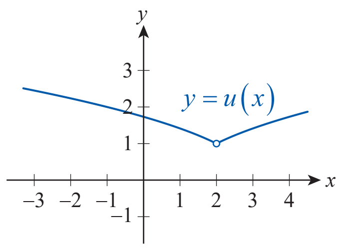
42. **a.** $\lim_{x\to -1^-} k(x)$ **b.** $\lim_{x\to -1^+} k(x)$



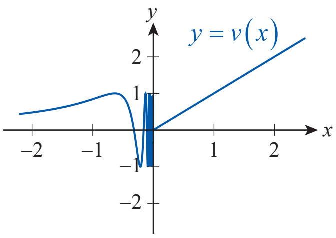
43. **a.** $\lim_{x\to 0^-} H(x)$ **b.** $\lim_{x\to 0^+} H(x)$



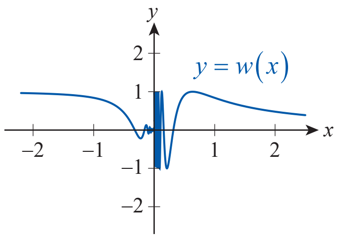
44. **a.** $\lim_{x\to 2^-} u(x)$ **b.** $\lim_{x\to 2^+} u(x)$



45. **a.** $\lim_{x\to 0^-} v(x)$ **b.** $\lim_{x\to 0^+} v(x)$



46. **a.** $\lim_{x\to 0^-} w(x)$ **b.** $\lim_{x\to 0^+} w(x)$



47–58 Use limit notation to describe the unbounded behavior of the given function as x approaches ∞ and/or $-\infty$.

47. $f(x) = x^3$

48. $g(x) = x^2 + 2.1x - 1$

49. $h(x) = -x^4 + 0.2x^3$

50. $k(x) = -0.35x^5 + x + 1.35$

51. $F(x) = \sqrt{x+2}$

52. $G(x) = \sqrt[3]{x+1} - 2.3$

53. $H(x) = |x+2|$

54. $K(x) = -|x+2| - 1$

55. $u(x) = |x-1| + |x+2|$

56. $v(x) = e^{x+2}$

57. $s(x) = -10^{-x} + 1$

58. $t(x) = \ln x - 1$

Concept Check

59–63 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

59. If $\lim_{x\to c} f(x)$ does not exist, then $f(x)$ is undefined at $x = c$.

60. If $f(x)$ is undefined at $x = c$, then $\lim_{x\to c} f(x)$ does not exist.

61. If $f(x)$ is defined on $(0,\infty)$ and $y = 0$ is a horizontal asymptote for $f(x)$, then there exists a number $M > 0$ such that if $x > M$ then $f(x) < 1/10^6$.

62. If $f(x)$ has a vertical asymptote at $x = c$, then either $\lim_{x\to c} f(x) = \infty$ or $\lim_{x\to c} f(x) = -\infty$.

63. If $\lim_{x\to c} f(x)$ does not exist, then $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$ or at least one of $\lim_{x\to c^-} f(x)$ or $\lim_{x\to c^+} f(x)$ does not exist.

2.2 Technology Exercises

64–71 Use a graphing utility to decide whether the given limit exists by evaluating the function at several x-values approaching the indicated c-value. Then graph the function to confirm your findings. Do you obtain misleading graphs when choosing small viewing windows?

64. $\lim_{x\to 2} \frac{x^2 - 5x + 6}{x - 2}$

65. $\lim_{x\to 3} \frac{x - 2}{x^2 - 5x + 6}$

66. $\lim_{x\to -1.5} \frac{x^2 - 2.25}{x + 1.5}$

67. $\lim_{x\to 1} \frac{x - 1}{\sqrt{x} - 1}$

68. $\lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x - 3}$

69. $\lim_{x\to 0} \frac{\sin 3x}{2x}$

70. $\lim_{x\to 0^+} \cos\frac{1}{x}$

71. $\lim_{x\to 0^+} x\cos\frac{1}{x}$

72. Evaluate the function $f(x) = \left(1 + \frac{1}{x}\right)^x$ for several consecutive positive integers, and try to observe a tendency. Then use a graphing utility to graph $f(x)$ in a large viewing window and try to guess $\lim_{x\to\infty} f(x)$. Have you seen that number before?

73. Write a program in a graphing calculator or computer algebra system to estimate the limit of an input function as x approaches c. (Calculate $f(x)$ successively at x-values increasingly close to c and display the results.) Try your program on Exercises 64–71.

74–81. Use the `Limit` command specific to your computer algebra system to evaluate the limits in Exercises 64–71. Are your previous results confirmed?