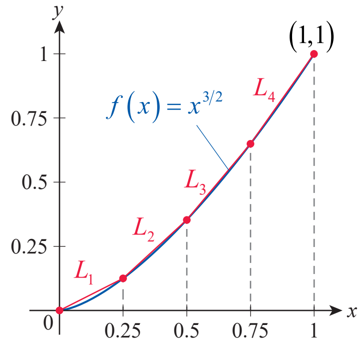


Figure 12

b. As far as the arc length of the curve connecting $(0,0)$ with $(1,1)$, we will illustrate the calculations by first dividing $[0,1]$ into four subintervals, as we did in part a.

If we label the endpoints as $x_1 = 0,\ x_2 = 0.25,\ \ldots,\ x_5 = 1$, and connect the points $(x_i, f(x_i))$ on the graph with $(x_{i+1}, f(x_{i+1}))$ for $i = 1, \ldots, 4$, a "crude" first approximation for the arc length will simply be the sum of the lengths of the four resulting line segments (see Figure 12). This can be calculated using the Pythagorean Theorem as follows.

$$\begin{aligned} s_4 &= L_1 + L_2 + L_3 + L_4 \\ &= \sqrt{(0.25)^2 + \left[(0.25)^{3/2}\right]^2} + \sqrt{(0.25)^2 + \left[(0.5)^{3/2} - (0.25)^{3/2}\right]^2} \\ &\quad + \sqrt{(0.25)^2 + \left[(0.75)^{3/2} - (0.5)^{3/2}\right]^2} + \sqrt{(0.25)^2 + \left[1 - (0.75)^{3/2}\right]^2} \\ &\approx 1.4362 \end{aligned}$$

Upon dividing $[0,1]$ into 10 equal parts, a similar, but a bit longer, calculation yields the much better approximation of

$$s_{10} \approx 1.4389.$$

As before, with the help of a computer or programmable calculator we can generate a table of values such as the following.

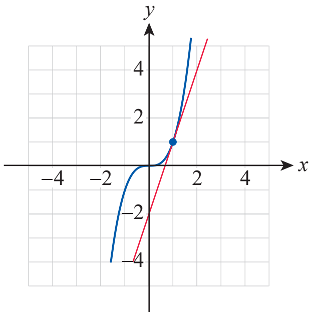
n	50	100	1000	10,000
s_n	1.43966	1.43970	1.43971	1.43971

From the table above, we conclude that the true value of the arc length is approximately $s \approx 1.43971$.

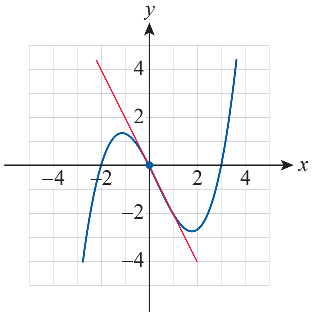
2.1 Exercises

1–6 Estimate the slope of the tangent line shown in the given graph.

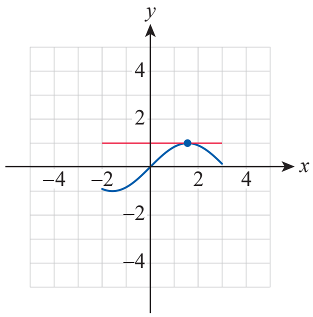
1.



2.



3.



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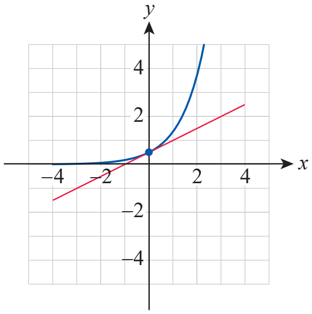
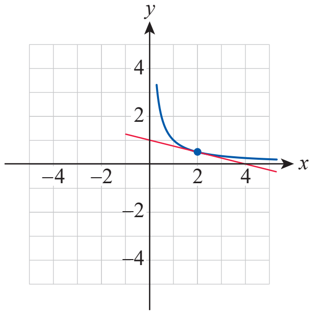
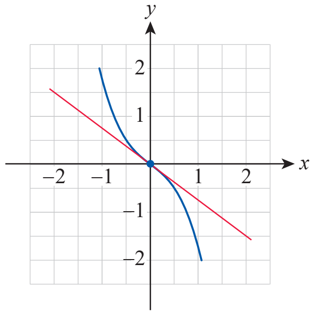


5.



6.



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7–18 Use difference quotients to approximate the slope of the tangent to the graph of the function at the given point. Use at least five different *h*-values that are decreasing in magnitude. (Answers will vary.)

7. $f(x)=1-2x;\quad (1,-1)$

8. $g(x)=\frac{5}{4}x-8;\quad (8,2)$

9. $h(x)=\frac{1}{3}x^2-1;\quad (3,2)$

10. $F(x)=3+x-\frac{x^2}{2};\quad (4,-1)$

11. $G(x)=\frac{1}{4}x^3-x+1;\quad (-2,1)$

12. $k(x)=10-x^{3/2};\quad (4,2)$

13. $H(x)=\ln x+1;\quad (e,2)$

14. $u(x)=\cos x;\quad \left(\frac{\pi}{2},0\right)$

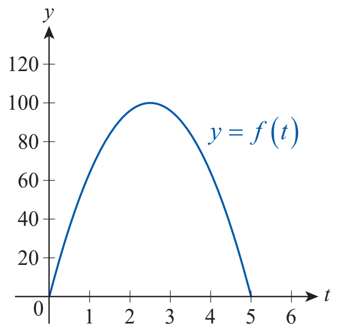
15. $v(x)=\log 2x-1;\quad (5,0)$

16. $w(x)=\tan x;\quad (0,0)$

17. $p(x)=-x^4+1;\quad (1,0)$

18. $q(x)=x^5-x+3;\quad (0,3)$

19. An arrow is shot into the air and its height in feet after t seconds is given by the function $f(t)=-16t^2+80t$. The graph of the curve $y=f(t)$ is shown.



a. Find the height of the arrow when $t=2$ seconds.

b. Find the instantaneous velocity of the arrow when $t=2$ seconds.

c. Find the slope of the line tangent to the curve at $t=2$ seconds.

d. Find the time it takes the arrow to reach its peak.

20. Suppose that a sailboat is observed, over a period of 5 minutes, to travel a distance from a starting point according to the function $s(t)=t^3+60t$, where t is time in minutes and s is the distance traveled in meters.

a. How far will it travel during the first 6 seconds?

b. What is the average velocity during the first 6 seconds?

c. Estimate how fast the boat is moving at the starting point.

d. Estimate how fast the boat is moving at the end of 3 minutes.

21. A model rocket is fired vertically upward. The height after t seconds is $h(t)=192t-16t^2$ feet.

a. What will be its height at the end of the first second?

b. What is the average velocity of the rocket during the first second?

c. Estimate the instantaneous velocity at $t=0$ seconds.

d. Estimate the instantaneous velocity at $t=4$ seconds.

e. When will the velocity be 0? (**Hint:** Start with the initial velocity you found in part c. and use the fact that under the influence of gravity, when air resistance is ignored, vertical upward velocity decreases by 32 ft/s every second. Once you have a guess, test it using a table of difference quotients.)

22. A particle moving in a straight line is at a distance of $s(t)=2.5t^2+18t$ feet from its starting point after t seconds, where $0\le t\le 12$. Estimate the instantaneous velocity at **a.** $t=6$ seconds and **b.** $t=9$ seconds.

23. The distance, in meters, traveled by a moving particle in t seconds is given by $d(t)=3t(t+1)$. Estimate the instantaneous velocity at **a.** $t=0$ seconds, **b.** $t=2$ seconds, and **c.** at time t_0. (**Hint:** Write the difference quotient corresponding to $t=t_0$, simplify, and try to find the value being approached by the expression as h decreases.)

24. The distance, in meters, traveled by a moving particle in t seconds is given by $d(t)=t^2-3t$. Estimate the instantaneous velocity at **a.** $t=0$ seconds, **b.** $t=4$ seconds, and **c.** at time t_0. (See the hint given in Exercise 23c.)

25. After start, on a straight stretch of the track, a race car's velocity changes according to the function $v(t) = -1.8t^2 + 18t$, when $0 \le t \le 10$, t is measured in seconds, and $v(t)$ is measured in meters per second.

a. When does peak velocity occur and what is it? (**Hint:** The graph of $v(t)$ may be helpful.)

b. When does peak deceleration occur?

c. Use difference quotients to estimate peak deceleration. Approximately what multiple of $g \approx 9.81 \text{ m/s}^2$ have you obtained?

26. If we ignore air resistance, a falling body will fall $16t^2$ feet in t seconds.

a. How far will it fall between $t = 2$ and $t = 2.1$?

b. What is its average velocity between $t = 2$ and $t = 2.1$?

c. Estimate its instantaneous velocity at $t = 2$.

27. A student dropped a textbook from the top floor of his dorm and it fell according to the formula $S(t) = -16t^2 + 8\sqrt{t}$, where t is the time in seconds and $S(t)$ is the distance in feet from the top of the building.

a. If the textbook hit the ground in exactly 2.5 seconds, how high is the building?

b. What was the average speed for the trip?

c. What was the instantaneous velocity at $t = 1$ second?

d. What was the velocity of impact?

28–31 Approximate the area of the region between the graph of the function and the x-axis on the given interval. Use **a.** $n = 4$ and **b.** $n = 5$. (Round your answers to four decimal places.)

28. $f(x) = x^2$ on $[0,1]$

29. $g(x) = 16x - x^3$ on $[0,4]$

30. $h(x) = \sin x$ on $[0,\pi]$

31. $F(x) = e^x + 1$ on $[-10,0]$

32–35 Approximate the arc length of the graph of the function on the given interval. Use **a.** $n = 4$ and **b.** $n = 5$. (Round your answers to four decimal places.)

32. $f(x) = \sqrt{x}$ on $[0,1]$

33. $g(x) = x^3 + x^2$ on $[-1,0]$

34. $F(x) = \cos x$ on $\left[0, \frac{\pi}{2}\right]$

35. $G(x) = \ln x + 1$ on $[1,2]$

2.1 Technology Exercises

36–39 Use a graphing utility to graph $f(x)$ along with three secant lines at the indicated x-value, corresponding to the difference quotients with h-values of 0.2, 0.1, and 0.01, respectively. Can you come up with a possible equation for the tangent line? Use technology to test your conjecture.

36. $f(x) = x^2; \quad x = 2$

37. $f(x) = -x^3 + x + 1; \quad x = \frac{\sqrt{3}}{3}$

38. $f(x) = \sin x + \cos x; \quad x = 0$

39. $f(x) = 3\sqrt{x}; \quad x = 4$

40–43 Use a graphing utility to graph the given function $f(x)$ along with $D(x) = \frac{f(x+0.001) - f(x)}{0.001}$ in the same coordinate system. Explain how the function values of $D(x)$ are reflected on the graph of $f(x)$.

40. $f(x) = x^4$

41. $f(x) = x(3 - x)$

42. $f(x) = \sin x$

43. $f(x) = \ln x$

44–47 Use a graphing utility to find the x-values at which the graph of $f(x)$ does not have a tangent line. Explain.

44. $f(x) = -|x-1| + 1$

45. $f(x) = |x^2 - 4|$

46. $f(x) = |\ln x|$

47. $f(x) = (x-1)^{2/3}$

48–51. Use a computer algebra system to find approximations for the areas in Exercises 28–31 by using **a.** $n = 100$ and **b.** $n = 1000$. (Round your answers to four decimal places.)

52–55. Use a computer algebra system to find approximations for the arc lengths in Exercises 32–35 by using **a.** $n = 100$ and **b.** $n = 1000$. (Round your answers to four decimal places.)

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