13.1 Exercises

1–4 Evaluate the given multivariable function at the indicated points.

1. $f(x,y) = xy - y^3x^2$; $(0,0)$, $(2,1)$
2. $f(u,v) = (2u+1)(v-u^2)$; $(3,-1)$, $(1,1)$
3. $f(x,y,z) = \dfrac{xz}{2y^2+z^4}$; $(5,0,-1)$, $(-4,2,1)$
4. $f(t,u,v,w) = \dfrac{(4t-v)^w}{(u^2+3)}$; $(1,1,3,1)$, $(2,-1,7,12)$

5–12 Determine the domain and range of the given function and evaluate the function at the indicated point.

5. $f(x,y) = xy + yx^3$; $(3,-1)$
6. $f(x,y) = 2y^2\left(x + \dfrac{y}{2}\right)$; $(0,2)$
7. $f(x,y) = \sqrt{x^2y}$; $(2,9)$
8. $f(x,y) = \dfrac{\sqrt{36-4x^2-9y^2}}{6}$; $(2,0)$
9. $f(x,y) = \ln|xy|$; $(e,-1)$
10. $f(x,y) = \sqrt{1-x^2y^2}$; $\left(-2,\dfrac{1}{4}\right)$
11. $f(x,y,z) = \arctan(xz - y^2)$; $(5,3,2)$
12. $f(x,y,z) = \dfrac{ze^{x/y}}{\sqrt{x-z}}$; $(0,-1,-2)$

13–20 Describe in words the graph of the function. (**Hint:** It is helpful to review quadric surfaces from Section 11.6.)

13. $f(x,y) = 3x + y - 2$
14. $f(x,y) = \dfrac{\sqrt{144-9x^2-16y^2}}{6}$
15. $f(x,y) = \dfrac{x^2}{4} + \dfrac{(y-1)^2}{9}$
16. $f(x,y) = \dfrac{\sqrt{9x^2+4y^2-36}}{6}$
17. $f(x,y) = \sqrt{x^2+2y^2}$
18. $f(x,y) = \sqrt{x^2+2y^2+1}$
19. $f(x,y) = 2x^2+3y^2$
20. $f(x,y) = \dfrac{3x^2-6y^2}{4}$

21–26 A region R and a point P in the Cartesian plane $\mathbb{R}^2$ are given. Classify the point as an interior point of R, a boundary point, or neither.

21. $R = \{(x,y) \mid y > |x|\}$; $P(1,1)$
22. $R = \{(x,y) \mid x^2+y^2 \le 1\}$; $P\left(\dfrac{1}{2},\dfrac{1}{2}\right)$
23. $R = \{(x,y) \mid 1 < x^2+y^2 < 2\}$; $P(0,1)$
24. $R = \{(x,y) \mid y \le x^2\}$; $P(1,2)$
25. $R = \{(x,y) \mid |x+2| + |x-3| < y\}$; $P(1.5,5)$
26. $R = \{(x,y) \mid y - |2-x| \ge 0\}$; $P(3,0)$

27–34 Classify the given subset R of $\mathbb{R}^2$ as open, closed, or neither.

27. $R = \{(x,y) \mid (x-2)^2 + y^2 \le 4\}$
28. $R = \{(x,y) \mid x^2+2y^2 \ge 3\}$
29. $R = \{(x,y) \mid 0 < x^2+y^2 < 9\}$
30. $R = \{(x,y) \mid y \ne 2x-3\}$
31. $R = \{(x,y) \mid xy \ne 0\}$
32. $R = \{(x,y) \mid |x| + |y| \le 1\}$
33. $R = \{(x,y) \mid \sqrt{x} + \sqrt{y} < 1\}$
34. $R = \{(x,y) \mid x > 0 \text{ or } y \ge 0\}$

35–42 The graphs of the given equations are quadric surfaces as seen in Exercises 27–34 of Section 11.6. Express each as a function of two variables other than the pair x and y. (Note that the graph of the resulting function may not be the entire surface. Can you see why?)

35. $3z^2 + 2y^2 = \dfrac{3x}{2}$
36. $3z^2 + 2y^2 = 1 - \dfrac{3x^2}{2}$
37. $2x^2 + 2y^2 = z$
38. $2x^2 + 2y^2 = z^2$
39. $x^2 + 2x - y^2 + z^2 = 0$
40. $x^2 + 2x + 2y^2 + 3z^2 = 8y$
41. $x^2 + 2x - 2y^2 - 3z = 8y + 7$
42. $-x^2 + 2x + 2y^2 - 3z^2 = 8y + 2$

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43–48 Match the function with its graph (labeled A–F).

43. $f(x,y)=\dfrac{(x+y)^2}{2}$

44. $f(x,y)=\cos|xy|$

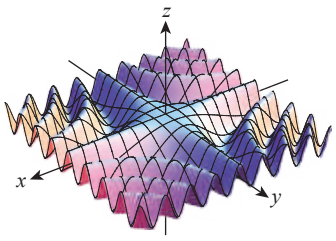
45. $f(x,y)=\dfrac{3}{4x^2+3y^2+1}$

46. $f(x,y)=|x-1|+|y|$

47. $f(x,y)=\sin(x+2y)$

48. $f(x,y)=\cos(x^2+y^2)$

A.



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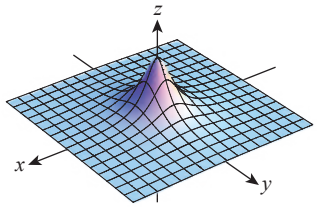

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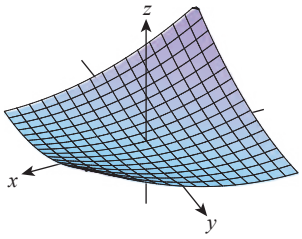

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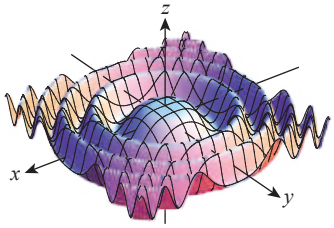

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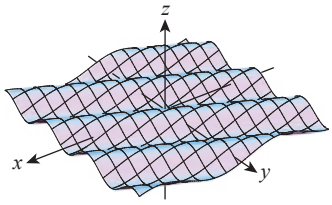

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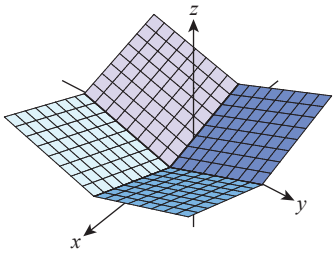
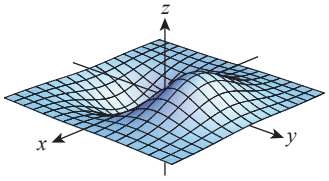


49–54 Match the graph with its contour map (labeled A–F).

49.



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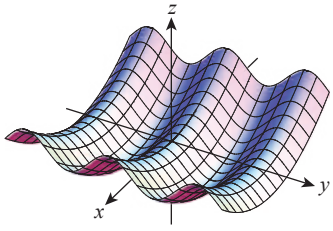

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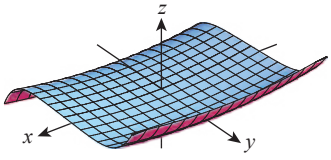

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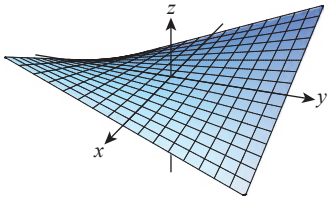

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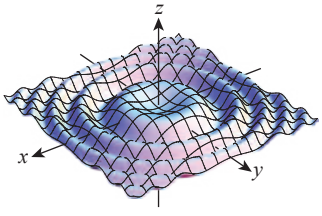

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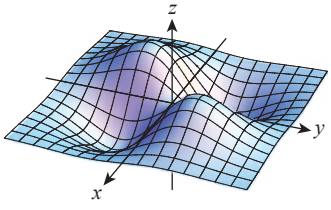
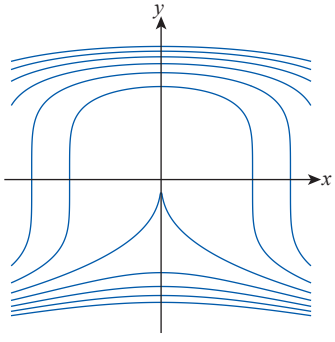


A.



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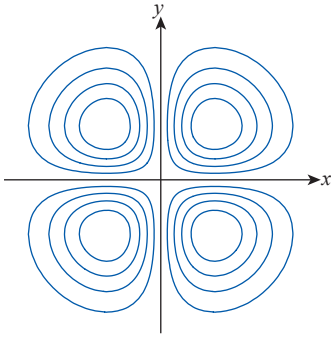

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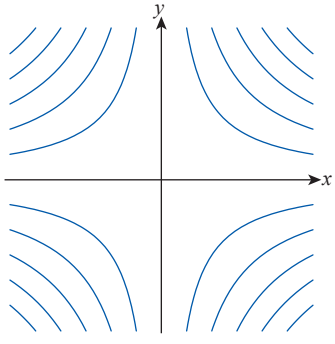

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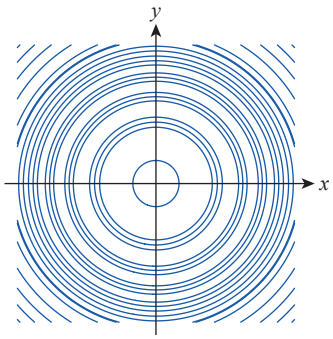
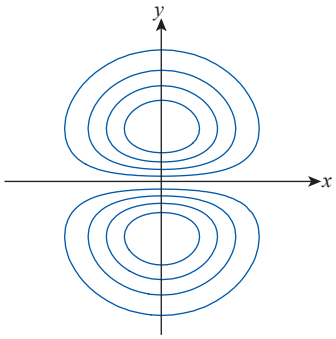
D.



E.



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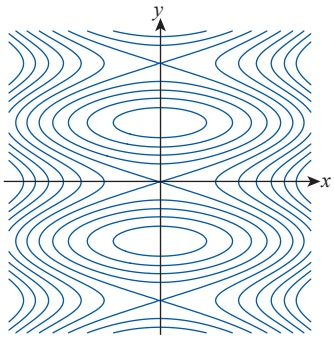


55–60 Sketch a rough graph of the given function by hand. Then sketch its contour map by selecting a few representative contours.

55. $f(x,y) = x - 3y + 1$

56. $f(x,y) = \dfrac{x^2}{9} + \dfrac{y^2}{4}$

57. $f(x,y) = y^2 - x$

58. $f(x,y) = \ln\left(x^2 + y^2\right)$

59. $f(x,y) = \dfrac{y}{x^2 + y^2 + 1}$

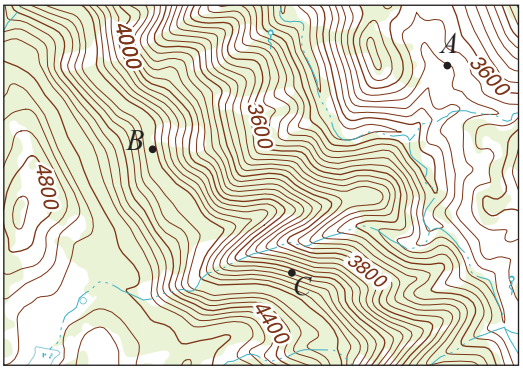
60. $f(x,y) = \sin\sqrt{x^2 + y^2}$

61–62 Describe the level surfaces of the given three-variable function.

61. $f(x,y,z) = x^2 + 2y^2 + 3z^2$

62. $f(x,y,z) = x^2 + 2y^2 - 3z^2$

63. The figure below shows a portion of a topographical map of an area near Julian, California. Examine the map and answer the following.



Source: www.usgs.gov

a. Estimate the direction of the steepest slope from point A.

b. Find a possible "steepest path" from point A to point B.

c. Estimate the elevation of point C.

d. Find a point D where the northern direction is uphill, while it is downhill to the southwest.

64.* Prove that if S is a finite subset (i.e., a set consisting of finitely many points) of $\mathbb{R}^2$, then $\mathbb{R}^2 - S$ is an open subset of $\mathbb{R}^2$. (**Hint:** Pick an arbitrary point P in $\mathbb{R}^2 - S$ and prove that it is an interior point.)

65.* Prove that the open interval $(0,1)$, when viewed as a subset of $\mathbb{R}^2$, is neither open nor closed.

13.1 Technology Exercises

66–71. Use a computer algebra system to generate the graphs and contour maps of the functions in Exercises 55–60.

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