

11.5 Exercises

1–3 Determine which of the given points lie on the given line L .

- L is the line $\mathbf{r}(t) = \langle 2 + 3t, -1 + t, 4 + 2t \rangle$;
 $A(5, 0, 6)$, $B(0, 0, 2)$, $C(-1, -2, 2)$
- L is the line through the point $(-1, 0, 5)$ and parallel to $\mathbf{i} - 3\mathbf{j} + \mathbf{k}$; $A(1, -6, 3)$, $B(-3, 6, 3)$, $C(-\frac{1}{2}, -\frac{3}{2}, \frac{11}{2})$
- L is the line through the points $(-6, -2, 1)$ and $(-4, 1, 3)$; $A(-2, 4, 5)$, $B(4, 13, 11)$, $C(0, 7, 2)$

4–11 Find the vector form and the parametric equations of the given line.

- The line through the point $(2, 5, -3)$ and parallel to $\mathbf{i} + \mathbf{j} + 3\mathbf{k}$
- The line through the point $(1, 2, 3)$ and parallel to $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$
- The line through the point $(14, 0, -\frac{13}{4})$ and parallel to $\langle -1, \frac{1}{2}, \frac{1}{3} \rangle$
- The line through the point $(0, -5, 8)$ and parallel to $\langle -4, -15, -19 \rangle$
- The line through the points $(0, -1, 0)$ and $(4, 1, 1)$
- The line through the point $(-9, 11, -6)$ and parallel to $4\mathbf{i} + 3\mathbf{k}$
- The line through the points $(3, -5, 1)$ and $(0, 2, 7)$
- The line through the points $(16, 21, 28)$ and $(-10.5, 32, 12)$

12–19. By changing the initial point and the tracing rate, find alternative parametrizations for each of the lines in Exercises 4–11.

20–24 Find parametric equations for the line segment between the two given points. (**Hint:** First determine a direction vector and be sure to appropriately restrict the possible values of the parameter t .)

- The line segment between $(-1, 0, 1)$ and $(3, 2, 0)$
- The line segment between $(1, 2, 3)$ and $(3, 2, 1)$
- The line segment between $(-2, 4, 7)$ and $(-1, 5, 8)$
- The line segment between $(-3, \frac{2}{3}, -11)$ and $(5, -2, -7)$
- The line segment between $(-\frac{1}{2}, 1, \frac{2}{3})$ and $(\frac{4}{3}, -6, \frac{3}{2})$

25–32. Find the intersection points of the lines in Exercises 4–11 with the coordinate planes.

33–37 If possible, determine the point of intersection of the given pair of lines.

- $\mathbf{r}(t) = \langle 1 + t, -2 + 5t, 1 \rangle$ and $\mathbf{s}(u) = \langle 4 + 3u, 2u, 2 + u \rangle$
- $\mathbf{r}(t) = \langle 4 + 2t, -1 - t, 2 + 3t \rangle$ and $\mathbf{s}(u) = \langle \frac{1}{2} + 7u, \frac{1}{2} - 3u, \frac{3}{2} + u \rangle$
- $\mathbf{r}(t) = \langle 3 + t, 3t, 1 + 2t \rangle$ and $\mathbf{s}(u) = \langle 1 + u, -1 - 2u, -2 + u \rangle$
- $\mathbf{r}(t) = \langle 3 + 2t, 3 + 2t, \frac{3}{2} + \frac{1}{2}t \rangle$ and $\mathbf{s}(u) = \langle 5 - 2u, 6u - 19, \frac{43}{2} - 7u \rangle$
- $\mathbf{r}(t) = \langle 1 - t, 2t - 1, 4t \rangle$ and $\mathbf{s}(u) = \langle 2 + u, 5 + 2u, 2 \rangle$

38–42 Find a scalar equation for the plane containing the given point and having the indicated normal vector. Then describe the plane as a two-parameter set of points in \mathbb{R}^3 .

- The plane through the origin, with normal vector $\mathbf{n} = \langle -1, 3, 1 \rangle$
- The plane through the point $(2, 0, 9)$, with normal vector $\mathbf{n} = \langle 5, -4, -1 \rangle$
- The plane through the point $(7, 2, 4)$, with normal vector $\mathbf{n} = \langle -1, 2, -5 \rangle$
- The plane through the point $(1, 2, 3)$, with normal vector $\mathbf{n} = \langle \frac{1}{2}, \frac{3}{4}, \frac{5}{6} \rangle$
- The plane through the point $(-2, 3, 5)$, with normal vector $\mathbf{n} = \langle 7, -\frac{2}{5}, -\frac{5}{2} \rangle$

43–47 Find an equation for the plane containing the given points.

- $A(2, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, -3)$
- $A(2, 0, -1)$, $B(3, -4, 4)$, $C(-1, 5, 2)$
- $A(1, 2, 0)$, $B(3, 0, 4)$, $C(0, 5, 6)$
- $A(-3, \frac{1}{2}, 1)$, $B(-1, 0, \frac{5}{2})$, $C(-4, -1, 2)$
- $A(4, 0, -\frac{3}{5})$, $B(1, 0, 1)$, $C(2, -1, -5)$

48–51 Find the parametric equations of the line as described.

48. The line through the point $(-5, 7, 8)$ that is perpendicular to the plane $2x - y + 3z = 4$
49. The line through $(-3, -1, 4)$ that is parallel to the line $\mathbf{r}(t) = \langle 1+t, -2+4t, 1-3t \rangle$
50. The line through the origin that is perpendicular to the vectors $\mathbf{i} + 2\mathbf{j}$ and $\mathbf{i} - \mathbf{k}$
51. The line through the point $(-1, -1, -5)$ that forms a right angle with the line $\mathbf{r}(t) = \langle 1+t, 1+2t, 1-t \rangle$

52–56. Solving each of the parametric equations $x = x_0 + at$, $y = y_0 + bt$, and $z = z_0 + ct$ for t and equating the results yields the so-called **symmetric equations** for a line.

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

Use this method to find the symmetric equations for the lines in Exercises 4–8.

57–65 Find the equation of the plane as described.

57. The plane through the point $(-2, 1, 5)$, parallel to the plane $3x - 2y + 5z = 1$
58. The plane through the point $(-5, 0, 1)$, perpendicular to the vector $\langle 1, -1, 7 \rangle$
59. The plane through the point $(6, -1, 2)$, perpendicular to the line $\mathbf{r}(t) = \langle 1-t, 3+4t, 2+t \rangle$
60. The plane that contains the z -axis and makes an angle of $\pi/6$ with the positive x -axis
61. The plane through the point $(3, 5, -2)$ that contains the line $x = 2 - t$, $y = 4 + 2t$, $z = t - 3$
62. The plane containing the lines $\mathbf{r}(t) = \langle 1+t, -2+5t, 1 \rangle$ and $\mathbf{s}(u) = \langle 4+3u, 2u, 2+u \rangle$
63. The plane containing the lines $\mathbf{r}(t) = \langle 4+2t, -1-t, 2+3t \rangle$ and $\mathbf{s}(u) = \langle \frac{1}{2} + 7u, \frac{1}{2} - 3u, \frac{3}{2} + u \rangle$
64. The plane through the points $(4, -1, 1)$ and $(1, 0, -2)$ that is perpendicular to the plane $2x - y + z = 9$
65. The plane that contains the line $\mathbf{r}(t) = \langle t-2, 4+t, 3-2t \rangle$ and the intersection of the planes $x - y + 2z = 0$ and $2x + 2y - z = 1$

66–69 Find the point of intersection between the given line and plane.

66. The line $\mathbf{r}(t) = \langle t-1, 4t, t \rangle$ and the plane $3x - y + 4z = 3$
67. The line $\mathbf{r}(t) = \langle 2+5t, 3t-2, 4+t \rangle$ and the plane $x + 2y - 6z = 2$
68. The line $\mathbf{r}(t) = \langle 8+t, 4+2t, 3+\frac{t}{3} \rangle$ and the plane $x + 2y - 2z + 3 = 0$
69. The line $\mathbf{r}(t) = \langle 1+\frac{t}{9}, 1-\frac{t}{12}, 1+\frac{t}{4} \rangle$ and the plane $5y + z = 15 + x$

70–73 Find the parametric equations for the line formed by the intersection of the two given planes.

70. $x - 2y + z = 4$, the xz -plane
71. $x + 3y - z = 2$, $2x - y + z = 1$
72. $x - 2y - z = 3$, $-x + y + 2z = 0$
73. $3x + y - 2z + 1 = 0$, $x + y - 4z = 5$
74. Find an equation for the set of points that are equidistant from the points $(2, -1, 6)$ and $(-4, 5, 2)$.
75. Find the equation of the plane having x -, y -, and z -intercepts of $(a, 0, 0)$, $(0, b, 0)$, and $(0, 0, c)$, respectively.
76. Find three planes that intersect along the line $\mathbf{r}(t) = \langle t-1, 4t, t \rangle$.

77–80 Find the shortest distance between the point and the plane.

77. $A(0, 0, 0)$; $4x - y + 5z = 1$
78. $A(-2, -1, 7)$; $x - 3y + 2z = 0$
79. $A(-3, 1, -1)$; $2x - 2y + z = 3$
80. $A(1, 2, 3)$; $x + 2y + 3z + 4 = 0$

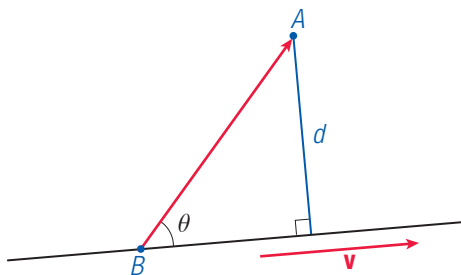
81–84 Find the shortest distance d between the given skew lines.

81. $\mathbf{r}(t) = \langle 1, 1+t, t \rangle$ and $\mathbf{s}(u) = \langle -2u, 1-u, 2-2u \rangle$
82. $\mathbf{r}(t) = \langle 1+4t, 0, -2t \rangle$ and $\mathbf{s}(u) = \langle 4u-4, 2u, u \rangle$
83. $\mathbf{r}(t) = \langle 2t, t-1, -1 \rangle$ and $\mathbf{s}(u) = \langle u-1, 2u, 2u-1 \rangle$
84. $\mathbf{r}(t) = \langle 1+2t, 2-9t, 1-3t \rangle$ and $\mathbf{s}(u) = \langle 2u, 16u-7, 2u \rangle$

85–88 We can find the shortest distance in three-dimensional space from a point A to a line containing a point B by the following argument. Denoting the direction vector of the line by \mathbf{v} , and the angle formed by \mathbf{v} and the vector \overrightarrow{BA} by θ , note that the distance d we are seeking is $|\overrightarrow{BA}| \cdot \sin\theta$ (see figure below). Recalling that $|\mathbf{v} \times \overrightarrow{BA}| = |\mathbf{v}| \cdot |\overrightarrow{BA}| \cdot \sin\theta$, d can be expressed as

$$d = \frac{|\mathbf{v} \times \overrightarrow{BA}|}{|\mathbf{v}|}.$$

(Note that this is an improvement over Exercise 86 of Section 11.3 where both the point and the line were lying in the xy -plane. However, it is worth comparing the above formula with that of Exercise 68 of Section 11.4.)



In Exercises 85–88, use the above formula to find the shortest distance between the point and the line.

85. $A(2, -1, 1)$; $\mathbf{r}(t) = \langle 3t, 1-t, 1+t \rangle$
 86. $A(-2, 1, 0)$; $\mathbf{r}(t) = \langle 1+t, t, 2-t \rangle$
 87. $A(1, 0, 0)$; $\mathbf{r}(t) = \langle 1-2t, 1-t, 1+3t \rangle$
 88. $A\left(\frac{1}{2}, 5, -4\right)$; $\mathbf{r}(t) = \left\langle \frac{3}{2}-t, 6+t, -3-2t \right\rangle$

Concept Check

89–116 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

89. If the corresponding coefficients of x , y , and z in the equations of two planes are equal, then the planes are parallel. (Here we assume that both equations are in the standard form $ax + by + cz = d$.)
90. If two planes are parallel, then the corresponding coefficients of x , y , and z (as in Exercise 89) in their equations are equal.
91. If the corresponding coefficients in the equations of two planes (as in Exercise 89) are negative reciprocals, then the planes are perpendicular.
92. If two lines are parallel to a common line, then they are parallel.
93. If two lines are parallel to a common plane, then they are parallel.
94. If two lines are perpendicular to a common plane, then they are parallel.
95. If two lines are perpendicular to a common line, then they are parallel.
96. If two planes are perpendicular to a common plane, then they are parallel.
97. If two planes are perpendicular to a common line, then they are parallel.
98. If two planes are parallel to a common line, then they are parallel.
99. If two planes are parallel to a common plane, then they are parallel.
100. If two lines do not intersect, then they are parallel.
101. If two planes do not intersect, then they are parallel.
102. If a line and a plane do not intersect, then they are parallel.
103. The intersection of a line and a plane is empty or a single point.
104. The intersection of two distinct planes is empty or it is a line.
105. If plane P is parallel to the xz -plane, then $\langle 1, 0, -1 \rangle$ is normal to P .
106. If plane P is parallel to the xz -plane, then $\langle 0, -1, 0 \rangle$ is normal to P .
107. If plane P is parallel to the xz -plane, then the coefficient of y in its equation is 0.
108. If plane P is parallel to the xz -plane, then the coefficients of both x and z in its equation are 0.
109. The plane $y = 0$ contains the y -axis.
110. The plane $2z - x = 3$ contains the y -axis.
111. The plane $2z - x = 0$ contains the y -axis.
112. The plane with equation $ax + by + cz = d$ goes through the origin if and only if $d = 0$.
113. If \mathbf{n}_1 and \mathbf{n}_2 are normal vectors to the planes P_1 and P_2 , respectively, then $\mathbf{n}_1 \times \mathbf{n}_2$ is a direction vector for their line of intersection.

114. If L_1 and L_2 are skew lines, then there is a pair of parallel planes containing them.
115. The vector equation $\mathbf{r}(t) = \langle 3+t, 5-2t, 1-4t \rangle$ and the parametric equations $x = 5 - \frac{t}{2}$, $y = 1 + t$, and $z = -7 + 2t$ describe the same line.
116. The equations $4(1-x) + 2(y-7) - 14(z-1) = 0$ and $2x - y + 7z = 2$ describe the same plane.

11.5 Technology Exercises

- 117–121. Use a computer algebra system or programmable calculator to write a program that returns the equation of a plane through three given points in three-dimensional space. Use your program to check your answers for Exercises 43–47.
- 122–130. Use a computer algebra system or programmable calculator to write a program that decides whether two lines intersect in three-dimensional space. If they do, the program should return the coordinates of the intersection point, otherwise the message “skew lines” should appear, along with the shortest distance between them. Use your program to revisit Exercises 33–37 and 81–84.