11.2 Exercises

1–8 Quantities are given that arise in everyday life. Decide whether they are vectors or scalars.

1. The speed of your car

2. The cost of a telephone call

3. The velocity of your car

4. The mass of a golf cart

5. The weight of a gallon of milk

6. The displacement of a particle that moved from $(2,-1)$ to $(5,8)$ in the xy-system

7. The distance covered by a flight from Houston to San Diego

8. The restoring force exerted by a vertical spring when a mass is hung on it

9–14 Decide whether the following is a vector.

9. $(2,\pi)$

10. $(1,0,-4)$

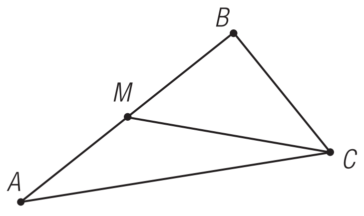
11. $\langle 0,0\rangle$

12. $\left\langle 0,\dfrac{\pi}{2},\cos\dfrac{\pi}{2}\right\rangle$

13. $\langle 2,-1,3\rangle-\langle 1,5,10\rangle$

14. $\left|\langle 2,-1,3\rangle\right|$

15–20 Use the figure below to express the indicated sum or difference as a single vector. (M is the midpoint of $\overline{AB}$.)



15. $\overrightarrow{CA}+\overrightarrow{CB}$

16. $\overrightarrow{CA}-\overrightarrow{CB}$

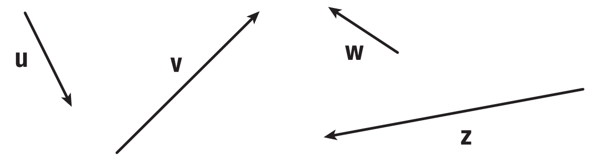
17. $\overrightarrow{BC}-\overrightarrow{BM}$

18. $\overrightarrow{AB}+\overrightarrow{BM}+\overrightarrow{MC}$

19. $3\overrightarrow{AM}-\overrightarrow{AB}-\overrightarrow{AC}$

20. $2\overrightarrow{BM}+\overrightarrow{AC}$

21–24 Geometrically construct the indicated linear combination from the given vectors.



21. $\mathbf{u}-\mathbf{z}$

22. $\mathbf{v}+2\mathbf{w}$

23. $2\mathbf{u}+\mathbf{v}-\mathbf{w}$

24. $3\mathbf{w}+\dfrac{1}{2}\mathbf{z}-\mathbf{u}$

25–30 Find the linear combination in component form if the vectors **u** and **v** are given in component form as $\mathbf{u}=\langle 4,-1\rangle$ and $\mathbf{v}=\langle 5,8\rangle$.

25. $3\mathbf{u}$

26. $2\mathbf{u}+\mathbf{v}$

27. $4\mathbf{v}-\mathbf{u}$

28. $\dfrac{1}{4}\mathbf{u}+\dfrac{3}{4}\mathbf{v}$

29. $\mathbf{u}-\dfrac{1}{2}\mathbf{v}$

30. $\dfrac{2}{5}\mathbf{v}-\pi\mathbf{u}$

31–34 Find the coordinates of the endpoint of the indicated vector with the given initial point if $\mathbf{u}=\langle 3,-1,2\rangle$ and $\mathbf{v}=\langle 2,1,-1\rangle$.

31. $2\mathbf{u}$ with initial point $(4,0,-1)$

32. $-4\mathbf{v}$ with initial point $(2,5,-3)$

33. $\mathbf{u}-2\mathbf{v}$ with initial point $(3,7,-10)$

34. $5\mathbf{v}-3\mathbf{u}$ with initial point $(-3,6,11)$

35–36 Find a vector **v** that solves the vector equation.

35. $2\mathbf{v}+\langle 4,-8,2\rangle=\langle 2,2,2\rangle$

36. $\langle 4,-7,1\rangle-4\mathbf{v}=\langle 5,-1,0\rangle$

37–38 Use vectors to determine whether the given points are collinear (i.e., whether they fall on the same line).

37. $P(2,0,-1),\quad Q(-4,3,1),\quad R(8,-3,-3)$

38. $P\left(1,\dfrac{3}{2},-5\right),\quad Q(3,-1,-1),\quad R\left(9,-\dfrac{17}{2},13\right)$

39–41 Express the vector **w** as a linear combination of **u** and **v**. (**Hint:** Using undetermined coefficients, start with the vector equation $\mathbf{w}=a\mathbf{u}+b\mathbf{v}$, and solve the resulting linear system.)

39. $\mathbf{w}=\langle 1,2\rangle;\quad \mathbf{u}=\langle 2,3\rangle,\quad \mathbf{v}=\langle 0,1\rangle$

40. $\mathbf{w}=\langle 4,-7\rangle;\quad \mathbf{u}=\langle 1,5\rangle,\quad \mathbf{v}=\langle 2,1\rangle$

41. $\mathbf{w} = \langle -13,6 \rangle$; $\mathbf{u} = \langle -3,1 \rangle$, $\mathbf{v} = \langle 4,-8 \rangle$

42–45 Find the magnitude of the given vector.

42. $\mathbf{u} = \langle -4,3 \rangle$

43. $\mathbf{v} = \left\langle \frac{1}{5}, \frac{\sqrt{3}}{5} \right\rangle$

44. $\mathbf{w} = \left\langle 3, \frac{9}{4} \right\rangle$

45. $\mathbf{z} = \left\langle -\frac{1}{2}, -\sqrt{2} \right\rangle$

46–49 Find the indicated magnitude given that $\mathbf{u} = \langle -2,-5 \rangle$ and $\mathbf{v} = \langle -1,3 \rangle$.

46. $|\mathbf{u} + 2\mathbf{v}|$

47. $|\mathbf{v} - 4\mathbf{u}|$

48. $\left| \frac{\mathbf{u} - \mathbf{v}}{3} \right|$

49. $\left| -\frac{1}{10}\mathbf{u} + \frac{2}{5}\mathbf{v} \right|$

50–55 Determine whether the vectors $\overrightarrow{P_1Q_1}$ and $\overrightarrow{P_2Q_2}$ are equal.

50. $P_1(0,1)$, $Q_1(5,1)$, $P_2(-1,-2)$, $Q_2(4,-2)$

51. $P_1(-2,0)$, $Q_1(-2,-3)$, $P_2(1,-1)$, $Q_2(1,2)$

52. $P_1(0,0)$, $Q_1(0,\sqrt{2})$, $P_2(1,1)$, $Q_2(2,2)$

53. $P_1(-5,0,2)$, $Q_1(1,1,-4)$, $P_2(0,-1,7)$, $Q_2(6,0,1)$

54. $P_1(0,0,0)$, $Q_1(3,5,-1)$, $P_2(4,1,-3)$, $Q_2(7,6,2)$

55. $P_1(2,1,3)$, $Q_1(-1,-4,4)$, $P_2(3,2,1)$, $Q_2(4,-1,-4)$

56–61 Find the component form and magnitude of the vector $\overrightarrow{PQ}$.

56. $P(4,0,-3)$, $Q(0,0,0)$

57. $P(2,9,-5)$, $Q(2,9,-5)$

58. $P(-1,3,4)$, $Q(-5,0,4)$

59. $P(-2,-1,3)$, $Q(1,-4,1)$

60. $P\left(-5,1,\frac{\sqrt{7}}{2}\right)$, $Q\left(-1,-\frac{1}{2},0\right)$

61. $P\left(\frac{1}{3},8,\frac{\sqrt{5}+1}{3}\right)$, $Q\left(-\frac{1}{3},5,\frac{1}{3}\right)$

62–67 Decide whether the points determine a parallelogram in three-dimensional Cartesian space.

62. $A(-1,2)$, $B(-2,-3)$, $C(6,-2)$, $D(7,3)$

63. $A(-1,4)$, $B(-3,1)$, $C(2,-5)$, $D\left(5,-\frac{1}{2}\right)$

64. $A(2,0,-3)$, $B(-4,1,0)$, $C(-1,2,7)$, $D(5,1,4)$

65. $A(-1,1,2)$, $B(5,0,-2)$, $C(9,-3,0)$, $D(3,-2,4)$

66. $A(0,1,0)$, $B(-3,0,4)$, $C(-1,1,4)$, $D(2,1,1)$

67. $A(-1,-2,-3)$, $B(4,1,1)$, $C(-5,-2,-4)$, $D(0,-1,0)$

68–71 Use the technique seen in Example 5 to find the coordinates of the indicated point.

68. The point one-third of the way from $P(12,-3,0)$ to $Q(0,6,-9)$

69. The point four-fifths of the way from $P(4,2,-5)$ to $Q\left(\frac{1}{4},2,-10\right)$

70. The point one percent of the way from $P(-3.8,-2.2,1.5)$ to $Q(2.4,-5.6,10)$

71. The point(s) on the line $\overleftrightarrow{PQ}$ with a distance from $Q(-3,1,7)$ equaling three times the distance from $P(1,5,3)$

72–77 For the given vector $\mathbf{v}$, find the unit vector $\mathbf{u}$ pointing in the same direction. Express your answer in terms of the standard basis vectors.

72. $\mathbf{v} = \langle -8,6 \rangle$

73. $\mathbf{v} = \langle 2,9 \rangle$

74. $\mathbf{v} = \langle 2,0,-1 \rangle$

75. $\mathbf{v} = \langle -4,-5,2 \rangle$

76. $\mathbf{v} = \left\langle 3, -\frac{5}{4}, \frac{13}{4} \right\rangle$

77. $\mathbf{v} = \left\langle 1, -\frac{3}{2}, -\frac{1}{2} \right\rangle$

78. Find the unit vector $\mathbf{u}$ in $\mathbb{R}^2$ that makes a directed $2\pi/3$ radian angle with the positive x-axis. Express your answer as a linear combination of the standard basis vectors in $\mathbb{R}^2$.

79. Show that any vector $\mathbf{u}$ in $\mathbb{R}^2$ that can be written as $\mathbf{u} = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$, for some $0 \le \theta \le 2\pi$, is a unit vector.

80. Find an appropriate scalar a for the vector $\mathbf{v} = \langle 2,1,-1 \rangle$ so that $a\mathbf{v}$ has a magnitude of 3.

81. Find an appropriate scalar a for the vector $\mathbf{w} = \langle -\sqrt{3},4,-5 \rangle$ so that $a\mathbf{w}$ has a magnitude of 5.

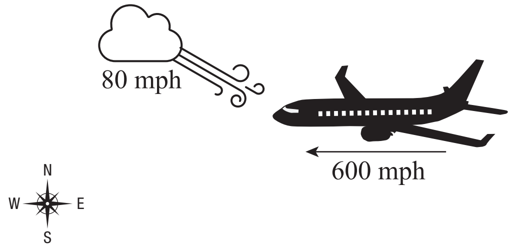
82. Find the vector $\mathbf{s}$ in $\mathbb{R}^2$ that makes a directed $7\pi/6$ radian angle with the positive x-axis and has a magnitude of 2. Express your answer in component form.

83. By first solving the linear system of Example 7 for $|\mathbf{T}_1|$ and $|\mathbf{T}_2|$, verify that $\mathbf{T}_1 \approx \langle -4.55, 2.12 \rangle$ and $\mathbf{T}_2 \approx \langle 4.55, 7.88 \rangle$.

84. Suppose that in Example 7 the ten-pound weight is suspended from two ropes that form angles of 75° and 51°, respectively, with the horizontal direction (refer to Figure 10). Find the tension forces $\mathbf{T}_1$ and $\mathbf{T}_2$ under these conditions.

85. A baseball bounces off a bat with an initial velocity that has a vertical component of 35.2 m/s and a horizontal component of 8 m/s. Ignoring air resistance, determine its velocity at time *t*, and sketch its position function. (Assume the initial height is 1 m. See Example 2.)

86. A jetliner flying at 600 mph due west encounters an 80 mph headwind that blows 30° south of east. If the captain wants to keep both his ground speed and direction, how much increase in speed will be needed and in what direction should he steer the plane?



87. A jetliner flying at 500 mph due southeast encounters a 60 mph tailwind that blows from the north. At the same time, a 20 mph updraft is affecting the flight. Find the actual ground speed of the jetliner under these conditions.

88. A guest in the restaurant section of a passenger train gets up from his table and cuts across toward the bar walking at a steady pace of 2 mph. If the train is moving at a constant eastward velocity of 80 mph, and the passenger is walking northeast (45° north of east), what is his resultant (ground) velocity?

89.* A plane is taking off with a flight plan that calls for a takeoff velocity of 250 mph in the direction of $\langle 1,4,2 \rangle$. However, the plane experiences a tailwind, blowing at 15 mph in the direction $\langle 2,1,0 \rangle$. Calculate the plane's actual direction and ground speed at takeoff. (Express your direction vector in a form where the first component is 1.)

90. Suppose the following three forces are acting simultaneously upon an object of mass 2 kilograms: $\mathbf{F}_1 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{F}_2 = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, and $\mathbf{F}_3 = 7\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ (units are in newtons). Find a unit vector $\mathbf{u}$ pointing in the direction of acceleration as well as the magnitude of the acceleration of the object. (**Hint:** Use Newton's Second Law of Motion.)

91. Use the component definition of scalar multiplication to verify the five scalar multiplication properties of vectors, as listed immediately preceding Example 3.

92. Use the component definition of vector addition to verify the four vector addition properties listed immediately preceding Example 3.

93.* Suppose that for some scalars *a*, *b* and *c*, $a\mathbf{i} + b\mathbf{j} + c\mathbf{k} = \mathbf{0}$, where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the standard basis vectors in $\mathbb{R}^3$, and $\mathbf{0}$ is the zero vector. Prove that $a = b = c = 0$.

94–98 Generalize the rules of this section to answer the following problems for vectors in higher dimensions.

94. Find $|\mathbf{u}|$ for $\mathbf{u} = \langle 1,-4,2,3 \rangle$.

95. Find $3\mathbf{u} - 2\mathbf{v}$ for $\mathbf{u} = \langle 1,-4,2,3 \rangle$ and $\mathbf{v} = \langle 2,-5,0,-1 \rangle$.

96. Find $|2\mathbf{u} + \mathbf{v}|$ for $\mathbf{u} = \langle 3,1,\frac{1}{2},5 \rangle$ and $\mathbf{v} = \langle -2,0,1,-4 \rangle$.

97. Find $\mathbf{u} - \dfrac{\mathbf{v}}{3}$ for $\mathbf{u} = \langle 2,0,-4,6,-8,0 \rangle$ and $\mathbf{v} = \langle 0,-3,9,-12,1,6 \rangle$.

98. Find $\left|\dfrac{\mathbf{u}}{2} + 3\mathbf{v}\right|$ for $\mathbf{u} = \langle 4,-2,0,8,0,-6 \rangle$ and $\mathbf{v} = \langle \frac{2}{3},-2,1,0,-\frac{1}{3},1 \rangle$.