

Example 4 Finding the Distance between Points in Cartesian Three-Dimensional Space

Find the distance between the points $(-2, 5, 1)$ and $(3, 0, -1)$.

Solution

Applying the previous formula, the distance is as follows.

$$\sqrt{(-2-3)^2 + (5-0)^2 + [1-(-1)]^2} = \sqrt{54} = 3\sqrt{6}$$

Example 5 Using the Distance Formula to Construct an Equation of a Sphere

Construct an equation whose graph is the sphere of radius 2 centered at the point $(2, 3, -1)$.

Solution

We seek an equation in x , y , and z that describes all those points (x, y, z) that are 2 units from $(2, 3, -1)$. So using the distance formula, we need

$$\sqrt{(x-2)^2 + (y-3)^2 + [z-(-1)]^2} = 2,$$

or, squaring both sides,

$$(x-2)^2 + (y-3)^2 + (z+1)^2 = 4.$$

The graph of such a sphere is shown in Figure 11.

More generally, the graph of the equation $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ is a sphere of radius r centered at the point (a, b, c) .

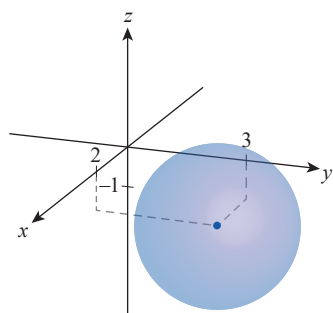
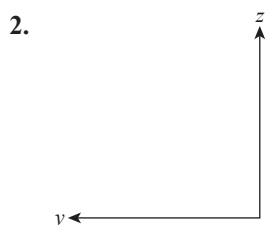
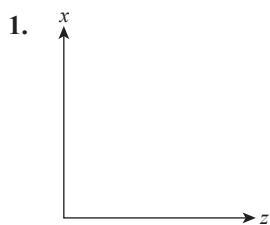


Figure 11
Sphere of Radius 2
Centered at $(2, 3, -1)$

11.1 Exercises

1–2 Describe the orientation of the third axis if the resulting system is to be a right-handed system. (**Hint:** Feel free to use the words “out of the page” or “into the page.”)



3–8 Plot the given point in the three-dimensional coordinate system.

3. $(1, 0, 0)$

4. $(-2, 0, 1)$

5. $(0, 1, \frac{3}{2})$

6. $(3, 4, -1)$

7. $(1, -2, \frac{5}{2})$

8. $(-2, -1, \frac{7}{4})$

9–16 Explain what you know about the coordinates of point P if its location in the three-dimensional Cartesian system is as described below.

9. P is in the xy -plane

10. P is in the first octant

11. P is on the y -axis

12. P is in the yz -plane

13. P is on the negative x -axis

14. P is in the third quadrant of the xz -plane

15. P is in the plane $y = -1$

16. P is in the second quadrant of the plane $x = 3$

17–20 Find the distance from the point to the indicated plane.

17. $(1, 2, 3)$; the xy -plane

18. $(2, -2, -5)$; the xz -plane

19. $(0, -7, -2)$; the yz -plane

20. $(1, -3, 4)$; the plane $z = -2$

21–25 Find the coordinates of the projection of the given point.

21. The projection of the point $(5, 3.2, -2)$ onto the xy -plane

22. The projection of the point $(-5, 1, 4)$ onto the xz -plane

23. The projection of the point $(3, 0, 0)$ onto the yz -plane

24. The projection of the point $(1, 0, -7)$ onto the plane $z = -2$

25. The projection of the point $(1.3, \pi, -10)$ onto the plane $y = 4.5$

26–35 Describe the set of points represented by the given equation(s).

26. $z = 0$

27. $y = 1$

28. $x = 3 - y$

29. $x + y = 1, z = 0$

30. $x = y = z$

31. $xyz = 0$

32. $y = 0, z = 4x - 2$

33. $z = 2y - 1, x = 5$

34. $z = |x|$

35. $z = \lceil x \rceil$

36–41 Find the coordinates of the described point.

36. A point in the plane $y = -2$ whose projection onto the xz -plane is $(1, 0, -5)$

37. A point on the line $x = y = z$, equidistant from the xy -plane and the plane $z = 5$

38. A point on the x -axis, equidistant from the planes $z = x$ and $z = x - 2$

39. A point P with an x -coordinate of -1 , so that the line through P and the point $(1, -2, 5)$ is parallel to one of the coordinate axes

40. The point on the sphere $x^2 + y^2 + 6y + z^2 - 7 = 0$ that is closest to the plane $y = 5$

41. The point on the sphere $x^2 + y^2 - 2y + z^2 - 6z + 6 = 0$ that is closest to the point $(2\sqrt{3}, 1, 5)$

42–45 Match the equation to its graph (labeled A–D).

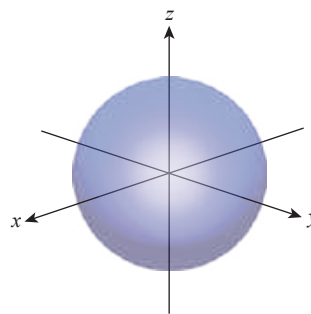
42. $z - x = 1$

43. $x^2 + y^2 + z^2 = 10$

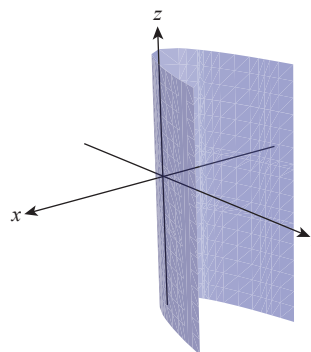
44. $x^2 + z^2 = 5$

45. $y - x^2 = 0$

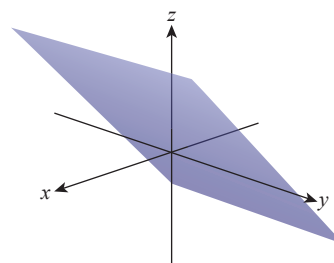
A.



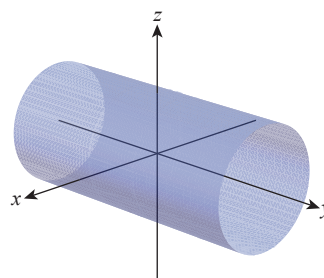
B.



C.



D.

**46–59** Describe the set of points represented by the given relation(s). (**Hint:** A sketch may be helpful.)

46. $z^2 \geq 1$

47. $x^2 + y^2 > 1$

48. $x^2 + y^2 + z^2 \leq 4$

49. $(x-3)^2 + y^2 + z^2 = 9$

50. $y^2 + z^2 < 9$

51. $x^2 + y = 1$

52. $x^2 + y^2 + (z-3)^2 = 25, z = 0$

53. $x - z^2 = 2, y = 1$

54. $xz \geq 0, y = 0$ 55. $xyz = 0$

56. $x^2 - 4x + y^2 + z^2 + 6z + 12 = 0$

57. $x^2 + 4x + y^2 - 8y + z^2 - 10z = 0, z \geq 5$

58. $x^2 - 6x + y^2 + z^2 + 4z \leq 3$

59. $x^2 + y^2 + 8y + z^2 - 2z + 8 = 0, z = -2$

60–77 Find the equations or inequalities that define the indicated set.

60. The plane through $(2, 0, -1)$ that is parallel to the xy -plane

61. The plane through $(1, 0, 0)$ that is perpendicular to the x -axis

62. The plane through $(-4, 7, 2)$ that is perpendicular to the y -axis

63. The plane through $(\frac{4}{5}, 1, e)$ that is perpendicular to the z -axis

64. The line through $(1, 2, 3)$ that is parallel to the z -axis

65. The line through $(0, -1, 2)$ that is parallel to the x -axis

66. The line through $(-4, 1, 5)$ that is parallel to the y -axis

67. The sphere of radius 2, centered at the point $(0, 2, 0)$

68. The sphere of radius $\sqrt{5}$, centered at the point $(-1, 3, 5)$

69. The sphere centered at $(1, 0, -2)$, passing through the point $(3, -3, 4)$

70. The circle of radius 1 in the xy -plane, centered at $(1, 0, 0)$

71. The circle of radius 3 in the yz -plane, centered at $(0, -4, 1)$

72. The circle of radius $\sqrt{2}$ in the plane $y = -1$, centered at $(-1, -1, 1)$

73. The sphere centered at $(-1, 4, -3)$ that is tangent to the yz -plane

74. The sphere with a diameter joining the points $(0, -2, 5)$ and $(2, 4, 1)$

75. The intersection of the first octant and the sphere centered at $(3, 2, -1)$, passing through the point $(5, 3, 1)$

76. A horizontal circular cylinder of inner radius $r = 4$ that is tangent to the xy -plane along the line $y = -1$

77. The set of points equidistant from $(-1, 2, 3)$ and $(7, 2, 3)$

78–81 Find the distance between the given pair of points.

78. $(0, 4, 2)$ and $(-1, 1, 0)$

79. $(-3, 4, 10)$ and $(5, -3, 6)$

80. $(\frac{\sqrt{2}}{2}, 3, -1)$ and $(\sqrt{2}, 3, 1)$

81. $(2, 3\sqrt{3}, -4)$ and $(-1, \sqrt{3}, -2)$

82–85 A triangle is given by the coordinates of its vertices. For the triangle, select all that apply from the following list: **a.** isosceles, **b.** equilateral, **c.** scalene, **d.** right triangle.

82. $(0, 6, 2)$, $(3, 4, 1)$, and $(1, 3, 4)$

83. $(-1, 2, 7)$, $(1, 1, 3)$, and $(5, 3, 2)$

84. $(2, 10, 4)$, $(1, 7, 0)$, and $(-3, 1, -1)$

85. $(2, 1, 1)$, $(4, 3, 1)$, and $(3, 1, 2)$