1.4 Exercises

1–12 Graph the inverse of the given relation, and state its domain and range.

1. $R = \{(-4,2),(3,2),(0,-1),(3,-2)\}$
2. $S = \{(-3,-3),(-1,-1),(0,1),(4,4)\}$
3. $y = x^3$
4. $y = |x| + 2$
5. $x = |y|$
6. $x = -\sqrt{y}$
7. $y = \frac{1}{2}x - 3$
8. $y = -x + 1$
9. $y = [\![x]\!]$
10. $T = \{(4,2),(3,-1),(-2,-1),(2,4)\}$
11. $x = y^2 - 2$
12. $y = 2\sqrt{x}$

13–22 Determine if the given function has an inverse function. If not, suggest a domain to restrict the function so that it would have an inverse function. (Answers will vary.)

13. $f(x) = x^2 + 1$
14. $g(x) = (x-2)^3 - 1$
15. $h(x) = \sqrt{x+3}$
16. $s(x) = \frac{1}{x^2}$
17. $G(x) = 3x - 5$
18. $F(x) = -x^2 + 5$
19. $r(x) = -\sqrt{x^3}$
20. $b(x) = [\![x]\!]$
21. $m(x) = \frac{13x-2}{4}$
22. $H(x) = |x - 12|$

23–37 Find the inverse of the given function.

23. $f(x) = x^{1/3} - 2$
24. $g(x) = 4x - 3$
25. $r(x) = \frac{x-1}{3x+2}$
26. $s(x) = \frac{1-x}{1+x}$
27. $F(x) = (x-5)^3 + 2$
28. $G(x) = \sqrt[3]{3x-1}$
29. $V(x) = \frac{x+5}{2}$
30. $W(x) = \frac{1}{x}$
31. $h(x) = x^{3/5} - 2$
32. $A(x) = (x^3+1)^{1/5}$
33. $J(x) = \frac{2}{1-3x}$
34. $k(x) = \frac{x+4}{3-x}$
35. $h(x) = x^7 + 6$
36. $F(x) = \frac{3-x^5}{-9}$
37. $r(x) = \sqrt[5]{2x}$

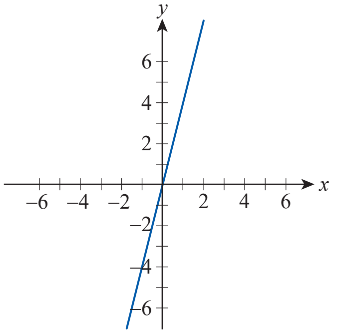
38–45 Show that $f^{-1}(f(x)) = x$ and that $f(f^{-1}(x)) = x$.

38. $f(x) = 2x - 3$; $f^{-1}(x) = \frac{x+3}{2}$
39. $f(x) = x^2,\ x \ge 0$; $f^{-1}(x) = \sqrt{x}$
40. $f(x) = \frac{3x-1}{5}$; $f^{-1}(x) = \frac{5x+1}{3}$
41. $f(x) = \frac{x-5}{2x+3}$; $f^{-1}(x) = \frac{3x+5}{1-2x}$
42. $f(x) = (x-2)^2,\ x \ge 2$; $f^{-1}(x) = \sqrt{x} + 2,\ x \ge 0$
43. $f(x) = \sqrt[3]{x+2} - 1$; $f^{-1}(x) = (x+1)^3 - 2$
44. $f(x) = \frac{1}{x}$; $f^{-1}(x) = \frac{1}{x}$
45. $f(x) = \frac{1}{1+x},\ x \ge 0$; $f^{-1}(x) = \frac{1-x}{x},\ 0 < x \le 1$

46–51 Match the function with the graph of the inverse of the function (labeled A–F).

46. $f(x) = x^3$
47. $f(x) = x - 5$
48. $f(x) = \sqrt{x-4}$
49. $f(x) = x^2$
50. $f(x) = \frac{x}{4}$
51. $f(x) = \sqrt[3]{x+1}$

A.



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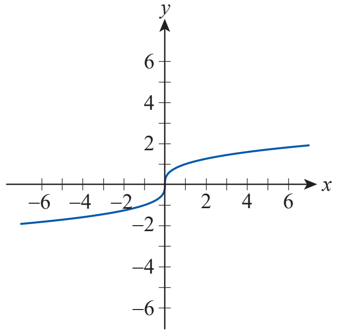

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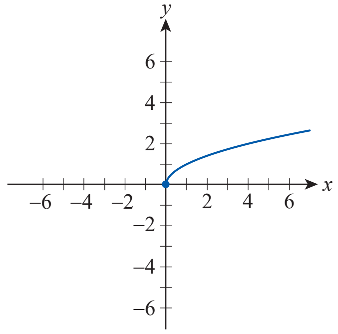

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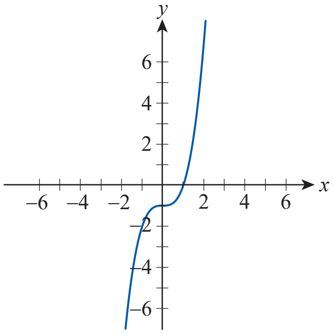
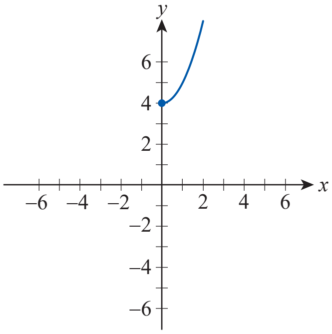
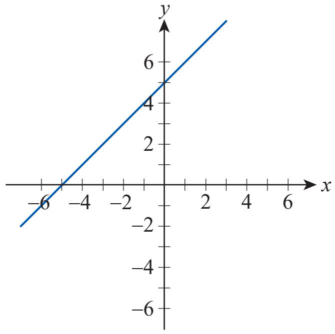


E.



F.



52–60 Match the logarithmic function with its graph (labeled A–I).

52. $f(x)=\log_2 x-1$

53. $f(x)=\log_2(2-x)$

54. $f(x)=\log_2(-x)$

55. $f(x)=\log_2(x-3)$

56. $f(x)=1-\log_2 x$

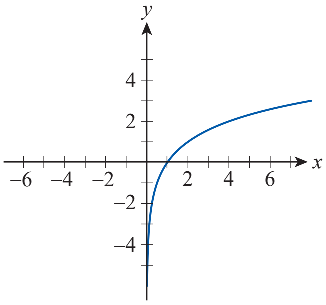
57. $f(x)=-\log_2 x$

58. $f(x)=-\log_2(-x)$

59. $f(x)=\log_2 x$

60. $f(x)=\log_2 x+3$

A.



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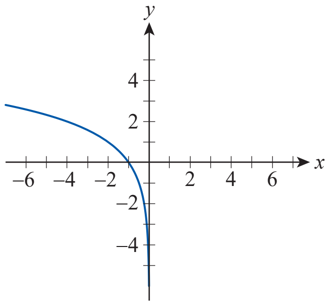

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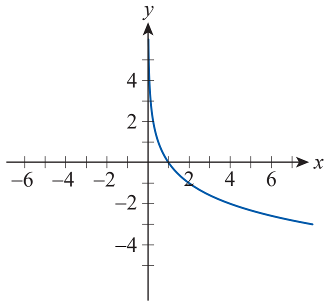

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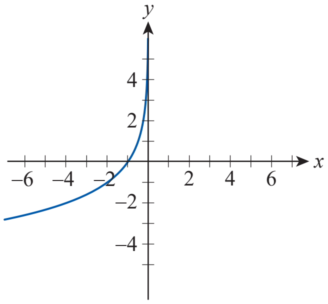

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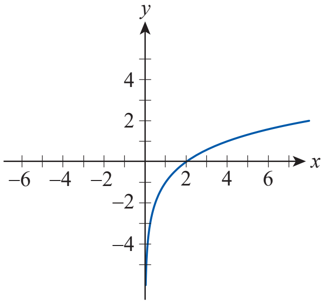
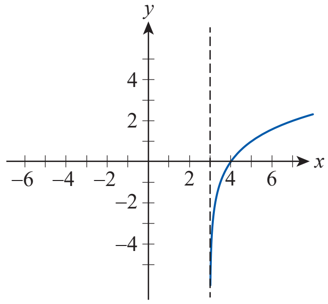
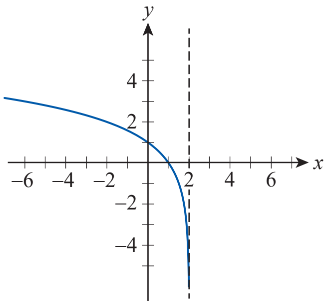


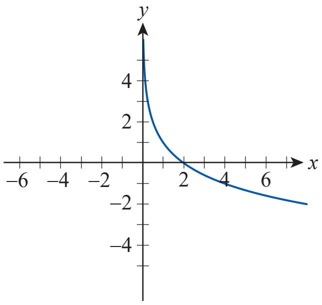
F.



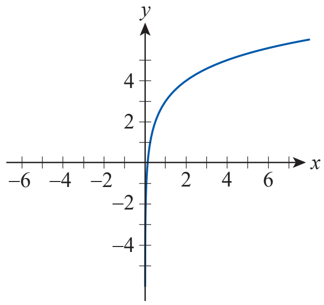
G.



H.



I.



61–72 Sketch the graph of the given function.

61. $f(x)=\log_3(x-1)$

62. $g(x)=\log_5(x+2)-1$

63. $r(x)=\log_{1/2}(x-3)$

64. $p(x)=3-\log_2(x+1)$

65. $q(x)=\log_3(2-x)$

66. $s(x)=\log_{1/3}(5-x)$

67. $h(x)=\log_7(x-3)+3$

68. $m(x)=\log_{1/2}(1-x)$

69. $f(x)=\log_3(6-x)$

70. $p(x)=4-\log_{10}(x+3)$

71. $s(x)=-\log_{1/3}(-x)$

72. $g(x)=\log_5(2x)-1$

73–78 Evaluate the given expression without using a calculator.

73. $\log_4 16$

74. $\log_5(25^3)$

75. $\ln(e^4)+\ln(e^3)$

76. $\log_4\frac{1}{64}$

77. $\ln(e^{1.5})-\log_4 2$

78. $\log_2\left(8^{2\log_2 4-\log_2 4}\right)$

79–84 Evaluate the given logarithmic expression to two decimal places. (**Hint:** Use the change of base formula.)

79. $\log_6(3^4)$

80. $\log_7 14.3$

81. $\log_{1/2}(\pi^{-2})$

82. $\log_{1/5} 626$

83. $\ln(\log 123)$

84. $\log_{17} 0.041$

85–90 Use the properties of logarithms to rewrite the given expression as a single term that does not contain a logarithm.

85. $5^{2\log_5 x}$

86. $\log_4 16\cdot\log_x(x^2)$

87. $e^{2-\ln x+\ln p}$

88. $e^{5\left(\ln\sqrt[5]{3}+\ln x\right)}$

89. $10^{\log(x^3)-4\log y}$

90. $a^{\log_a b+4\log_a\sqrt{a}}$

91–99 Use the properties of logarithms to expand the given expression as much as possible; that is, decompose the expression into sums or differences of the simplest possible terms. Simplify any numerical expressions that can be evaluated without a calculator.

91. $\ln\frac{\sqrt{x^3}\,pq^5}{e^7}$

92. $\log_a\sqrt[5]{\frac{a^4b}{c^2}}$

93. $\log\left(\log\left(100x^3\right)\right)$

94. $\log_3(9x+27y)$

95. $\log\frac{10}{\sqrt{x+y}}$

96. $\ln\left(\ln\left(e^{ex}\right)\right)$

97. $\log_2\frac{y^2+z}{16x^4}$

98. $\log\left(\log\left(100{,}000^{2x}\right)\right)$

99. $\log_b\sqrt{\frac{x^4y}{z^2}}$

100–105 Use the properties of logarithms to condense the given expression as much as possible, writing the answer as a single term with a coefficient of 1.

100. $\frac{1}{5}\left(\log_7(x^2)-\log_7(pq)\right)$

101. $\ln 3+\ln p-2\ln q$

102. $2\left(\log_5\sqrt{x}-\log_5 y\right)$

103. $\log(x-10)-\log x$

104. $2\log(a^2b)-\log\frac{1}{b}+\log\frac{1}{a}$

105. $3\left(\ln\sqrt[3]{e^2}-\ln(xy)\right)$

106–111 Evaluate the given expression, if possible.

106. $\cos^{-1}\left(\cos\frac{2\pi}{4}\right)$

107. $\sin^{-1}\left(\sin\frac{3\pi}{2}\right)$

108. $\tan\left(\tan^{-1}(0.5)\right)$

109. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

110. $\cos\left(\cos^{-1}(-0.8)\right)$

111. $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

112–117 Most calculators are not equipped with arccosecant, arcsecant, and arccotangent buttons, but expressions involving these functions can still be evaluated. For example, to evaluate $\csc^{-1}x$, let $\theta=\csc^{-1}x$.

$$\csc\theta=x$$

$$\frac{1}{\sin\theta}=x$$

$$\sin\theta=\frac{1}{x}$$

$$\theta=\sin^{-1}\frac{1}{x}$$

Use the method described above to evaluate the given expression. (Round your answer to four decimal places.)

112. $\csc^{-1} 5$

113. $\sec^{-1}(-0.5)$

114. $\cot^{-1} 150$

115. $\cot^{-1}(-0.2)$

116. $\csc^{-1}(-8.9)$

117. $\sec^{-1} 2$

118–123 Find the value of the given expression without using a calculator.

118. $\sin\left(\arctan\sqrt{3}\right)$

119. $\cos\left(\sec^{-1}(-2)\right)$

120. $\tan(\operatorname{arccot} 1)$

121. $\csc\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right)$

122. $\tan\left(\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

123. $\sec\left(\csc^{-1}\frac{2\sqrt{3}}{3}\right)$

124–129 Rewrite the given function as a purely algebraic function.

124. $\tan\left(\cos^{-1} x\right)$

125. $\cot\left(\sin^{-1}\frac{2}{x}\right)$

126. $\sec\left(\tan^{-1} 3x\right)$

127. $\tan\left(\sin^{-1}\frac{x}{\sqrt{x^2+3}}\right)$

128. $\sin\left(\sec^{-1} x\right)$

129. $\cos\left(\tan^{-1}\frac{x}{4}\right)$

130–133 Sketch the graph of the given function. Then graph the function using a graphing utility to check your answer.

130. $f(x) = \sin^{-1}(x-3)$

131. $f(x) = \sec^{-1} 2x$

132. $f(x) = \arctan\frac{x}{2}$

133. $f(x) = 2\arccos x$

134–137 An inverse function can be used to encode and decode words and sentences by assigning each letter of the alphabet a numerical value (A = 1, B = 2, C = 3, …, Z = 26). Example: Use the function $f(x) = x^2$ to encode the word CALCULUS. The encoded message would be 9 1 144 9 441 144 441 361. The word can then be decoded by using the inverse function $f^{-1}(x) = \sqrt{x}$. The inverse values are 3 1 12 3 21 12 21 19, which translates back to the word CALCULUS.

Encode or decode the given message using the numerical values A = 1, B = 2, C = 3, …, Z = 26.

134. Encode the message SANDY SHOES using the function $f(x) = 4x - 3$.

135. Encode the message WILL IT RAIN TODAY using the function $f(x) = x^2 - 8$.

136. The following message was encoded using the function $f(x) = 8x - 7$. Decode the message.

41 137 65 145 9 33 33 169 113 89 89 33 193 9 1 89 89

1 105 25 57 113 137 145

33 145 57 113 33 145

137. The following message was encoded using the function $f(x) = 5x + 1$. Decode the message.

91 26 66 26 66 11 26 91 126 76 106 91

96 106 71 11 61 76 16 56

138–139 The energy released during earthquakes can vary greatly, but logarithms provide a convenient way to analyze and compare the intensity of earthquakes. Earthquake intensity is measured on the Richter scale (named for the American seismologist Charles F. Richter, 1900–1985). In the formula that follows, I_0 is the intensity of a just-discernible earthquake, I is the intensity of an earthquake being analyzed, and R is its ranking on the Richter scale.

$$R = \log\frac{I}{I_0}$$

By this measure, earthquakes range from a classification of minor ($R < 4$), to light ($4 \le R < 5$), to moderate ($5 \le R < 6$), to strong ($6 \le R < 7$), to major ($7 \le R < 8$), and finally to great ($R \ge 8$).

Use this information to solve the problem.

138. The 1994 Northridge, California earthquake measured 6.7 on the Richter scale. What was the intensity, relative to a 0-level earthquake, of this event?

139. The April, 2009 Abruzzo earthquake in Italy was 2,000,000 times as intense as a 0-level earthquake. What was the Richter ranking of this tragic event?

140–141 Sound intensity is another quantity that varies greatly, and the measure of how the human ear perceives intensity, in units called decibels, is very similar to the measure of earthquake intensity. If I_0 is the intensity of a just-discernible sound, I is the intensity of the sound being analyzed, and D is its decibel level, we have the formula $D = 10\log(I/I_0)$. Decibel levels range from 0 for a barely discernible sound, to 40 for the level of a quiet conversation, to 80 for heavy traffic, to 120 for a loud rock concert, and finally (as far as humans are concerned) to around 160, at which point the eardrum is likely to rupture.

Use the decibel formula given above to answer the question.

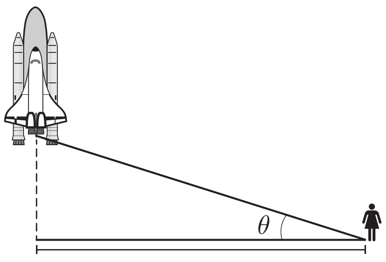
140. A construction worker operating a jackhammer would experience noise with an intensity of 20 watts/meter^2 if it weren't for ear protection. Given that $I_0 = 10^{-12} \text{ watts/meter}^2$, what is the decibel level for such noise?

141. The intensity of a cat's soft purring is measured to be 2.19×10^{-11} watts/meter^2. Given that $I_0 = 10^{-12}$ watts/meter^2, what is the decibel level of this noise?

142–143 Use inverse trigonometric functions to solve the problem. (Round your answer to four decimal places.)

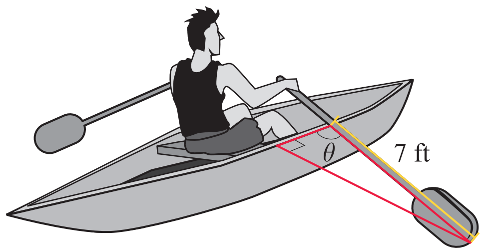
142. Kim is watching a space shuttle launch from an observation spot 2 miles away from the launchpad. Find the angle of elevation to the shuttle for each of the following heights.

a. 0.5 miles **b.** 2 miles **c.** 2.8 miles



143. Jesse is rowing in the men's singles race. The length of the oar from the side of the shell to the water is 7 feet. At what angle is the oar from the side of the boat when the blade is at the following distances from the boat?

a. 2 feet **b.** 3 feet **c.** 5 feet



Concept Check

144–151 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

144. All exponential functions are one-to-one.

145. $\sin(\arcsin x) = x$ for all $x \in [-1,1]$

146. $\arcsin(\sin x) = x$ for all $x \in \mathbb{R}$

147. $\tan(\arctan x) = x$ for all $x \in \mathbb{R}$

148. $\arccos(\cos(3\pi/2)) = 3\pi/2$

149. The domain of $\arcsin x$ is $[-\pi/2, \pi/2]$.

150. The domain of $f(x) = \cot^{-1} x$ is $\mathbb{R}$.

151. The function $f(x) = \sin(\tan^{-1} x)$ can be represented as an algebraic function.