0.9 and 1.1 in distance from the origin. The rules for coloring other complex numbers in the plane are as follows. Given an initial complex number z not on the gold ring, $f(z)$ is calculated. If the complex number $f(z)$ lies somewhere on the gold ring, the original number z is colored the deepest shade of green. If not, the iterate $f^2(z)$ is calculated. If this result lies in the gold ring, the original z is colored a bluish shade of green. If not, the process continues up to the 12th iterate $f^{12}(z)$, using a different color each time. If $f^{12}(z)$ lies in the gold ring, z is colored red, and if not the process halts and z is colored black.

The idea of recursion can be used to generate any number of similar images, with the end result usually striking and often surprising even to the creator.

1.3 Exercises

1–23 Sketch the graph of the given function by first identifying the more basic function that has been shifted, reflected, stretched, or compressed. Then determine the domain and range of the function.

1. $f(x)=(x+2)^3$ **2.** $G(x)=|x-4|$ **3.** $p(x)=-(x+1)^2+2$ **4.** $g(x)=\sqrt{x+3}-1$

5. $q(x)=(1-x)^2$ **6.** $r(x)=-\sqrt[3]{x}$ **7.** $s(x)=\sqrt{2-x}$ **8.** $F(x)=\frac{|x+2|}{3}+3$

9. $w(x)=\frac{1}{(x-3)^2}$ **10.** $v(x)=\frac{1}{3x}-2$ **11.** $f(x)=\frac{1}{2-x}$ **12.** $k(x)=\sqrt{-x}+2$

13. $b(x)=[\![x-4]\!]+4$ **14.** $R(x)=4-|2x|$ **15.** $S(x)=(3-x)^3$ **16.** $g(x)=-\frac{1}{x+1}$

17. $h(x)=\frac{x^2}{2}-3$ **18.** $W(x)=1-|4-x|$

19. $g(x)=x^2-6x+9$ (**Hint:** Find a better way to write the function.)

20. $h(x)=\frac{|x|}{x}$ (**Hint:** Evaluate h at a few points to understand its behavior.)

21. $W(x)=\frac{x-1}{|x-1|}$ **22.** $S(x)=[\![x-2]\!]$ **23.** $V(x)=-3\sqrt{x-1}+2$

24–29 Write an equation for the function described.

24. Use the function $f(x)=x^2$. Move the function 4 units to the right and 2 units up.

25. Use the function $f(x)=x^2$. Reflect the function across the x-axis and move it 6 units up.

26. Use the function $f(x)=x^3$. Move the function 1 unit to the left and reflect across the y-axis.

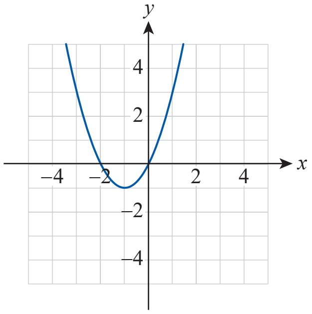
27. Use the function $f(x)=\sqrt{x}$. Move the function 5 units to the left and reflect across the x-axis.

28. Use the function $f(x)=\sqrt{x}$. Reflect the function across the y-axis and move it 3 units down.

29. Use the function $f(x)=|x|$. Move the function 7 units to the left, reflect across the x-axis, and reflect across the y-axis.

30–33 Use your knowledge about transformations to find a possible formula for the function $f(x)$ given by its graph.

30.



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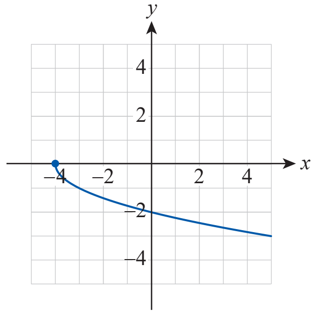

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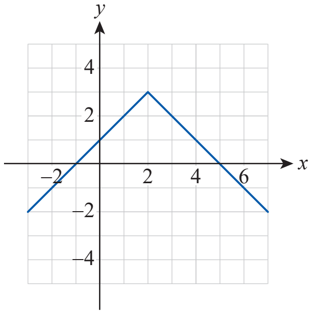

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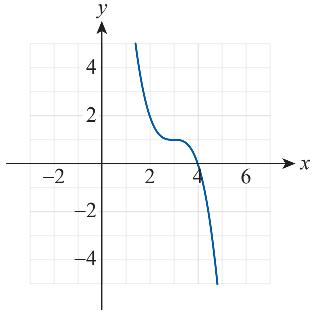


34–45 Use the information given to determine **a.** $(f+g)(-1)$, **b.** $(f-g)(-1)$, **c.** $(fg)(-1)$, and **d.** $(f/g)(-1)$.

34. $f(-1)=-3;\quad g(-1)=5$

35. $f(-1)=0;\quad g(-1)=-1$

36. $f(x)=x^2-3;\quad g(x)=x$

37. $f(x)=\sqrt[3]{x};\quad g(x)=x-1$

38. $f(-1)=15;\quad g(-1)=-3$

39. $f(x)=\dfrac{x+5}{2};\quad g(x)=6x$

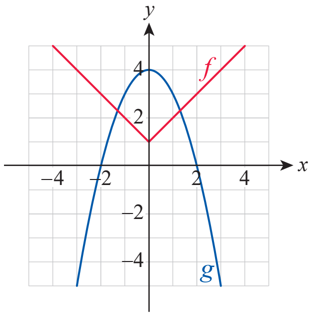
40. $f(x)=x^4+1;\quad g(x)=x^{11}+2$

41. $f(x)=\dfrac{6-x}{2};\quad g(x)=\sqrt{\dfrac{x}{-4}}$

42. $f=\{(5,2),(0,-1),(-1,3),(-2,4)\};$
$g=\{(-1,3),(0,5)\}$

43. $f=\{(3,15),(2,-1),(-1,1)\};\quad g(x)=-2$

44.



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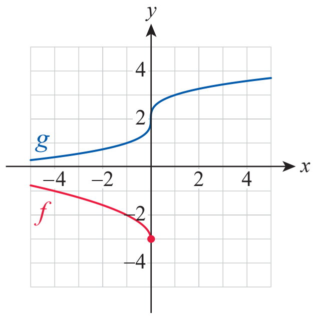


46–53 Find the formula and domain for **a.** $f+g$ and **b.** f/g.

46. $f(x)=|x|;\quad g(x)=\sqrt{x}$

47. $f(x)=x^2-1;\quad g(x)=\sqrt[3]{x}$

48. $f(x)=x-1;\quad g(x)=x^2-1$

49. $f(x)=x^{3/2};\quad g(x)=x-3$

50. $f(x)=3x;\quad g(x)=x^3-8$

51. $f(x)=x^3+4;\quad g(x)=\sqrt{x-2}$

52. $f(x)=-2x^2;\quad g(x)=[\![x+4]\!]$

53. $f(x)=6x-1;\quad g(x)=x^{2/3}$

54–63 Evaluate the expression, if possible, given $f(x)=1/x^2$ and $g(x)=2x+3$.

54. $(f+g)(-7)$

55. $(f+g)(-10)$

56. $(f-g)(-5)$

57. $(f-g)(0)$

58. $(fg)(4)$

59. $(fg)(-3)$

60. $\left(\dfrac{f}{g}\right)(-2)$

61. $\left(\dfrac{f}{g}\right)(0)$

62. $\left(\dfrac{g}{f}\right)(1)$

63. $\left(\dfrac{g}{f}\right)(-6)$

64–73 Use the information given to determine $(f \circ g)(3)$.

64. $f(-5) = 2; \quad g(3) = -5$

65. $f(\pi) = \pi^2; \quad g(3) = \pi$

66. $f(x) = x^2 - 3; \quad g(x) = \sqrt{x}$

67. $f(x) = \sqrt{x^2 - 9}; \quad g(x) = 1 - 2x$

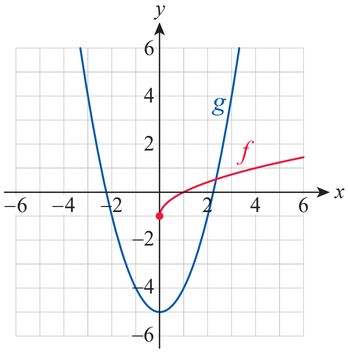
68. $f(x) = 2 + \sqrt{x}; \quad g(x) = x^3 + x^2$

69. $f(x) = x^{3/2} - 3; \quad g(x) = \left[\!\left[\frac{3x}{2} \right]\!\right]$

70. $f(x) = \sqrt{x + 6}; \quad g(x) = \sqrt{4x - 3}$

71. $f(x) = \sqrt{\frac{3x}{14}}; \quad g(x) = x^4 - x^3 - x^2 - x$

72.



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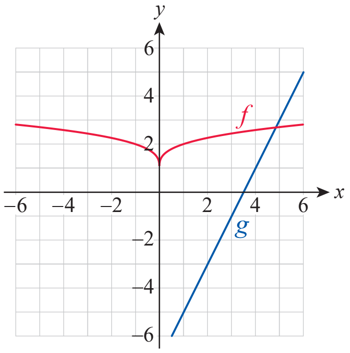


74–87 Find the formula and domain for **a.** $f \circ g$ and **b.** $g \circ f$.

74. $f(x) = \sqrt{x - 1}; \quad g(x) = x^2$

75. $f(x) = \frac{1}{x}; \quad g(x) = x - 1$

76. $f(x) = \frac{4x - 2}{3}; \quad g(x) = \frac{1}{x}$

77. $f(x) = 1 - x; \quad g(x) = \sqrt{x}$

78. $f(x) = [\![x - 3]\!]; \quad g(x) = x^3 + 1$

79. $f(x) = x^2 + 2x; \quad g(x) = 3x^2 + 5$

80. $f(x) = x^2 + 1; \quad g(x) = 3x^2 + 5$

81. $f(x) = \sqrt{x}; \quad g(x) = 2x$

82. $f(x) = \frac{1}{x + 7}; \quad g(x) = \frac{2}{x}$

83. $f(x) = \frac{1}{x}; \quad g(x) = \frac{1}{x}$

84. $f(x) = x^2; \quad g(x) = 3x + 1$

85. $f(x) = \sqrt[3]{x}; \quad g(x) = x^3$

86. $f(x) = \sqrt{x - 4}; \quad g(x) = x^2 + 2$

87. $f(x) = \frac{3}{1 - x}; \quad g(x) = 3x^2$

88–93 Write the given function as a composition of two functions. (Answers will vary.)

88. $f(x) = \sqrt[3]{3x^2 - 1}$

89. $f(x) = \frac{2}{5x - 1}$

90. $f(x) = |x - 2| + 3$

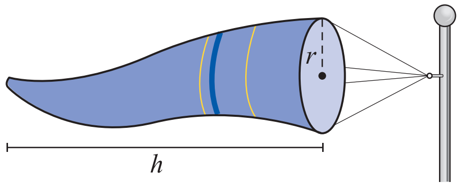
91. $f(x) = x + \sqrt{x + 2} - 5$

92. $f(x) = |x^3 - 5x| + 7$

93. $f(x) = \frac{\sqrt{x - 3}}{x^2 - 6x + 9}$

94. The volume of a right circular cylinder is given by the formula $V = \pi r^2 h$. If the height h is three times the radius r, show the volume V as a function of r.

95. The surface area of a wind sock is defined by the formula $S=\pi r\sqrt{r^2+h^2}$ where r is the radius of the base of the wind sock and h is the height of the wind sock. As the wind sock is being knitted by an automated knitter, the height h is increasing with time t as defined by the formula $h(t)=\frac{1}{4}t^2, t\geq 0.$ Find the surface area S of the wind sock as a function of time t.



96. The volume of the wind sock described in the previous exercise is given by the formula $V=\frac{1}{3}\pi r^2 h$ where r is the radius of the wind sock and h is the height of the wind sock. If the height h is increasing with time t as defined by the formula $h(t)=\frac{1}{4}t^2,\ t\geq 0$, find the volume V of the wind sock as a function of time t.

97. A widget factory produces n widgets in t hours of a single day. The number of widgets the factory produces is given by the formula $n(t)=10{,}000t-25t^2, 0\leq t\leq 9.$ The cost c in dollars of producing n widgets is given by the formula $c(n)=2040+1.74n.$ Find the cost c as a function of time t that the factory is producing the widgets.

98. Suppose that $H(x)$ represents the percentage of income spent on a home loan in the year x and $C(x)$ represents the percentage of income spent on a car loan in the year x. If $I(x)$ represents the income in year x, determine the function L that represents the total loan expenses in year x.

99. Given two odd functions f and g, show that $f\circ g$ is also odd. Then verify this fact with the particular functions $f(x)=\sqrt[3]{x}$ and $g(x)=-x^3/(3x^2-9)$. (**Hint:** Recall that a function is odd if $f(-x)=-f(x)$ for all x in the domain of f.)

100. Given two even functions f and g, show that the product is also even. Then verify this fact with the particular functions $f(x)=2x^4-x^2$ and $g(x)=1/x^2$. (**Hint:** Recall that a function is even if $f(-x)=f(x)$ for all x in the domain of f.)

101–108 As mentioned in the Interlude, a given complex number c is said to be in the Mandelbrot set if, for the function $f(z)=z^2+c$, the sequence of iterates $f(0), f^2(0), f^3(0),\ldots$ stays close to the origin (which is the complex number $0+0i$). It can be shown that if any single iterate falls more than 2 units in distance (magnitude) from the origin, then the remaining iterates will grow larger and larger in magnitude. In practice, computer programs that generate the Mandelbrot set calculate the iterates up to a predecided point in the sequence, such as $f^{50}(0)$, and if no iterate to this point exceeds 2 in magnitude the number c is admitted to the set. The magnitude of a complex number $a+bi$ is the distance between the point (a,b) and the origin, so the formula for the magnitude of $a+bi$ is $\sqrt{a^2+b^2}$.

Use the above criterion to determine, without a calculator or computer, if the given complex number is in the Mandelbrot set.

101. $c=0$ **102.** $c=1$ **103.** $c=i$ **104.** $c=-1$

105. $c=1+i$ **106.** $c=-2$ **107.** $c=1-i$ **108.** $c=-1-i$

Concept Check

109–112 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

109. The graph of any quadratic polynomial is a transformation of the prototypical parabola.

110. The graphs of $y=f(x)$ and $y=f(-x)$ are reflection images of each other.

111. A cubic function can have up to three x-intercepts.

112. If $f(x)$ is an algebraic function and c is a nonzero constant, then $f(cx)=cf(x)$.

1.3 Technology Exercises

113–118 Mentally sketch the graph of the given function by identifying the basic shape that has been shifted, reflected, stretched, or compressed. Then use a graphing utility to graph the function and check your reasoning.

113. $f(x) = -2(3-x)^3 + 5$

114. $f(x) = \dfrac{3}{x+5} - 1$

115. $f(x) = \dfrac{-1}{(x-2)^2} - 3$

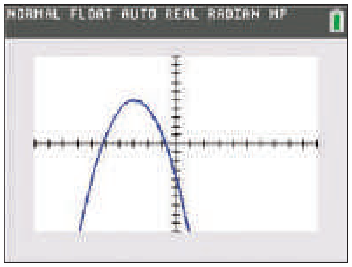
116. $f(x) = -3|x+2| - 4$

117. $f(x) = -\sqrt{1-x} + 2$

118. $f(x) = \sqrt[3]{2+x} - 1$

119–124 Write a possible equation for the function depicted on the graphing calculator. The function is shown in a $[-10,10]$ by $[-10,10]$ window.

119.



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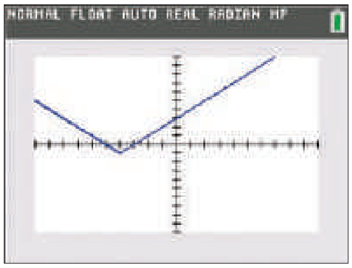

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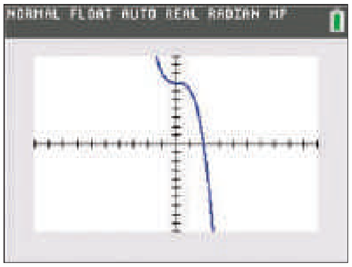

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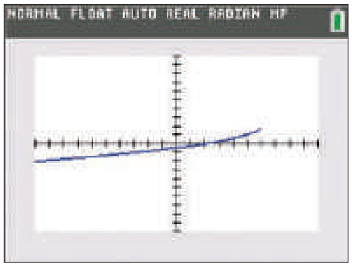

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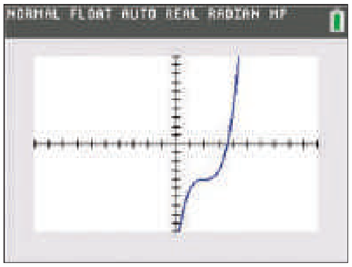

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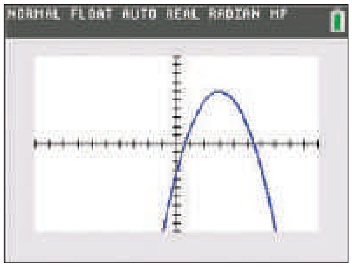

125–127 Use a graphing utility to determine $(f+g)(x)$, $(fg)(x)$, $(f \circ g)(x)$, and $(g \circ f)(x)$ for the given pair of functions.

125. $f(x) = (3x+2)^2$; $g(x) = \sqrt{x^2+5}$

126. $f(x) = \dfrac{1}{3x-5}$; $g(x) = (x+2)^3$

127. $f(x) = \dfrac{x+1}{x-1}$; $g(x) = \dfrac{x-1}{x}$