```
In[1]:= f[x_] := Piecewise[{{x^2, x < 0}, {3 x + 1, 0 ≤ x < 2},
            {7 - Sqrt[x - 2], x ≥ 2}}]

In[2]:= f[-2]

Out[2]= 4

In[3]:= f[0]

Out[3]= 1

In[4]:= Plot[f[x], {x, -3, 3}]
```

Out[4]=

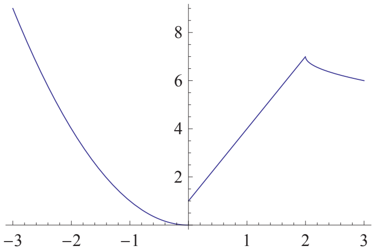



**Figure 18**

# 1.2 Exercises

**1–8** Identify the degree, leading coefficient, intercepts, and range of the given polynomial function, and then graph the function.

**1.** $f(x) = \frac{1}{2}x - \frac{3}{2}$

**2.** $g(x) = -1.2x + 4.8$

**3.** $h(x) = 2x^2 - 3x - 2$

**4.** $u(x) = \frac{1}{2}x^2 + x - \frac{3}{2}$

**5.** $v(x) = x^3 - 7x + 6$

**6.** $F(x) = 10 - 8x + \frac{x^2}{2} + \frac{x^3}{2}$

**7.** $G(x) = \frac{x^4}{4} - 2x^2$

**8.** $H(x) = 2x^4 + 12x^3 + 2x^2 - 48x - 40$

**9–16** Find all asymptotes and intercepts of the given rational function and then sketch the graph of the function.

**9.** $f(x) = \frac{5}{x-1}$

**10.** $g(x) = \frac{x^2 - 4}{2x - x^2}$

**11.** $h(x) = \frac{x^2 + 3}{x + 3}$

**12.** $u(x) = \frac{x + 2}{x^2 - 9}$

**13.** $v(x) = \frac{x^2 - 2x - 3}{2x^2 - 5x - 3}$

**14.** $F(x) = \frac{3x^2 + 1}{x - 2}$

**15.** $G(x) = \frac{x^2 + 2x}{x + 1}$

**16.** $H(x) = \frac{x^3 - 27}{x^2 + 5}$

**17–24** Construct the algebraic function in a finite number of steps. (Answers will vary.)

**17.** $f(x) = \frac{\sqrt{x^2 - 1}}{x + 1}$

**18.** $g(x) = \sqrt[3]{\frac{x - 1}{-2 + x + x^2}}$

**19.** $h(x) = \sqrt{2x^2 + x + 1} + 3x(2x + 1)$

**20.** $u(x) = 13x^3(2 - x) + 3\sqrt{x} - 5x^2(2x - x^2)$

**21.** $v(x) = \sqrt{2} + \frac{x + 3}{\sqrt[5]{2x^2 + 2x - 12}}$

**22.** $F(x) = \frac{(x^3 - 4x^2 - 7x + 10)^{2/3}}{\sqrt[5]{x - 5}}$

**23.** $G(x) = \left(x + \left(x + \left(x + (x + 1)^3\right)^3\right)^3\right)^3$

**24.** $H(x) = \sqrt{2x + \sqrt{2x + \sqrt{2x + \sqrt{2x}}}}$

**25–34** Simplify the given trigonometric expression.

**25.** $\dfrac{1-\cos^2\left(\frac{\pi}{2}-x\right)}{\cos x}$

**26.** $\dfrac{1}{\sec^2 x}+\sin x\cos\left(\dfrac{\pi}{2}-x\right)$

**27.** $\sin\alpha(\csc\alpha-\sin\alpha)$

**28.** $\dfrac{1}{1+\cos\alpha}+\dfrac{1}{1-\cos\alpha}$

**29.** $\cot^2\theta-\cos^2\theta\cot^2\theta$

**30.** $\cos x(1+\tan^2 x)$

**31.** $\dfrac{\sin\beta}{1+\cos\beta}+\cot\beta$

**32.** $\dfrac{1}{\cos(-t)\csc(-t)}$

**33.** $\dfrac{1-\tan^2 x}{\cot^2 x-1}$

**34.** $\dfrac{\sin x\tan\left(\frac{\pi}{2}-x\right)}{\cos x}$

**35–38** Graph the given piecewise-defined function. Use open or closed circles as appropriate at the endpoints of the intervals of definition.

**35.** $F(x)=\begin{cases}-x^2 & \text{if } x\le 0\\ \frac{1}{2}x+1 & \text{if } x>0\end{cases}$

**36.** $G(x)=\begin{cases}-2x-4 & \text{if } x\le -2\\ \frac{1}{2}x+\frac{3}{2} & \text{if } -2<x\le 1\\ \frac{1}{x-1} & \text{if } x>1\end{cases}$

**37.** $H(x)=\begin{cases}-x & \text{if } x\le 0\\ \sin x & \text{if } 0<x\le\frac{\pi}{2}\\ \sqrt[3]{x-\frac{\pi}{2}} & \text{if } x>\frac{\pi}{2}\end{cases}$

**38.** $u(x)=\begin{cases}-\sqrt{-x-1} & \text{if } x\le -1\\ \sqrt{1-x^2} & \text{if } -1<x\le 0\\ x^2 & \text{if } x>0\end{cases}$

**39–42** Rewrite the given function as a piecewise-defined function, and then graph the function. Use open or closed circles as appropriate at the endpoints of the intervals of definition.

**39.** $f(x)=|x-1|$

**40.** $g(x)=\dfrac{x}{|x|}$

**41.** $h(x)=|\sin x|$

**42.** $v(x)=|x+2|+|x-3|$

**43–45** The greatest integer function is defined as follows: For $x\in\mathbb{R}$, $[\![x]\!]$ is the greatest integer less than or equal to $x$. For example, $[\![\pi]\!]=3$, $[\![1]\!]=1$, $[\![-1.5]\!]=-2$, and so on.

Use the greatest integer function to sketch the graph of the given function.

**43.** $f(x)=x-[\![x]\!]$

**44.** $g(x)=[\![x]\!]-x$

**45.** $h(x)=[\![\sin x]\!]$

**46–48** Simple polynomial functions are used to model real-life phenomena. (**Hint:** See Example 2 for guidance as you work through these problems.)

**46.** Suppose that while vacationing in Europe, one day you feel a bit dizzy and your host hands you a metric thermometer. Upon checking your temperature, the reading is 39.5 °C. Would you call the doctor? (**Hint:** Recall that the conversion formula between the Fahrenheit and Celsius scales is the linear function $C=\frac{5}{9}(F-32)$. Express $F$ from this formula to answer the question.)

**47.** Two trains are 630 miles apart, heading directly toward each other. The first train is traveling at 95 mph, and the second train is traveling at 85 mph. How long will it be before the trains pass each other?

**48.** Jessica started a candle business a few weeks ago and noticed that the relationship between her total cost in producing the candles and the number of candles produced can be modeled by a linear function. She was able to make 3 candles for a total cost of \$29, while 7 candles cost her a total of \$41 to produce.

**a.** Find a formula for the total investment as a function of the number of candles produced.

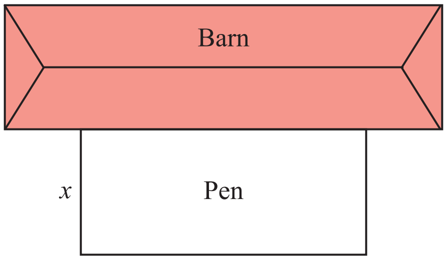
**b.** Graph the function found in part a. What are the practical meanings of the slope and $y$-intercept in this particular situation?

**c.** How much will be Jessica's total cost in producing 50 candles?

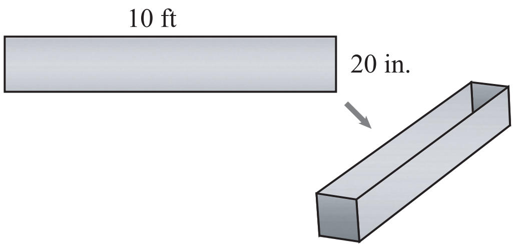
**d.** If Jessica plans to invest a total of \$320 in the next 3 months, how many candles will she be able to produce?

**49–60** Find a formula for the quantity to be optimized, and use the location of the vertex of its graph to solve the problem.

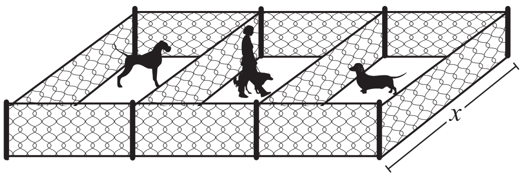
**49.** A farmer has a total of 200 yards of fencing to enclose a rectangular pen, so that one of the four sides will be the existing wall of a barn. What should the length and width be in order to maximize the enclosed area? (**Hint:** Let $x$ represent the width, and find an expression for the length in terms of $x$. Then write an expression for the area and analyze the resulting function.)




**50.** A rancher has a rectangular piece of sheet metal that is 20 inches wide by 10 feet long. He plans to fold the metal into a three-sided channel and weld two rectangular pieces of metal to the ends to form a watering trough 10 feet long. How should he fold the metal in order to maximize the volume of the resulting trough?




**51.** Cindy wants to construct three rectangular dog-training arenas side by side using a total of 400 feet of fencing. What should the overall length and width be in order to maximize the area of the three combined arenas? (**Hint:** Let $x$ represent the width, as shown, and find an expression for the overall length in terms of $x$.)




**52.** Among all the pairs of numbers with a sum of 10, find the pair whose product is maximum.

**53.** Find the point on the line $2x + y = 5$ that is closest to the origin. (**Hint:** Instead of trying to minimize the distance between the origin and points on the line, minimize the square of the distance.)

**54.** Among all the pairs of numbers $(x, y)$ such that $2x + y = 20$, find the pair for which the sum of the squares is minimum.

**55.** Find a pair of numbers whose product is maximum when two times the first number plus the second number is 48.

**56.** The total revenue for Morris' Studio Apartments is given as the function $R(x) = 100x - 0.1x^2$, where $x$ is the number of apartments rented. What is the number of apartments rented that produces the maximum revenue?

**57.** The total cost of manufacturing golf clubs is given as the function $C(x) = 800 - 10x + 0.20x^2$, where $x$ is the number of sets of golf clubs produced. How many sets of golf clubs should be manufactured to incur minimum cost?

**58.** A rock is thrown upward with a velocity of 48 feet per second from the top of a 64-foot-high cliff. What is the maximum height attained by the rock? (**Hint:** Use $h(t) = -16t^2 + 48t + 64$ to describe the height of the rock as a function of time $t$.)

**59.** Jason is driving his Mustang GT down a two-lane highway one night, carefully observing the posted speed limit sign of 55 mph. His headlights suddenly illuminate a white-tailed deer, about 120 ft in front of his car, and he immediately hits the brakes. Suppose that the coefficient of friction between his car's tires and the pavement is $\mu = 0.9$. Using the quadratic model from Example 5b in Section 1.1, do you think he will hit the deer? What if he had traveled at 60 mph?

**60.** A student is throwing a small rubber ball during physical education class at an upward angle so that the horizontal component of the ball's initial velocity is 40 feet per second. If the vertical position function of the ball is given by $h(t) = -16t^2 + 24t + 7$, how far from the student will the ball hit the ground? (**Hint:** First determine how long it will take for the ball to hit the ground. The vertical position $h$ is measured in feet, $t$ in seconds. Ignore air resistance.)

**61–72** Trigonometric and exponential functions are used to model real-life situations. (**Hint:** See Examples 6 and 7 for guidance as you work through these problems.)

**61.** Suppose several potatoes are dumped into the basket of a grocer's scale, which then proceeds to bounce up and down with an amplitude of 4 cm. As discussed in Example 6a, a first approximation to this motion can be given by a trigonometric model. Supposing that the constant $\omega$ for the above motion is $6\pi$ and that $t = 0$ when the potatoes land in the basket, find the position function for this motion. How long does it take for the basket to complete a full period?

**62.** The size of a local coyote population in a certain California national forest is estimated to cycle annually according to the function $P(t) = 250 + 20\sin(\pi t/6)$, where $t$ is measured in months, starting on March 1st of each year.

**a.** What is the approximate size of the population on July 1st?

**b.** When is the population expected to be the smallest, and what is its size then?

**63.** A certain species of fish is to be introduced into a new man-made lake, and wildlife experts estimate that the population will grow according to $P(t) = (1000)2^{t/3}$, where $t$ represents the number of years from the time of introduction.

**a.** What is the doubling time for this population of fish?

**b.** How long will it take for the population to reach 8000 fish, according to this model?

**64.** Assuming a current world population of 8 billion people, and exponential growth at an annual rate of 0.9%, what will the world population be in **a.** 10 years and **b.** 50 years?

**65.** Suppose that a new virus has broken out in an isolated region, and it is spreading exponentially through the villages. The growth of this new virus can be mapped using the following formula where $P$ stands for the number of people in a village, $V$ for the number of infected individuals, and $d$ for the number of days since the virus first appeared.

$$V = P\left(1 - e^{-0.18d}\right)$$

According to this equation, how many people in a village of 300 will be infected after 5 days?

**66.** The radioactive element polonium-210 decays according to the function $A(t) = A_0e^{-0.004951t}$, where $A_0$ is the mass at time $t = 0$, and $t$ is measured in days. The fact that $A(140) = A_0/2$ means that the half-life of polonium-210 is 140 days. What percentage of the original mass of a sample of polonium-210 remains after one year?

**67.** The half-life of a radioactive material is the time required for an initial quantity to decrease to half its original value. In the case of radium, this is approximately 1600 years.

**a.** Determine $a$ so that $A(t) = A_0a^t$ describes the amount of radium after $t$ years, where $A_0$ is the initial amount at $t = 0$.

**b.** How much of a 1-gram sample of radium would remain after 100 years?

**c.** How much of a 1-gram sample of radium would remain after 1000 years?

**68.** When continuous compounding is used in banking, the balance after $t$ years is described by the formula $A(t) = Pe^{rt}$, where $P$ is the initial amount (or principal) at $t = 0$, and $r$ is the annual interest rate. Suppose Mario made a deposit two years ago, which is compounded continuously at an annual rate of 4.5%. If his current balance is \$1094.17, how much was his initial deposit? How much longer would he have to wait until his initial deposit doubles?

**69.** The function $f(t) = C(1+r)^t$ models the rise in the cost of a product that has a cost of $C$ today, subject to an average yearly inflation rate of $r$ for $t$ years. If the average annual rate of inflation over the next decade is assumed to be 3%, what will the inflation-adjusted cost of a \$150,000 house be in 10 years?

**70.** The concentration of a certain drug in the bloodstream after $t$ minutes is given by the formula $C(t) = 0.05\left(1 - e^{-0.2t}\right)$. What is the concentration after 10 minutes?

**71.** Carbon-11 has a radioactive half-life of approximately 20 minutes, and is useful as a diagnostic tool in certain medical applications. Because of the relatively short half-life, time is a crucial factor when conducting experiments with this element.

**a.** Determine $a$ so that the formula $A(t) = A_0a^t$ describes the amount of carbon-11 left after $t$ minutes, where $A_0$ is the amount at time $t = 0$.

**b.** How much of a 2-kilogram sample of carbon-11 would be left after 30 minutes?

**c.** How much of a 2-kilogram sample of carbon-11 would be left after 6 hours?

**72.** Charles has recently inherited \$8000, which he wants to deposit in a savings account. He has determined that his two best bets are an account that compounds annually at a rate of 3.20% and an account that compounds continuously at an annual rate of 3.15%. Which account would pay Charles more interest?

## Concept Check

**73–82** Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

**73.** The slope of the graph of $y = Ax + B$ is $A$.

**74.** The slope of the graph of $y = Ax^2 + Bx + C$ is $B$.

**75.** The lines with equations $y = Ax + B$ and $y = -Bx + A$ are perpendicular to each other.

**76.** A quadratic function can have up to two $y$-intercepts.

**77.** If line $L_1$ has positive slope and $L_2$ is perpendicular to $L_1$, then the slope of $L_2$ is negative.

**78.** If a polynomial has even degree, then its graph always rises to both the right and the left.

**79.** All rational functions of the form $p(x)/q(x)$, where $q(x)$ is nonconstant, have at least one asymptote of some kind.

**80.** Trigonometric functions are transcendental.

**81.** Logarithmic functions are transcendental.

**82.** If a population of bacteria grows without restriction from 1000 to 2000 in one hour, then it will grow to 3000 during the next hour.