Chapter 1
Review Exercises

1–4 Find the domain and range of the given relation and determine whether the relation is a function.

1. $R=\{(-2,9),(-3,-3),(-2,2),(-2,-9)\}$

2. $3x-4y=17$

3. $x=y^2-6$

4. $x=\sqrt{y-4}$

5–8 Identify the domain, codomain, and range of the given function.

5. $f:\mathbb{N}\to\mathbb{R},\quad f(x)=\dfrac{3x}{4}$

6. $g:\mathbb{R}\to\mathbb{R},\quad g(x)=5x+1$

7. $h:\mathbb{R}\to\mathbb{R},\quad h(x)=\dfrac{1}{x^2+1}$

8. $k:\mathbb{N}\to\mathbb{R},\quad k(x)=2+\sqrt{x-1}$

9–12 Find the value of the given function for **a.** $f(x-1)$, **b.** $f(x^2)$, and **c.** $\dfrac{f(x+h)-f(x)}{h}$.

9. $f(x)=(x+5)(2x)$

10. $f(x)=\sqrt[3]{x}+6(x+4)$

11. $f(x)=\dfrac{3}{x+2}$

12. $f(x)=\sin 2x$

13–14 Find all open intervals of monotonicity (intervals where the function is increasing or decreasing) for the given function.

13. $f(x)=(x-2)^4-6$

14. $R(x)=\begin{cases}(x+2)^2 & \text{if } x<-1\\ -x & \text{if } x\ge -1\end{cases}$

15–18 Determine if the function is even, odd, or neither and then graph it.

15. $f(x)=\dfrac{1}{3}x^3$

16. $f(x)=\sqrt{x}$

17. $f(x)=-2\sin x$

18. $g(x)=-2\sin^2 x$

19–20 Discuss the symmetry of the given equation and then graph it.

19. $y=|5x|$

20. $x^2+y^2=25$

21–34 Graph the given function. Locate the x- and y-intercepts, if any.

21. $f(x)=7x-2$

22. $f(x)=\dfrac{2x-6}{3}$

23. $f(x)=(x-1)^2-1$

24. $f(x)=-x^2-6x-11$

25. $f(x)=4x^3$

26. $f(x)=-\dfrac{2}{x^2}$

27. $f(x)=\dfrac{-x^3+7x+6}{2}$

28. $f(x)=\dfrac{x+1}{x^2-4}$

29. $f(x)=\dfrac{\sqrt[3]{x}}{2}$

30. $f(x)=5|-x|$

31. $f(x)=\left[\!\left[\dfrac{2x}{3}\right]\!\right]$

32. $f(x)=\begin{cases}x^2 & \text{if } x<1\\ \dfrac{1}{x} & \text{if } x\ge 1\end{cases}$

33. $f(x)=-\cot x$

34. $f(x)=\log_{1/2} x$

35–36 Sketch the graph of the given function by first identifying the more basic function that has been shifted, reflected, stretched, or compressed. Then determine the domain and range of the function.

35. $f(x)=(x-1)^3+2$

36. $f(x)=4|x+3|$

37–38 Write an equation for the function described.

37. Use the function $f(x)=1/x$. Move the function 2 units to the right and 3 units up.

38. Use the function $f(x)=\sqrt[3]{x}$. Move the function 1 unit to the left and reflect across the y-axis.

39–40 Find the formula and domain for **a.** $f+g$ and **b.** f/g.

39. $f(x)=x^2;\quad g(x)=\sqrt{x}$

40. $f(x)=\dfrac{1}{x-2};\quad g(x)=\sqrt[3]{x}$

41–42 Use the information given to determine **a.** $(f\circ g)(x)$, **b.** $(g\circ f)(x)$, and **c.** $(f\circ g)(3)$.

41. $f(x)=-x+1;\quad g(x)=-x-1$

42. $f(x)=\dfrac{x^{-1}}{18}-3;\quad g(x)=\dfrac{x-4}{x^3}$

©HAWKES LEARNING

43–44 Write the given function as a composition of two functions. (Answers will vary.)

43. $f(x)=\dfrac{\sqrt{x+3}+2}{x^2+6x+9}$

44. $f(x)=|x+2|+x^2+4x+4$

45–46 Graph the inverse of the given relation, and state its domain and range.

45. $R=\{(-3,5),(2,1),(0,-5),(-1,-2)\}$

46. $y=\dfrac{1}{3}x^2$

47–50 Find the inverse of the given function.

47. $f(x)=\dfrac{2}{7x-1}$ **48.** $f(x)=\dfrac{4x-3}{x}$

49. $f(x)=x^{1/5}-6$ **50.** $f(x)=\dfrac{6x-7}{2-x}$

51. Show that $f^{-1}(f(x))=x$ and that $f(f^{-1}(x))=x$ for the functions given in Exercises 49 and 50.

52–53 Rewrite the given function as a purely algebraic function.

52. $\cos(\sin^{-1}x)$ **53.** $\tan\left(\sec^{-1}\dfrac{x}{2}\right)$

54–55 Use the properties of logarithms to expand the given expression as much as possible; that is, decompose the expression into sums or differences of the simplest possible terms.

54. $\ln\dfrac{x^2y-xz}{5y^3}$ **55.** $\log\sqrt[3]{\dfrac{yz^2}{x^4}}$

56. If a pebble is shot upward with an initial (vertical) velocity of $56\ \text{ft/s}$, how high does it go? (**Hint:** Use the height function $h(t)=-16t^2+56t$.)

57. The half-life of actinium-225 is 10 days. Assuming that A_0 is the mass at time $t=0$, find the function $A(t)$ that gives the mass remaining after t days. What percentage of the original mass of a sample of Ac-225 remains after 25 days?

58. Prove that the product of an odd function and an even function is odd.

59. Among all the pairs of numbers with a sum of 15, find the pair whose product is maximum.

60. The January 2010 Haiti earthquake was initially reported as a 7.2-magnitude quake on the Richter scale, while the March 2011 earthquake off the eastern shores of Japan was magnitude 9.0. According to these numbers, about how many times more intense was the Japanese earthquake? (**Hint:** See the directions preceding Exercise 138 in Section 1.4.)

61–62 Determine if the given complex number is in the Mandelbrot set. (**Hint:** See the directions preceding Exercises 101–108 in Section 1.3.)

61. $c=-i$ **62.** $c=2$

Concept Check

63–76 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

63. The graph of any function can always be represented by a curve that passes the vertical line test.

64. Any linear function whose graph is a line with negative slope always has an inverse.

65. If f is a function, and $f(a)=f(b)$, then $a=b$.

66. If $(2,5)$ is a point on the graph of an odd function, then $(-2,-5)$ is also on the graph.

67. The graph of a quadratic function is a parabola, so quadratic functions are even functions.

68. In general, $(f\circ g)(x)=(g\circ f)(x)$ holds for all x if both compositions can be formed.

69. If a function can be decomposed into three functions, it can only be done in one way.

70. The graph of $f(x)+a$ is that of $f(x)$ translated vertically by $|a|$ units.

71. If $c\in\mathbb{R}$ is a constant, and f is any function, then the graphs of $f(cx)$ and $cf(x)$ are identical.

72. When the graph of any function $f(x)$ is reflected about the line $y=x$, the graph of the inverse is obtained.

73. The graph of any function must pass the horizontal line test.

74. If (a,b) is a point on the graph of an invertible function $f(x)$, then (b,a) is on the graph of its inverse.

75. The common logarithm and $y=10^x$ are inverses.

76. The function $f(x)=\tan^{-1}x$ has no asymptotes.

©HAWKES LEARNING

Chapter 1
Technology Exercises

77–79 Mentally sketch the graph of the given function by identifying the basic shape that has been shifted, reflected, stretched, or compressed. Then use a graphing utility to graph the function and check your reasoning.

77. $f(x) = \ln(x+1) + 2$

78. $f(x) = -\dfrac{2}{x-3} + 1$

79. $f(x) = \sin \pi x - 1$

80. The annual expenditures (in millions of dollars) for a corporation are given in the table below.

Annual Expenditures

Year	2017	2018	2019	2020	2021	2022
Expenditures (in millions)	$16.2	$17.1	$18.8	$19.6	$21.1	$22.9

a. Find the least-squares line of best fit for the data. (Let $x = 0$ correspond to the year 2017.)

b. Estimate the expenditures for 2023.

81–82 Use a graphing utility to approximate the solution(s) of the given equation, rounded to four decimal places. (**Hint:** Zoom in on the x-intercepts or points of intersection as appropriate for each equation.)

81. $x^5 - x^3 - 3 = 0$

82. $x^2 + 6 = 2^{x+1}$

83–84 Use a graphing utility to graph the given function, and describe the characteristics of the graph as c varies. Use different viewing windows.

83. $u(x) = \dfrac{1 - e^{c/x}}{1 + e^{c/x}}$

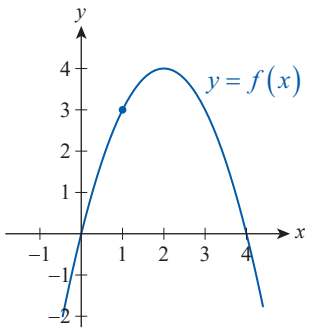
84. $v(x) = \dfrac{x}{c^2}\sqrt[4]{c^4 - x^4}$

©HAWKES LEARNING

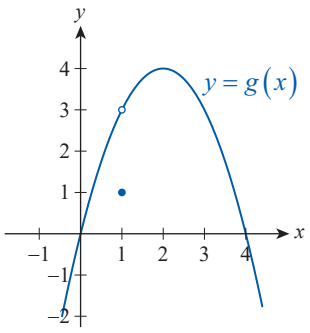
Chapter 2
Review Exercises

1–4 Use the graph of the function to find the indicated (possibly one-sided) limit, if it exists. Also examine the continuity of the function at the indicated point and classify any discontinuities.

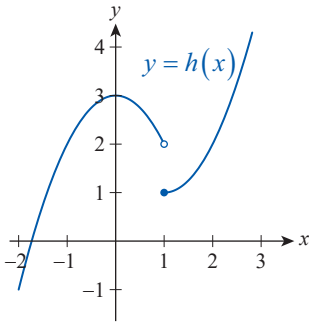
1. $\lim_{x\to 1} f(x)$



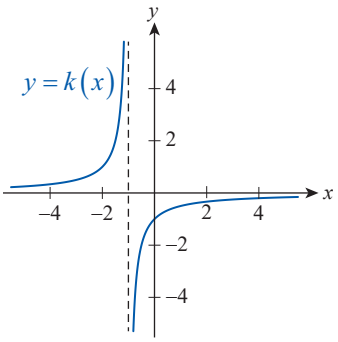
2. $\lim_{x\to 1} g(x)$



3. $\lim_{x\to 1^-} h(x)$



4. $\lim_{x\to -1^+} k(x)$



5–8 Use difference quotients to approximate the slope of the tangent to the graph of the function at the given point. Use at least five different *h*-values that are decreasing in magnitude. (Answers will vary.)

5. $f(x) = 3x + 2; \quad (0,2)$

6. $g(x) = 2 - x^2; \quad (1,1)$

7. $h(x) = \sqrt{x-1}; \quad (2,1)$

8. $k(x) = \sin x; \quad (0,0)$

9. A pellet is shot vertically upward from an initial height of 6 feet. Its height after t seconds is given by $h(t) = 6 + 608t - 16t^2$ feet. Use difference quotients to answer the questions below.

a. What will be the pellet's height at the end of the first second?

b. What is the average velocity of the pellet during the first two seconds?

c. Estimate the instantaneous velocity at $t = 0$ seconds.

d. Estimate the instantaneous velocity at $t = 2$ seconds.

e. When will the velocity be 0?

10–11 Approximate the area of the region between the graph of $f(x)$ and the *x*-axis on the given interval. Use $n = 4$, as in Example 5a of Section 2.1. (Round your answer to four decimal places.)

10. $f(x) = x^3$ on $[0,1]$ **11.** $f(x) = \ln x$ on $[1,3]$

©HAWKES LEARNING

12–13 Approximate the arc length of the graph of $g(x)$ on the given interval. Use $n = 5$, as in Example 5b of Section 2.1. (Round your answer to four decimal places.)

12. $g(x) = x^{2/3}$ on $[3,8]$ **13.** $g(x) = e^x$ on $[-2,3]$

14–17 Create a table of values to estimate the value of the indicated limit without graphing the function. Choose the last x-value so that it is no more than 0.001 units from the given c-value.

14. $\lim\limits_{x\to 1} \dfrac{x^3-1}{x-1}$ **15.** $\lim\limits_{x\to 0} x^x$

16. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{4x}$ **17.** $\lim\limits_{x\to 0} \left(2x \sin \dfrac{1}{4x}\right)$

18. Use one-sided limit notation to describe the behavior of $f(x) = \dfrac{1}{x-1}$ near $x = 1$.

19–20 Find a $\delta > 0$ that satisfies the limit claim corresponding to $\varepsilon = 0.01$, that is, such that $0 < |x - c| < \delta$ would imply $|f(x) - L| < 0.01$.

19. $\lim\limits_{x\to 0}(3-2x) = 3$ **20.** $\lim\limits_{x\to 4} \sqrt{x} = 2$

21–24 Give the formal definition of the limit claim. Then use the definition to prove the claim.

21. $\lim\limits_{x\to 1}(3x+1) = 4$ **22.** $\lim\limits_{x\to 1} x^2 = 1$

23. $\lim\limits_{x\to 1} \sqrt{x} = 1$ **24.** $\lim\limits_{x\to 2} \dfrac{2}{x} = 1$

25–41 Use algebra and/or appropriate limit laws to evaluate the given limit (one-sided limit where indicated). If the limit is unbounded, use the symbol ∞ or $-\infty$ in your answer.

25. $\lim\limits_{x\to 3}(2x^2 - 3x + 5)$

26. $\lim\limits_{x\to -2}\left(\dfrac{x^3}{4} + 2x^2 - x + 1\right)$

27. $\lim\limits_{x\to 3} \sqrt{x^3 + 2x^2 + 4}$ **28.** $\lim\limits_{x\to -2} \dfrac{2x+1}{x^2 - x}$

29. $\lim\limits_{t\to 1} \left(\dfrac{3t + 5t^3}{t^2 + 1}\right)^{3/2}$ **30.** $\lim\limits_{x\to 4} \dfrac{x^2 - 16}{x - 4}$

31. $\lim\limits_{x\to -5} \dfrac{x+5}{x^2 - 25}$ **32.** $\lim\limits_{x\to -5^-} \dfrac{x+5}{x^2 - 25}$

33. $\lim\limits_{x\to 1^+} \dfrac{x^2 + 1}{x^4 - 1}$ **34.** $\lim\limits_{x\to 1} \dfrac{x^2 - 1}{x^4 - 1}$

35. $\lim\limits_{x\to 3} \dfrac{\sqrt{x+1} - 2}{x - 3}$ **36.** $\lim\limits_{x\to 0} \dfrac{\dfrac{1}{2+x} - \dfrac{1}{2}}{x}$

37. $\lim\limits_{x\to 0^-} \dfrac{2|x|}{x}$ **38.** $\lim\limits_{x\to -2^+} \sqrt{4 - x^2}$

39. $\lim\limits_{x\to 2^-} (\llbracket x \rrbracket + 2x)$ **40.** $\lim\limits_{x\to 1^+} \llbracket x \rrbracket x$

41. If $f(x) = x^2$, find $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

42–43 Use the Squeeze Theorem to prove the limit claim.

42. $\lim\limits_{x\to 0} x \cos \dfrac{1}{x} = 0$ **43.** $\lim\limits_{x\to \infty} \dfrac{\sin x}{\ln x} = 0$

44. Sketch a graph of a function (a formula is not necessary) that is not continuous at $x = 0$ from either direction, but both of its one-sided limits exist at $x = 0$. (Answers will vary.)

45. Sketch a graph of a function that is left-continuous at $x = 0$, but its right-hand limit at $x = 0$ doesn't exist. (Answers will vary.)

46–51 Find and classify the discontinuities (if any) of the given function as removable or nonremovable.

46. $f(x) = \dfrac{x-9}{\sqrt{x} - 3}, \quad x \geq 0$

47. $g(x) = \dfrac{\sqrt{x} + 2}{x - 4}, \quad x \geq 0$

48. $h(x) = \dfrac{1}{\sqrt{x^2 + 1}}$

49. $t(x) = 2 + 2\llbracket x \rrbracket$

50. $G(x) = \dfrac{x}{\sqrt{x+1} - 1}, \quad x \geq -1$

51. $k(x) = |x - 3| + |x + 1|$

52–53 Use the ε-δ definition to prove that the function is continuous.

52. $f(x) = 3x - 1$ **53.** $g(x) = 2x^2$

54. Find the values of a and b such that f is continuous on the entire real line.

$$f(x) = \begin{cases} -1 & \text{if } x \leq -3 \\ ax + b & \text{if } -3 < x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$$

55. Use the Intermediate Value Theorem to prove that the equation $2x^5 + x + 1 = 0$ has a solution on the interval $[-1,1]$.

©HAWKES LEARNING

56. Use the Intermediate Value Theorem to show that the graphs of $f(x) = x^3$ and $g(x) = e^{-x}$ intersect.

57–58 Find the equation of the tangent line to the graph of $f(x)$ at the given point.

57. $f(x) = x^2 + x; \quad (1,2)$

58. $f(x) = \sqrt{x}; \quad (4,2)$

59–60 Use the definition (also called the limit process) to find the derivative function f' of the given function f. Find all x-values (if any) where the tangent line is horizontal.

59. $f(x) = 2x - x^2$ **60.** $f(x) = \dfrac{3}{x-2}$

61–62 Sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

61. f is continuous at 0, $f(0) = 1$, $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$, and $f'(0)$ does not exist

62. $g(1) < 0$, $g'(1) > 0$, and $g(2) > 0$, but $g'(2) < 0$

63. Prove that if $f(x)$ is a quadratic function, then $f'(x)$ is linear.

64. A small object is thrown upward with an initial velocity of 12 m/s from the top of a 15 m high building.

a. How high does it go and when does it reach the ground?

b. What is the speed of impact?

(**Hint:** Use $h(t) = -5t^2 + 12t + 15$ as the position function, where h is in meters, t in seconds.)

65. The owner of a small toy manufacturer has determined that he can sell x toys if the price is $p = D(x) = 0.2x + 30$ dollars. The total cost as a function of x is given by $C(x) = 0.1x^2 + 15x + 247.5$ dollars.

a. Find the profit function $P(x)$.

b. Find any break-even points.

c. Find the marginal profit function.

Concept Check

66–73 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

66. Instantaneous velocity can be interpreted as the slope of a tangent line.

67. If $\lim_{x \to c} f(x)$ doesn't exist, then $f(x)$ has a vertical asymptote at $x = c$.

68. Any rational function has at least one vertical asymptote.

69. If $\lim_{x \to c} f(x) = A$ and $\lim_{x \to c} g(x) = B$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{A}{B}$.

70. If f is defined on $[a,b]$, L is a real number between $f(a)$ and $f(b)$, and $\lim_{x \to c} f(x)$ exists for all $x \in (a,b)$, then there is a c in the interval (a,b) such that $f(c) = L$.

71. If f is continuous at c, then $f(c)$ is equal to both one-sided limits at c.

72. If both one-sided limits of f exist at c, and if f is defined at c, then f is continuous at c.

73. If $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, and if $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$, then by the Squeeze Theorem $f(c) = L$ as well.

Chapter 2 Technology Exercises

74. Use a computer algebra system to find approximations for the areas in Exercises 10 and 11 by using $n = 100$. (Round your answers to four decimal places.)

75. Use a computer algebra system to find approximations for the arc lengths in Exercises 12 and 13 by using $n = 100$. (Round your answers to four decimal places.)

76. Use a graphing utility to verify your answers given for Exercises 14–17.

77. Use a graphing utility to approximate the solutions for Exercises 55 and 56. Round your answers to four decimal places.

78–81 Use a graphing utility to graph the function, and estimate from the graph the value of the given limit.

78. $\lim_{x \to \infty} x^{1/x}$ **79.** $\lim_{x \to 0} \dfrac{\arcsin x}{x}$

80. $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^{2x}$ **81.** $\lim_{x \to 1} \dfrac{\ln(x^3)}{x-1}$

©HAWKES LEARNING

Chapter 3
Review Exercises

1–2 Find the derivative of the given function at the specified point and express your answer using the differential notation due to Leibniz.

1. $f(x) = x^3 + x; \quad x = 1$

2. $g(x) = \dfrac{2}{x}; \quad x = 2$

3–4 Find the derivative of the function and use the differentiation operator D_x to express your answer.

3. $s(x) = \sqrt{x-2}$

4. $t(x) = \dfrac{1}{x^2+1}$

5–6 Find the first, second, and third derivatives of the function.

5. $f(x) = x^2 - 1$

6. $g(x) = x^4$

7–10 Find all points where the function is not differentiable. For each of those points, find the one-sided derivatives (if they exist).

7. $f(x) = \sqrt[3]{x}$

8. $g(x) = |x+1| + |x-3|$

9. $h(x) = [\![x]\!] + x$

10. $F(t) = \begin{cases} t^2 \sin\dfrac{1}{t} & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases}$

11–20 Use differentiation rules to find the derivative of the function.

11. $f(x) = 0.2x^5 - 2x^4 + x^3 + 0.5x^2 + 2^{3/4}$

12. $g(x) = \sqrt{3}x + \sqrt{3x} + \dfrac{1}{\sqrt{3x}}$

13. $h(x) = (2x-1)(x^2+4)$

14. $k(x) = \dfrac{x-2}{x}e^{x-2}$

15. $F(t) = (t-1)\left(t^2 + \dfrac{1}{t}\right)\left(\sqrt{t} + t\right)$

16. $G(s) = \dfrac{1}{2s + s^2}$

17. $u(x) = \dfrac{2e^x + 1}{3e^x + 5}$

18. $t(x) = \dfrac{\dfrac{2}{x} + \dfrac{1}{x^2} + 4}{\dfrac{2}{x^2} - \dfrac{1}{x}}$

19. $v(x) = \ln(x^2 + 2)$

20. $w(x) = \sin(\sin(\sin x))$

21–24 Find the first, second, and third derivatives of the function.

21. $f(x) = \dfrac{x}{x+1}$

22. $f(x) = 3\sqrt{x}$

23. $f(x) = \tan x$

24. $f(x) = \arctan x$

25–26 Find a function f that satisfies the given conditions. (**Hint:** A polynomial is the most natural choice. Answers will vary.)

25. $f(0) = 0$, $f'(1) = 1$, and $f''(2) = 4$.

26. $f(0) = 2$ and $y = 2x + 1$ is tangent to the graph at $x = -1$.

27–28 Find the equation of the line tangent to the graph of the function at the given point.

27. $f(x) = \dfrac{1}{\sqrt{3x^2+1}}; \quad \left(1, \dfrac{1}{2}\right)$

28. $f(x) = \tan(\sin x); \quad (\pi, 0)$

29. Find the equation(s) of the line(s) tangent to the graph of $f(x) = x^2 + 3x + 1$ through the point $(2,2)$, which is not on the graph of f.

30. Assuming f is differentiable, find the derivative of $y = \ln\sqrt{[f(x)]^2 + 1}$.

31–32 Find the indicated limit.

31. $\lim\limits_{x \to 0} \dfrac{-\sin 2x}{4x}$

32. $\lim\limits_{x \to 0^+} \dfrac{x \cos x}{1 - \cos x}$

33. The position function of a moving particle is given by $x(t) = \dfrac{50t}{t+1}$ feet at t seconds. Find its velocity and acceleration at $t = 1$ second.

34. An object is moving along a straight line so that its distance from the start at t seconds is given by $d(t) = 12t - t^3$ meters. Find its position and acceleration at the instant when its velocity changes directions.

35. The radius of a spherical balloon being inflated increases according to the function $r(t) = 3 + 4\sqrt[3]{t}$, where r is measured in centimeters and t in seconds. Find the rate of change of the balloon's volume and surface area with respect to time at $t = 1$ second.

©HAWKES LEARNING

36–37 Find dy/dx by implicit differentiation.

36. $x^3 + y^3 = 2$

37. $6\left(x^2 + y^2\right) = 15xy$

38–39 Find dx/dy by implicit differentiation.

38. $x\sin\left(x + y\right) = y^2 + 6$

39. $6\left(y^2 - x^2\right) = y^4$

40–43 Use implicit differentiation to find the equation of the line tangent to the curve at the indicated point.

40. $\dfrac{1}{x^3} + \dfrac{1}{y^3} = 2; \quad (1,1)$

41. $x^3 + y^2 = 2x + 1; \quad (0,1)$

42. $\dfrac{3\left(x + y\right)}{xy} = 16\sqrt{x + y}; \quad \left(\dfrac{3}{4}, \dfrac{1}{4}\right)$

43. $3\sqrt{x} + \dfrac{2}{\sqrt{y}} = xy; \quad (1,4)$

44. Find all points on the lemniscate

$$\left(x^2 + y^2\right)^2 = 2\left(x^2 - y^2\right)$$

where its graph has horizontal tangent lines.

45. Use implicit differentiation to find d^2y/dx^2 for $x^{2/3} + y^{2/3} = 1$.

46–49 Use the Derivative Rule for Inverse Functions to determine $\left(f^{-1}\right)'(a)$ for the indicated value of a. (The domain of f is assumed to have been restricted so that the inverse exists and is differentiable, whenever appropriate.)

46. $f\left(x\right) = x^3 + x; \quad a = 10$

47. $f\left(x\right) = \sqrt[4]{x + 1}; \quad a = 2$

48. $f\left(x\right) = \dfrac{2}{x^2}; \quad a = \dfrac{1}{2}$

49. $f\left(x\right) = e^x + x; \quad a = 1$

50–53 Determine the derivative of the given function.

50. $f\left(x\right) = \log\sqrt{x^2 + 1}$

51. $f\left(x\right) = \tan^{-1}\sqrt{x}$

52. $f\left(x\right) = e^{\arcsin x}$

53. $f\left(x\right) = \sec^{-1}\left(\ln x\right)$

54–55 Use logarithmic differentiation to find y'.

54. $y = \dfrac{\sqrt[3]{x^2 + 1}\left(x + 3\right)}{x^{2/3}\sqrt{2x^2 + 3}}$

55. $y = \left(\sqrt{x}\right)^{\ln x}$

56. A fast-growing population of bacteria doubles every half hour. If the initial count is 1000, how many bacteria are there in 100 minutes?

57. A 350 °F pizza is left on the counter and cools to 250 °F in 4 minutes. If the room temperature is 70 °F, determine the total time it takes for the pizza to cool down from 350 °F to 185 °F. (**Hint:** See Exercise 9 of Section 3.7.)

58. Find the rate of change of the distance from the origin of a point moving on the graph of $f\left(x\right) = x^3$ when $x = 1$ and $dx/dt = 2$ units per second.

59. A spherical balloon is being filled with helium at a rate of $20\ \text{in.}^3/\text{s}$. How fast is the radius increasing at the instant when the radius is 4 in.?

60. A small plane, flying at an altitude of 0.1 miles at a ground speed of 85 miles per hour, passes directly over an observer. How fast is the distance between the observer and the plane increasing a minute later?

61. Radar is tracking a rocket that was launched vertically upward. It is found that the rocket's distance from the radar is increasing at a rate of $1200\ \text{km/h}$ at the instant when that distance is 5 km. If the radar station is 4 km from the launch site, find the speed of the rocket.

62. A ship sailing west at 9 miles per hour passes a buoy 20 minutes before another ship sailing due north at 12 miles per hour passes the same buoy. How fast will they be separating an hour later?

63. Tiffany walks toward a light source that is 8 feet above ground. If the speed of the tip of her shadow is three times that of her walking speed, how tall is Tiffany?

64–65 Find the linearization of the function at the given value.

64. $f\left(x\right) = \dfrac{1}{\left(x - 1\right)^2}; \quad x = 2$

65. $f\left(x\right) = \sin x; \quad x = \dfrac{\pi}{4}$

66–67 Use linear approximation to approximate the given number. Round your answer to four decimal places.

66. $\sqrt[3]{8.2}$

67. $\arctan 0.9$

68. The diameter of a large bouncy ball was measured to be 65 cm with a possible error of 1 mm. Approximate the propagated errors in the calculated volume and surface area of the ball, respectively. Express your answers as percentage errors.

©HAWKES LEARNING

69. The proper dosage d of a certain over-the-counter medicine for children depends on body weight w according to the function $d(w) = \frac{5}{4}w^{3/5}$, where d is measured in milligrams and w in pounds. Use differentials to estimate how accurately (in terms of percentage error) we need to know a 32-pound child's weight if we cannot stray from the proper dosage by more than 6 percent.

70. A manufacturing business found its daily revenue to be $R(x) = 150x - \frac{1}{4}x^2$ dollars when x units are produced and sold.

a. Use linearization and marginal revenue to estimate the extra revenue when production is increased from 100 to 102 units.

b. Use the revenue function to calculate the actual revenue increase. Compare your answers.

71. Use the concept of the derivative function to explain why the graph of $y = x^a$, $a > 1$ curves upward, while the graph of $y = x^b$, $0 < b < 1$ curves downward.

Concept Check

72–82 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

72. If both one-sided derivatives of $f(x)$ either exist or are equal to $\pm\infty$ at c, then f is continuous at c.

73. If $p(x)$ is a polynomial of degree n, then all k^{th}-order derivatives of $p(x)$ for $k > n$ are 0.

74. If $y = \pi^n \sin x$, then $y' = n\pi^{n-1}\sin x + \pi^n \cos x$.

75. If $y = 1/(x^2 - 3x + 1)$, then $y' = 1/(2x - 3)$.

76. If $y = \ln(3x + 1)$, then $y' = 1/(3x + 1)$.

77. If $y = x^x$, then $y' = x \cdot x^{x-1}$.

78. Since $(e^x)' = e^x$, therefore $(e^{e^x})' = e^{e^x}$.

79. If $f(x) = x$, then $df = dx$.

80. If $f(x)$ is linear, then its linearization at any point is itself.

81. If $x \to 0$, then $\Delta x \to dx$ and $\Delta y \to dy$.

82. If $\Delta x \to 0$, then $\Delta y/\Delta x \to dy/dx$.

Chapter 3
Technology Exercises

83–85 Use a graphing utility to graph the function and identify all points where the function is not differentiable. Explain.

83. $f(x) = |x^2 - x|$

84. $f(x) = |x|(x + 2)$

85. $f(x) = \sqrt[4]{x^2 - 1}$

86. Use the differentiation capabilities of a graphing utility to find the derivative of $f(x) = 2\cos^2 x - \cos 2x$. Then find the derivative by hand, applying a trigonometric identity before differentiating. Does your answer agree with that of your technology? If not, what do you think is the reason? Can you "force" your graphing utility to represent its answer in a simpler form?

87. Repeat Exercise 86 for the function $f(x) = 2\sin(x/2)\cos(x/2)$.

88. Use a graphing utility to graph the functions $y = \ln x$, $y = a^x$, $a > 1$ and $y = x^b$, $0 < b < 1$ for various values of the parameters a and b. By zooming out appropriately, compare their relative growth rates; that is, conjecture "who wins the race toward infinity" in general among these three types of functions. Use the concept of the derivative to support your conjecture.

89. The displacement of a mass attached to a spring is given by the function $h(t) = e^{-t/6}\cos 2t$.

a. Use a graphing utility to graph the function and explain why it is realistic.

b. Use a graphing utility to graph the velocity and acceleration functions together with $h(t)$ on the same screen. What seems to be the position of the mass when velocity is maximum? When is velocity 0? When is acceleration maximum, and when is it 0?

©HAWKES LEARNING

Chapter 4
Review Exercises

1–4 Sketch by hand the graph of a function f on the specified domain, with the specified properties. (Answers will vary.)

1. Defined on $(0,2)$, absolute minimum at 1, no absolute maximum
2. Defined on $[-2,2]$, absolute maximum occurs twice, no absolute minimum
3. Defined on $(-1,1)$, no absolute or relative extrema
4. Differentiable on $(0,1)$, no critical points, no extrema

5–14 Find all absolute extrema of the function on the indicated domain.

5. $f(x)=x^2-\frac{4}{3}x^3;\quad D=[-1,1]$
6. $f(x)=-x^4+8x^3-16x^2;\quad D=[-1,5]$
7. $f(x)=|x^2-2x-8|;\quad D=[-3,5]$
8. $f(x)=(x+2)|x|;\quad D=[-2,2]$
9. $f(x)=\sqrt{x}(1-x);\quad D=[0,2]$
10. $f(x)=\frac{x^2+1}{x+1};\quad D=[0,1]$
11. $f(x)=3x^5-2x^3+1;\quad D=\mathbb{R}$
12. $f(x)=\sqrt{1-x^4};\quad D=(-1,1)$
13. $f(x)=\csc\frac{x}{2};\quad D=(0,2\pi)$
14. $f(x)=x^2-x[\![x]\!];\quad D=[1,2]$

15–16 Prove that the equation has exactly one real solution on the given interval.

15. $3x^3-x^4=1$ on $(0,1)$
16. $x\arcsin x=e^{-x}$ on $(0,1)$

17–18 Determine whether Rolle's Theorem applies to the function on the given interval. If so, find all possible values of c as in the conclusion of the theorem. If the theorem does not apply, state the reason.

17. $f(x)=x^3+x^2-8x-12$ on $[-2,3]$
18. $g(x)=x^4+2x^2-2$ on $[0,1]$

19–20 Determine whether the Mean Value Theorem applies to the function on the given interval. If so, find all possible values of c as in the conclusion of the theorem. If the theorem does not apply, state the reason.

19. $f(x)=|x^4-3x|$ on $[2,3]$
20. $f(x)=|x^4-3x|$ on $[0,2]$
21. If $|f'(x)|\le 3$ for all x, prove that $|f(10)-f(2)|\le 24$.
22. If $g(-5)=-1$ and $g'(x)\le 4$ for all x, what is the largest possible value of $g(1)$?
23. Find the function f that passes through $(0,3)$ and whose derivative is $\cos x+e^x$.
24. An object is moving along the x-axis, starting at $x_0=-4$ with velocity function $v(t)=3t^2-2t$ $(0\le t\le 5)$. Find the time t when it reaches the origin.
25. A car driving at 70 mph passes a mile marker, and then exactly 48 seconds later, still driving at 70 mph, passes the next mile marker.
    a. Prove that there was at least one instant when the car traveled at 75 mph between the markers.
    b. Prove that there was at least one instant when the car's acceleration was zero.

26–29 Use the first derivative to determine where the function is increasing and decreasing.

26. $f(x)=|2x-2|$
27. $g(x)=-x^2+6x+7$
28. $h(x)=x^3-6x^2+9x+1$
29. $k(x)=x^4-4x^3-20x^2+96x$

30–33 Determine the intervals of concavity of the given function.

30. $f(x)=\frac{x^3}{6}-x^2-2$
31. $g(x)=\frac{e^x}{x}$
32. $h(x)=(x-2)\sqrt[3]{x}$
33. $k(x)=\frac{x+3}{x^2-1}$

34–35 Use the first and second derivatives to identify the intervals of monotonicity, extrema, intervals of concavity, and inflection points of the given function.

34. $f(x)=3x^5-20x^3$
35. $g(x)=\arctan(x^2)$

©HAWKES LEARNING

36–37 The function $p(t)$ gives the position, relative to its starting point, of an object moving along a straight line. Identify the time intervals when the object is moving in the positive versus negative direction, as well as those intervals when it is accelerating or slowing down. Find the times when the object changes direction as well as when its acceleration is zero.

36. $p(t) = t^3 - 3t^2, \quad 0 \le t \le 5$

37. $p(t) = 3t^2 - \dfrac{t^4}{2}, \quad 0 \le t \le 3$

38–49 Check whether L'Hôpital's Rule applies to the given limit. If it does, use it to determine the value of the limit. If it does not, find the limit some other way. (When necessary, apply L'Hôpital's Rule several times.)

38. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

39. $\lim\limits_{x\to 0} \dfrac{x - \tan x}{\sec x - 1}$

40. $\lim\limits_{x\to \infty} \dfrac{e^{-x}}{\ln x}$

41. $\lim\limits_{x\to 0^+} \cot x \csc x$

42. $\lim\limits_{x\to 0} \left(1 + 4x^2\right)^{1/x^2}$

43. $\lim\limits_{x\to (1/2)^+} \left(\dfrac{1}{4x-2} - \dfrac{1}{\ln 2x}\right)$

44. $\lim\limits_{x\to 0^+} x^{\sqrt{x}}$

45. $\lim\limits_{x\to 0^+} \left(\sqrt{x}\right)^{\ln x}$

46. $\lim\limits_{x\to 0^-} x \cot x$

47. $\lim\limits_{x\to 0} \dfrac{\arctan x}{\arctan 2x}$

48. $\lim\limits_{x\to 0^+} \sin x \ln x$

49. $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \csc x\right)$

50. By examining the limits $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^a}$ and $\lim\limits_{x\to\infty} \dfrac{x^a}{b^x}$ using L'Hôpital's Rule, compare the relative growth rates of the functions $y = \ln x$, $y = x^a$, and $y = b^x$ ($a > 0$, $b > 1$). (See Exercise 88 in the Chapter 3 Review.)

51–60 Use the curve-sketching strategy to construct a graph of the function.

51. $f(x) = \dfrac{x^3}{3} - x^2 - 15x$

52. $g(x) = -x^3 + 12x - 16$

53. $h(x) = x^4 - 2x^3$

54. $m(x) = 3x^3 - 4x^5$

55. $f(x) = \dfrac{2x}{x^2+1}$

56. $f(x) = \dfrac{2x}{x^2-1}$

57. $f(x) = |x|(x-1)$

58. $f(x) = x\sqrt{9-x^2}$

59. $f(x) = \dfrac{x^3}{x^2-4}$

60. $f(x) = \sin^2 x \cos x$

61–62 Use Newton's method to approximate the given number to five decimal places.

61. $\sqrt[5]{30}$

62. $\log 11$

63–64 Use Newton's method to solve the equation on the given interval. Approximate the root to six decimal places.

63. $2x^5 = 1 - x^2$ on $(0,1)$

64. $\ln x = \cos x$ on $(0,\infty)$

65. Use Newton's method to approximate to four decimal places the fixed point(s) of $f(x) = 1 - \tan x$ on $(0, \pi/2)$. (See Exercise 62 of Section 4.2.)

66. Find a positive number that is greater than its own cube by the greatest possible amount.

67. Generalize Exercise 66 to the n^{th} power of a number ($n \ge 2$).

68. Find a number a so that for given $a_1, a_2, a_3 \in \mathbb{R}$, the quantity $S_3 = (a-a_1)^2 + (a-a_2)^2 + (a-a_3)^2$ is minimal.

69. Generalize Exercise 68 for n given numbers to minimize the following quantity: $S_n = (a-a_1)^2 + (a-a_2)^2 + \cdots + (a-a_n)^2$.

70. A wire of length l is bent into an L shape. Where should it be bent in order to minimize the distance between the two endpoints?

71. Find the length l and width w of the rectangle inscribed in the unit circle for which $l^2 w$ is maximal.

72. Find the dimensions of the rectangle whose diagonal is d units and whose area is maximum.

73. A book page of area 500 cm^2 is required to have 1 cm margins on the sides, while the margins on the top and bottom are to be 2 cm. Find the dimensions of the page that maximize the printed area.

74. Among all isosceles triangles whose legs are l units long, find the base angle that maximizes the area.

75. A vertex of a rectangle is at the origin and the opposite vertex sits in the first quadrant and on the graph of $y = \dfrac{2-x}{x+1}$. Find the maximum possible area for such a rectangle.

76. Find the point on the graph of $y = 1 - x^2$ that is closest to the point $(-3,1)$.

77. Among all isosceles triangles that can be inscribed in the circle of radius R, find the one with maximum area.

©HAWKES LEARNING

78. A vending machine sells 500 bars of a certain type of candy when the price is \$1.50. It was discovered that 10 fewer customers will buy the candy bar for each 5¢ increase in price. What is the price that will bring maximum revenue from the sales of this type of candy bar?

79. Maximize the surface area of the can in Example 3 of Section 4.6. Explain your findings.

80. Minimize the cost of producing the can in Example 3 of Section 4.6 if the top and bottom are produced using a material that is 50% more expensive than the material used for the side.

81. Nate needs to reach a restaurant that is 600 ft upstream on the other side of a 150 ft wide river. Find the point where he has to reach the other side in order to make the best time if he can swim at $5\ \text{ft/s}$ and walk at $9\ \text{ft/s}$. (Ignore the flow of the river.)

82–89 Find the general antiderivative of the given function, and check your answer by differentiation. (If necessary, rewrite the function before antidifferentiation.)

82. $f(x) = 2x^3 - 6x^2 + 3x$

83. $f(x) = 5x^4 - 4.8x^3 + e^2$

84. $f(x) = x(x+2)(2x-3)$

85. $f(x) = 0.4x\sqrt{x} - \dfrac{2}{\sqrt{x}}$

86. $f(x) = \dfrac{x^4 - 4x}{x^2}$

87. $f(x) = 2(x + \sec^2 2x)$

88. $f(x) = 6e^{3x}$

89. $f(x) = \dfrac{3}{4x^2 + 1}$

90–91 Find $f(x)$ that satisfies the specified conditions.

90. $f''(x) = x, \quad f'(1) = 1, \quad f(1) = 0$

91. $f'''(x) = 2, \quad f''(2) = -1, \quad f'(2) = 2, \quad f(2) = 3$

92. A tennis ball is thrown upward from an initial height of 4 feet with an initial velocity of 56 feet per second. How high will it go and for how long is it rising? (Ignore air resistance.)

93. With what initial velocity do we need to throw a golf ball vertically upward in order for it to rise 100 feet high? (Ignore the initial height and air resistance.)

94. A pebble is shot horizontally using a slingshot at 10 meters per second from the top of a building that is 20 meters high. If the terrain around the building is nearly flat, approximately how far from the building will the pebble hit the ground? (Use the approximation $g \approx 10\ \text{m/s}^2$ and ignore air resistance.)

Concept Check

95–101 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

95. A continuous function on a finite interval always attains its maximum and minimum.

96. If $f(x)$ has a relative maximum or minimum at $x = c$, then $f'(c) = 0$.

97. If $f(x)$ has a relative maximum or minimum at $x = c$, then c is a critical point of f.

98. A cubic polynomial has exactly one inflection point.

99. If $f(x)$ is a polynomial, then between two consecutive local extrema there must be an $x = c$ so that $f''(c) = 0$.

100. If $f(x)$ is a polynomial and c is a critical point, then there is a relative maximum or minimum at $x = c$.

101. If $f'''(c) = 0$, then $f'(x)$ has a point of inflection at $x = c$.

Chapter 4
Technology Exercises

102–111. Use a graphing utility to verify the answers you obtained for Exercises 51–60.

112–113. Use a graphing utility to verify the conclusions of Exercises 15 and 16.

©HAWKES LEARNING

Chapter 5
Review Exercises

1–2 Use $(O_4 + U_4)/2$ to estimate the area under the graph of the function and above the x–axis on the given interval.

1. $f(x) = \dfrac{x^2}{2}$ on $[0,2]$

2. $f(x) = \sin x$ on $\left[0, \dfrac{\pi}{2}\right]$

3–6 Write the given sum using sigma notation.

3. $\dfrac{1}{2} - \dfrac{1}{9} + \dfrac{1}{28} - \dfrac{1}{65} + \cdots - \dfrac{1}{1,000,001}$

4. $a_1 + a_5 + a_9 + a_{13} + \cdots + a_{97}$

5. $f\left(\dfrac{2}{n^2}\right) + f\left(\dfrac{4}{n^2}\right) + f\left(\dfrac{6}{n^2}\right) + \cdots + f\left(\dfrac{100}{n^2}\right)$

6. $g\left(t_{-2}^*\right)\Delta t + g\left(t_{-1}^*\right)\Delta t + g\left(t_0^*\right)\Delta t + \cdots + g\left(t_{2n}^*\right)\Delta t$

7–8 Assuming that $\sum_{i=0}^{n} a_i = 50$ and $\sum_{i=0}^{n} b_i = 80$, find the sum.

7. $\sum_{i=0}^{n} (a_i + 2b_i + 2)$

8. $\sum_{i=0}^{n} \left(\dfrac{a_i}{5} - \dfrac{b_i}{4}\right)$

9–10 Use summation formulas to find the value of the sum.

9. $\sum_{i=1}^{10} (3i^3 - 1)$

10. $\sum_{j=1}^{n} \dfrac{(2j+1)(j-2)}{2}$

11. Find the value of the sum $\sum_{i=1}^{n} \left[\dfrac{1}{i^2} - \dfrac{1}{(i+1)^2}\right]$.
(**Hint:** Write out the first few terms as well as the last few terms.)

12–13 Evaluate the geometric sum using the formula proven in Exercise 53 of Section 5.1.

12. $\sum_{i=0}^{10} \dfrac{3}{2^i}$

13. $\sum_{j=0}^{6} (-1)^j (0.3)^j$

14–15 Evaluate the given double sum.

14. $\sum_{i=3}^{11} \sum_{j=1}^{4} (2i - j)$

15. $\sum_{i=1}^{n} \sum_{j=1}^{m} i^2 j$

16–18 Find the area under the graph of $f(x)$ and above the x–axis on the given interval, by taking the limit of the associated Riemann sums.

16. $f(x) = x^2 + 1$ on $[0,2]$

17. $f(x) = x^3$ on $[0,1]$

18. $f(x) = \sqrt{x}$ on $[0,4]$
(**Hint:** Choose $x_i^* = \dfrac{4(i-1)^2}{n^2}$.)

19. Identify the region whose area is the limit given by $\lim_{n\to\infty} \dfrac{2}{n} \sum_{i=1}^{n} \left[\dfrac{4i}{n} - \left(\dfrac{2i}{n}\right)^2\right]$. Then use summation formulas to evaluate the limit.

20–21 Prove that the given function is not integrable on the interval $[0,1]$.

20. $f(x) = \dfrac{1}{x^2}$

21. $g(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$

22–25 Suppose that f is an even function, g is odd, both are integrable on $[-a,a]$, and we know that $\int_0^a f(x)\,dx = 2$, while $\int_0^a g(x)\,dx = 0.5$ $(a > 0)$. If possible, find the integral.

22. $\int_{-a}^{a} [5f(x) + 4g(x)]\,dx$

23. $\int_{-a}^{a} [f(x)]^2 g(x)\,dx$

24. $\int_{-a}^{a} f(x)[g(x)]^2\,dx$

25. $\int_{-a}^{0} [f(x) + g(x)]\,dx$

26–27 Find the average value of $f(x)$ over the given interval and identify all points in the domain where $f(x)$ assumes its average value.

26. $f(x) = 4x - x^2$ on $[0,4]$

27. $f(x) = |x-2| - 1$ on $[1,5]$

©HAWKES LEARNING

28. Use the Fundamental Theorem of Calculus to evaluate the limit

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\frac{i}{n}}-\frac{i}{n}\right)$$

by recognizing it as a Riemann sum of a function over an interval.

29–30 Use Part I of the Fundamental Theorem of Calculus to find the derivative of the given function.

29. $F(x)=\int_0^x\sqrt{1+t^2}\,dt$ **30.** $G(x)=\int_0^{x^2}e^{t^2}\,dt$

31–38 Use Part II of the Fundamental Theorem of Calculus to evaluate the definite integral.

31. $\int_1^2\left(2x^4+3x^2-2\right)dx$ **32.** $\int_0^2(3x+2)(5-x)\,dx$

33. $\int_1^4\left(\frac{1}{t}-\frac{2}{t^2}+1\right)dt$ **34.** $\int_1^9\frac{x^2-2\sqrt{x}+2}{x}\,dx$

35. $\int_0^1\frac{2}{\sqrt{1-x^2}}\,dx$

36. $\int_{\pi/4}^{3\pi/4}\left(2\csc^2 x-\cos x\right)dx$

37. $\int_2^3\frac{x+2}{x-1}\,dx$ **38.** $\int_0^1\frac{2x^2-1}{x^2+1}\,dx$

39–40 Find the area of the region between the graph of the given function and the x-axis on the indicated interval.

39. $y=\frac{1}{2x^2}$ on $[1,10]$

40. $y=2\sqrt{x}-x^2$ on $[0,2]$

41. Find a formula for $f(x)$ if $\int_0^{x^3}f(t)\,dt=\sin\left(x^3\right)$.

42. The velocity function of a particle moving along the x–axis is $v(t)=3t-t^2$ units per second. If it started at the origin, find **a.** the position of the particle at $t=5$ seconds and **b.** the total distance traveled by the particle in the time interval $[0,5]$.

43–54 Use an appropriate substitution (when necessary) to evaluate the indefinite integral.

43. $\int\frac{-2}{\sqrt{1-x^2}}\,dx$ **44.** $\int\frac{-2x}{\sqrt{1-x^2}}\,dx$

45. $\int\sec x(\sec x+\tan x)\,dx$

46. $\int\sec^2 x\tan x\,dx$

47. $\int 6x^2\left(2x^3-7\right)^9\,dx$ **48.** $\int x^4\sqrt{x^5-3}\,dx$

49. $\int\frac{4x}{\left(x^2+1\right)^2}\,dx$ **50.** $\int\frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$

51. $\int\frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$ **52.** $\int\frac{1}{x\ln\left(x^2\right)}\,dx$

53. $\int\frac{e^{2x}}{e^x+1}\,dx$ **54.** $\int\frac{1}{x^2}\sin\left(\frac{x+1}{x}\right)dx$

55–56 Find $y(x)$ that satisfies the given conditions.

55. $\frac{dy}{dx}=\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)^2};\quad y(9)=1$

56. $y''(x)=1-\sin x;\quad y'(0)=2;\quad y(0)=0$

57. A particle is moving along the x-axis in the positive direction with a velocity function of $v(t)=\frac{t}{t^2+1}$ units per second. If it started at the point $(1,0)$, what is the particle's position at $t=4$ seconds?

58–63 Evaluate the definite integral.

58. $\int_0^1 3x^5\left(x^6-1\right)^{12}\,dx$

59. $\int_0^4(x+1)\sqrt{x^2+2x}\,dx$

60. $\int_1^2\frac{x^3}{x^4+1}\,dx$ **61.** $\int_{-1/2}^0\frac{2^t}{\sqrt{1-4^t}}\,dt$

62. $\int_e^{e^2}\frac{1}{x(\ln x)^2}\,dx$ **63.** $\int_1^4\frac{dx}{\sqrt{x}\left(\sqrt{x}+1\right)^2}$

64–69 Find the area of the region bounded by the graphs of the given equations. (If convenient or necessary, integrate with respect to y rather than x.)

64. $y=x^3-4x,\quad 3y=15x,\quad x\ge 0$

65. $y=1-x^2,\quad y=1-x^6,\quad x\ge 0$

66. $y=2\sqrt[4]{x},\quad y=4-2x,\quad y=0$

67. $y=\ln x,\quad (e-1)y=x-1$

68. $y=\frac{1}{1+x^2},\quad 2y=1$

69. $y=\sin x,\quad y=\sin x\cos x,\quad 0\le x\le\pi$

70. Consider the region bounded by the graph of $y=1/x$ and the x-axis over the interval $[1,a]$ $(a>1)$. Find the vertical line $x=c$ that bisects the region in two subregions of equal area.

©HAWKES LEARNING

71. Consider the function $f(x) = 1/x^2$ defined on some interval $[a,b]$. Partition $[a,b]$ and in each subinterval $[x_{i-1}, x_i]$ choose the sample point $x_i^* = \sqrt{x_{i-1}x_i}$ (the geometric mean of the endpoints). Show that

$$\frac{1}{(x_i^*)^2}\Delta x_i = \frac{1}{x_{i-1}} - \frac{1}{x_i}$$

and use this observation to prove the following formula.

$$\int_a^b \frac{1}{x^2}\,dx = \frac{1}{a} - \frac{1}{b}$$

72. Prove that if the conditions of Part I of the Fundamental Theorem of Calculus are satisfied and $F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$, where $g(x)$ and $h(x)$ are differentiable, then $F'(x) = f(h(x))h'(x) - f(g(x))g'(x)$. (**Hint:** See Example 3 of Section 5.3.)

73. Prove that if f is a linear function, then its definite integral on an interval $[a,b]$ is the average of its left and right Riemann sums, that is,

$$\int_a^b f(x)\,dx = \frac{L_n + R_n}{2}.$$

What is your expectation regarding the integral and the average above if f is concave up? Concave down?

Concept Check

74–81 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

74. If $n_1 < n_2$, then the Riemann sum R_{n_2} is always a better approximation of the integral than R_{n_1}.

75. If f is piecewise continuous on a closed interval, then the limit of its Riemann sums always exists.

76. When applying the Fundamental Theorem of Calculus, we must choose the antiderivative with $C = 0$.

77. $\int \frac{1}{e^x}\,dx = \ln(e^x) + C = x + C$

78. The definite integral of the velocity function of a moving object on $[t_1, t_2]$ is equal to the total distance traveled by the object from time $t = t_1$ to $t = t_2$.

79. $\int_a^b f(x)\,dx > 0$ if and only if $f(x) > 0$ on $[a,b]$.

80. $\int \sec x\,dx = \sec x \tan x + C$

81. $\int_{-1}^{1} \frac{1}{x^3}\,dx = \frac{-1}{2x^2}\Big]_{-1}^{1} = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

Chapter 5
Technology Exercises

82. Use the summation feature of a graphing utility to verify your answers for Exercises 9–15.

83. Write a program for a graphing calculator or computer algebra system that calculates the n^{th} Riemann sum for a given function on a given interval, using subintervals of equal width and sample points of your choice. Use your program to verify your answers for Exercises 16–18.

84. Use a graphing utility to evaluate the limit of Exercise 28. What do you find? (Answer will vary depending on the capabilities of the particular software used.)

©HAWKES LEARNING

Chapter 6
Review Exercises

1–2 The base of a solid S is described in the xy-plane along with its cross-sections in a certain direction. Find the volume of S.

1. The base of S is the first-quadrant region of the unit disk centered at the origin; the cross-sections perpendicular to the y-axis are squares.

2. The base of S is the region bounded by the graph of $y = \sqrt{2 - x}$ and the coordinate axes; the cross-sections perpendicular to the x-axis are equilateral triangles.

3–6 Use the disk/washer method to find the volume of the solid generated by rotating the region R about the indicated axis.

3. R is bounded by the graph of $y = \sqrt{1 - x}$ and the coordinate axes, rotated about the x-axis.

4. R is the region of Exercise 3, rotated about the y-axis.

5. R is bounded by the graphs of $y = \operatorname{arcsec} x$, $x = \sqrt{2}$, and $y = 0$, rotated about the y-axis.

6. R is the first-quadrant portion of the region bounded by the graphs of $y = 4/x^2$ and $y = 5 - x^2$, rotated about $x = -2$.

7–10 Use the shell method to find the volume of the solid generated by rotating the region R about the indicated axis.

7. R is bounded by the graph of $y = 2x - x^2$ and the x-axis, rotated about the y-axis.

8. R is bounded by the graphs of $y = x$, $y = 3 - 2x$, and the y-axis, rotated about $x = -1$.

9. R is the first-quadrant region bounded by the graphs of $y = 4x$ and $y = x^3$, rotated about the x-axis.

10. R is bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$, rotated about $y = -2$.

11–14 The given integral represents the volume of a solid of revolution. Describe the solid. (Do not evaluate the integral.)

11. $\displaystyle\int_0^3 \pi x^2 \, dx$

12. $\displaystyle\pi \int_0^2 \left[1 - (y-1)^2\right] dy$

13. $\displaystyle\int_0^2 2\pi x\left(4 - x^2\right) dx$

14. $\displaystyle 2\pi \int_1^3 y\sqrt{1 - (y-2)^2} \, dy$

15–18 Use the shell method or the disk/washer method to find the volume of the solid obtained by revolving the region bounded by the graphs of the equations about the given axis. Choose the method that seems to work best.

15. $x = \sqrt{\sin y}$, $x = 0$, $0 \le y \le \pi$; about the y-axis

16. $y = x^3 - 2x^2 + x$, $y = 0$, $x = 0$, $x = 1$; about the x-axis

17. $y = \sqrt{x}$, $y = 2 - x$, $x = 0$; about the y-axis

18. $y = \dfrac{e^x}{x}$, $y = 0$, $x = 1$, $x = 2$; about the y-axis

19. Suppose the region bounded by the graph of $y = \sqrt{c^2 - x}$ $(c \neq 0)$ and the coordinate axes is revolved about the y-axis. Find the resulting solid's volume, and then consider the volume of the inscribed circular cone that results from rotating the line segment $y = c - \dfrac{x}{c}$, $0 \le x \le c^2$, about the y-axis. Show that the ratio of these two volumes is $\frac{8}{5}$. (This result is due to Pierre de Fermat, the great 17th-century mathematician.)

20–23 Set up, but do not evaluate, an integral defining the arc length of the graph of the equation over the given interval.

20. $y = x^2 - x + 1$; $0 \le x \le 3$

21. $x + 2y^2 = 3 - y$; $1 \le y \le 5$

22. $y = \tan x$; $-\pi/4 \le x \le \pi/4$

23. $e^x = \cos y$; $0 \le y \le \pi/3$

24–29 Determine the arc length L of the curve defined by the equation over the given interval.

24. $y = \sqrt{3}x + 1$; $0 \le x \le 4$

25. $y = \dfrac{4x^{3/2} + 1}{6}$; $0 \le x \le 3$

26. $y = \dfrac{x^4 + 3}{6x}$; $1 \le x \le 3$

27. $y = \dfrac{x^2}{4} - \ln\sqrt{x}$; $1 \le x \le e^2$

28. $y = \dfrac{\sqrt{x}}{6} - 2x^{3/2}$; $0 \le x \le 1$

©HAWKES LEARNING

29. $y = e^{x/2} + e^{-x/2}; \quad 0 \le x \le 2$

30–33 Set up, but do not evaluate, an integral defining the surface area of the solid obtained by revolving the given curve about the indicated axis.

30. $y = \sin x; \quad 0 \le x \le \pi; \quad$ about the x-axis

31. $y = \sqrt{\ln x}; \quad 1 \le x \le e; \quad$ about the x-axis

32. $y = \sqrt{\ln x}; \quad 1 \le x \le e; \quad$ about the y-axis

33. $y = x^2; \quad 0 \le x \le 2; \quad$ about $x = -1$

34–39 Find the surface area of the solid generated by revolving the given curve about the indicated axis.

34. $y = \frac{1}{3}x + 2; \quad 0 \le x \le 3; \quad$ about the x-axis

35. $y = \frac{x^3}{4}; \quad 0 \le x \le 1; \quad$ about the x-axis

36. $3y = 3x^{3/2} - \sqrt{x}; \quad 1 \le x \le 2; \quad$ about the x-axis

37. $x = \frac{y^3}{12} + \frac{1}{y}; \quad 1 \le y \le 2; \quad$ about the y-axis

38. $x = 1.5y^{5/3} - 0.3\sqrt[3]{y}; \quad 0 \le y \le 1; \quad$ about the y-axis

39. $12xy = 3y^4 + 4; \quad 1 \le y \le 2; \quad$ about $x = -\frac{1}{2}$

40–43 Find the centroid of the plane region bounded by the given curves. If possible, use symmetry to simplify your calculations.

40. $y = 5x - x^2, \quad y = 0$

41. $y = 2\sqrt{x}, \quad y = 2x$

42. $y = x^{3/5}, \quad x = 0, \quad y = 8$

43. $y = x, \quad y(x+1)^2 = 4, \quad x = 0$

44–47 Find the center of mass of the plane region of varying density that is bounded by the given curves.

44. $2y = 4 - x, \quad x = 0, \quad y = 0; \quad \rho(x, y) = x^2$

45. $2y = 4 - x, \quad x = 0, \quad y = 0; \quad \rho(x, y) = \sqrt{y}$

46. $y = x, \quad y = \sqrt{x}; \quad \rho(x, y) = 1 + x$

47. $x = 9 - y^2, \quad y = x - 3; \quad \rho(x, y) = y + 3$

48. Suppose the density of a baseball bat lying along the positive x-axis is given by the function $\rho(x) = (0.033x + 0.5)^2$ ounces per inch $(0 \le x \le 30)$. Find the center of mass of the bat.

49. Find the centroid of the upper semielliptical region bounded by the x-axis and the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (See Exercise 58 of Section 6.4.)

50. Use Pappus' Theorem for volumes to find the volume of the solid generated by revolving the region of Exercise 43 about the y-axis.

51.* Use Pappus' Theorem for volumes to find a second solution to Exercise 49 using the fact that the area of the semiellipse is $A = \pi ab/2$.

52. A 10 cm long unstressed spring requires a force of 2 N to stretch to 14 cm. How much work will be done in stretching the spring an additional 4 cm?

53. A 100 ft cable is lifting a 30 lb weight by 20 ft, so that the cable is wound on a cylindrical drum. Find the total work done if the cable weighs 2 lb/ft.

54. A 20 m cable is being wound up on a cylindrical drum at a rate of 25 cm/s. If the cable weighs 10 N/m, find the work done from $t = 4$ s to $t = 32$ s.

55. A leaky container is being lifted by a crane at 0.5 m/s for 20 s. If the full container originally weighed 300 kg, but is leaking at a rate of 0.1 kg/s, find the work done by the crane. Use $g \approx 9.81 \text{ m/s}^2$.

56. The shape of an underground gasoline tank is an inverted circular cone with a top radius of 6 ft and a depth of 18 ft. Find the work done in pumping the full content of the tank to 1 ft above the top of the tank. The weight density of gasoline is 44 lb/ft^3.

57. Find the total work done in Exercise 56 if the tank initially is filled to only half of its capacity.

58. The shape of a water tank can be approximated by rotating the graph of $y = x^2$, $-2 \le x \le 2$, about the y-axis (units in meters). Find the work required to fill up the empty tank through an opening at its lowest point. Use 9810 N/m^3 for the weight density of water.

59.* A cup of soda of height 5 inches can be approximated by a frustum of a circular cone, standing on its smaller base of radius 1.25 inches, with a top opening of radius 2 inches. Find the work done in drinking a full cup of soda through a straw, supposing that the end of the straw is 2 inches above the rim of the cup. Use the weight density of water, 62.4 lb/ft^3, for the soda.

©HAWKES LEARNING

60. A cylindrical underground tank of radius 3 ft is positioned horizontally, and is half full of gasoline. Find the force exerted by the gasoline on one end of the tank. (See Exercise 56 for the weight density of gasoline.)

61. Use the centroid approach (Example 8 in Section 6.5) to provide a second solution to Exercise 60. (**Hint:** See Exercise 52 of Section 6.4.)

62. Use the centroid approach to provide a second solution to Exercise 56. (**Hint:** See the discussion preceding Exercises 58–64 of Section 6.5 and use the fact that the centroid of a right circular cone is on the axis of the cone, one-fourth of the way from the base toward the vertex.)

63. One end of a dam is an isosceles trapezoid, standing vertically on its shorter base of 15 meters. The top width is 30 meters, while the depth of the dam is 20 meters. Find the force exerted by the water on one end of the dam. Use $9810\ \text{N/m}^3$ for the weight density of water.

64.* The water depth in an 18 ft by 35 ft pool increases uniformly from the shallow end of 3 ft to the deep end of 8 ft. Find the fluid force exerted on the bottom of the pool. Use $62.4\ \text{lb/ft}^3$ for the weight density of water.

65. Find the energy expended in moving a 1.5-ton satellite to an altitude of 220 miles above the surface of Earth. Assume the radius of Earth is approximately 4000 miles. Express your answer in the units foot-pounds. (**Hint:** Ignore the work done in accelerating the satellite, air resistance, as well as the weight of the launching vehicle and fuel.)

66–67 Find the indicated function value. If it doesn't exist, say so.

66. **a.** $\sinh 0$ **b.** $\operatorname{csch} 0$

67. **a.** $\coth^{-1} 0$ **b.** $\operatorname{sech} 0$

68. Use the definition to show that $\tanh x > 0$ if $x > 0$ and $\tanh x < 0$ if $x < 0$.

69. Prove that if $f(x) = \tanh x$ or $f(x) = \coth x$, then $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = -1$.

70. Suppose that $e^x = f(x) + g(x)$, where $f(x)$ is even and $g(x)$ is odd. Prove that in this case, we must necessarily have $f(x) = \cosh x$ and $g(x) = \sinh x$. (**Hint:** Start by expressing e^{-x} in terms of $f(x)$ and $g(x)$.)

71. Verify the following identity.

$$\cosh x + \cosh y = 2\cosh\frac{x+y}{2}\cosh\frac{x-y}{2}$$

72–81 Find the derivative or integral as indicated.

72. $\dfrac{d}{dx}\left[\sinh^2\left(\dfrac{6x-1}{2}\right)\right]$ **73.** $\dfrac{d}{dx}\left(e^{2x}\tanh x\right)$

74. $\dfrac{d}{dx}\left[\operatorname{csch}\left(x^2+2\right)\right]$

75. $\dfrac{d}{dx}\left[\ln\left(\sinh\left(5x-2\right)\right)\right]$

76. $\displaystyle\int \frac{\tanh\sqrt{x}}{2\sqrt{x}}\,dx$ **77.** $\displaystyle\int \frac{\sinh x}{\cosh^2 x}\,dx$

78. $\displaystyle\int \coth x \operatorname{csch}^2 x\,dx$ **79.** $\displaystyle\int \frac{dx}{\sqrt{x^2-4x+5}}$

80. $\displaystyle\int_0^1 \frac{2x}{\sqrt{1+9x^4}}\,dx$ **81.** $\displaystyle\int \frac{e^x}{\sqrt{e^{2x}-1}}\,dx;\quad x>0$

82. Prove the following formula for the derivative of the inverse hyperbolic cosecant function.

$$\frac{d}{dx}\left(\operatorname{csch}^{-1} x\right) = \frac{-1}{|x|\sqrt{1+x^2}}$$

83–85 In the 16th century, Galileo observed the additive property of velocities. For example, if a boat travels at 5 mph in the direction of flow on a river that in turn flows at 2 mph, then the velocity of the boat relative to the shore will be $u = 5 + 2 = 7$ mph. However, according to Albert Einstein's theory of special relativity, the relative velocity between any two objects can never exceed c, the speed of light. This contradicts Galileo's observation. (On a theoretical level, just think of a "river" flowing at $\frac{3}{4}c$, with a "boat" on it traveling at, say, $\frac{1}{2}c$. The velocity relative to shore would be $u = \left(\frac{3}{4}+\frac{1}{2}\right)c = \frac{5}{4}c$!) Instead of the Galilean $u = v_1 + v_2$, Einstein discovered the following relativistic addition formula for velocities.

$$u = \frac{v_1 + v_2}{1 + \dfrac{v_1 v_2}{c^2}}$$

In Exercises 83–85, refer to the above formula.

83. Use the addition formula for $\tanh x$ (Exercise 33 of Section 6.6) to verify the following identity.

$$\tanh^{-1}\left(\frac{u}{c}\right) = \tanh^{-1}\left(\frac{v_1}{c}\right) + \tanh^{-1}\left(\frac{v_2}{c}\right)$$

©HAWKES LEARNING

84. Suppose a fighter plane fires a missile at 500 mph in the forward direction at a moment when the plane itself is flying at 900 mph. Use Einstein's relativistic formula to find the missile's velocity relative to Earth and compare it with Galileo's prediction of $500 + 900 = 1400$ mph. Approximate the speed of light by 3×10^8 m/s.

85. In this exercise, we are going to up the numbers of Exercise 84 significantly. Suppose a rocket is traveling away from Earth at a speed of $0.7c$ and fires another rocket at $0.5c$. Use Einstein's formula to calculate the velocity of the second rocket relative to Earth.

Concept Check

86–92 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

86. The disk method is based on the idea of integrating slices.

87. When both the disk method and the shell method are applied to calculate the volume of a solid of revolution, the variable of integration is always the same.

88. If the area of the region bounded by $y = f(x)$ and $y = g(x)$ is A, then the volume of the solid obtained by revolving the same region about the x-axis is $V = \pi A^2$.

89. If the area of the region bounded by $y = f(x)$ and $y = g(x)$ is A, then the volume of the solid obtained by revolving the same region about the x-axis never equals πA^2.

90. $\lim\limits_{x\to-\infty} \tanh x = -1$

91. $\cosh 2x = 2\sinh^2 x + 1$

92. The work needed to pump fluid out of a tank through an opening on its top equals the total weight of the fluid multiplied by the distance traveled by its center of mass.

Chapter 6
Technology Exercises

93–96 Use a graphing utility to find (or approximate) the volume of the solid generated by rotating the region bounded by the graphs of the given equations about the indicated axis.

93. $y = \sin(x^2)$, $y = 0$, $x = 0$, $x = \sqrt{\pi}$; about the x-axis

94. $y = \arccos x$, $y = 0$, $x = 0$, $x = 1$; about the x-axis

95. $y = \sinh^{-1} x$, $y = 0$, $x = 4$; about the y-axis

96. $y = x^2 \sin^2 x$, $y = 0$, $x = 0$, $x = \pi$; about the y-axis

97–98 Use a graphing utility to find the arc length of the graph of the equation over the given interval.

97. $y = \dfrac{1}{x^2+1}$; $-1 \le x \le 1$

98. $y = \sin x$; $0 \le x \le \pi$

99–100 Use a graphing utility to find the surface area of the solid generated by revolving the given curve about the indicated axis.

99. $y = \sin x$; $0 \le x \le \pi$; about the x-axis

100. $y = \sqrt{\ln x}$; $1 \le x \le e$; about the y-axis

©HAWKES LEARNING

Chapter 7
Review Exercises

1–8 Use integration by parts to evaluate the given indefinite or definite integral.

1. $\int \sqrt{x}\ln x\,dx$

2. $\int x\sec^2 x\,dx$

3. $\int \frac{x}{\sqrt{x-1}}\,dx$

4. $\int \operatorname{arccot} x\,dx$

5. $\int t^2\cos t\,dt$

6. $\int 3t^2 e^{t+1}\,dt$

7. $\int_0^{\pi} x^3\sin x\,dx$

8. $\int_0^{\pi/2} e^{2t}\cos t\,dt$

9–10 Combine the method of integration by parts with substitution to evaluate the integral.

9. $\int \sin t\cos t\ln(\sin t)\,dt$

10. $\int \frac{\arctan\sqrt{t}}{\sqrt{t}}\,dt$

11. Use integration by parts to find the area of the region bounded by the graph of $y=\cos(\ln x)$ and the x-axis over the interval $\left[e^{-\pi/2}, e^{\pi/2}\right]$.

12. Consider the region bounded by the graph of $y=xe^x$ and the x-axis over the interval $[0,1]$.

a. Find the centroid of the region.

b. Use the shell method to find the volume of the solid generated by revolving the region about the y-axis.

13. Repeat Exercise 12 for the region bounded by the graph of $y=3x^2\ln x$ and the x-axis over the interval $[1,e]$.

14. Use the disk method to find the volume of the solid generated by revolving the region bounded by the graph of $y=\cos x,\ -\pi/2\le x\le\pi/2,$ about the x-axis.

15. Use integration by parts to prove the following formula for $a>0$, $a\neq 1$.

$$\int xa^x\,dx = a^x\left[\frac{x}{\ln a}-\frac{1}{(\ln a)^2}\right]+C$$

16–21 Use the partial fractions method to evaluate the given integral.

16. $\int \frac{4}{x^2-4}\,dx$

17. $\int \frac{x-7}{(x+1)(x-3)}\,dx$

18. $\int \frac{x+3}{x(x+1)(x+2)}\,dx$

19. $\int \frac{3x-4}{x^3(x-1)}\,dx$

20. $\int \frac{4}{2x^3+2x^2+x}\,dx$

21. $\int \frac{x^5+6x^3+7x}{(x^2+2)^3}\,dx$

22–23 Use the Heaviside cover-up method to evaluate the given integral.

22. $\int \frac{4x^2+4x-2}{x(x^2+3x+2)}\,dx$

23. $\int \frac{3x^2+16x+29}{(x+5)(x+3)(x-1)}\,dx$

24–25 Find the definite integral of the rational function over the given interval.

24. $\int_{-1}^{1} \frac{x^3+x^2-x-2}{x^2-4}\,dx$

25. $\int_0^1 \frac{2x^4+x^3-2x^2-3}{(x+1)(x^2+1)}\,dx$

26–27 Combine integration by substitution and the partial fractions method to evaluate the given integral.

26. $\int \frac{\sec^2 x}{\tan^2 x-\tan x}\,dx$

27. $\int \frac{dx}{\sqrt{x}(\sqrt{x}-2)(\sqrt{x}+3)}$

28. Use the disk method to find the volume of the solid generated by revolving the graph of $f(x)=2/\sqrt{x^2+8x+12}$, $0\le x\le 2$, about the x-axis.

29. Use the shell method to find the volume of the solid obtained by revolving about the y-axis the region bounded by the graph of $g(x)=4/(x^2+6x+5)$, the coordinate axes, and the line $x=5$.

30. If a and b are constants such that $a\neq b$, use the partial fractions method to prove the following formula.

$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b}\ln\left|\frac{x-a}{x-b}\right|+C$$

31–40 Evaluate the given indefinite or definite integral involving powers of sines and cosines.

31. $\int_0^{\pi} 4\sin^3 x\,dx$

32. $\int_0^{\pi/4} \frac{\sin x}{\sqrt{\cos x}}\,dx$

33. $\int \cos^2 x\sin^3 x\,dx$

34. $\int x^2\cos^2 x\,dx$

35. $\int_0^{\pi/4} \sin x\cos^{-5} x\,dx$

36. $\int_0^{\pi} \sin^9 x\,dx$

©HAWKES LEARNING

37. $\int \frac{1}{\sin^2 x \cos^2 x}\,dx$

38. $\int (\cos x - \sin x)^2\,dx$

39. $\int \sin 3x \sin x\,dx$

40. $\int \cos 2x \sin 5x\,dx$

41–48 Evaluate the given indefinite or definite integral involving powers of tangents and secants (or their cofunctions).

41. $\int_0^{\pi/4} \tan^3 x \sec^4 x\,dx$

42. $\int_0^{\pi/4} \tan^2 t \sec^3 t\,dt$

43. $\int \cot^3 x\,dx$

44. $\int \csc^4 x\,dx$

45. $\int \tan^4 x \sec^2 x\,dx$

46. $\int \frac{\csc^4 x}{\sqrt{\cot x}}\,dx$

47. $\int \frac{\tan^2 v}{\sec^3 v}\,dv$

48. $\int \frac{\sec^2 \theta}{\sqrt{\tan \theta}}\,d\theta$

49. Find the area of the region between the x-axis and the graph of $f(x) = 1/(\cos x + 1)$ from $x = -\pi/2$ to $x = \pi/2$.

50. Find the area of the region bounded by the graphs of $y = 16 \sin^2 x$ and $y = \csc^2 x$. (Restrict both functions to the interval $(0, \pi)$).

51. Find the volume of the solid obtained by revolving about the x-axis the region bounded by the graph of $y = \csc x + \sin x$ and the x-axis over the interval $(\pi/4, 3\pi/4)$.

52. Find the volume of the solid obtained by revolving about the x-axis the region bounded by the graph of $y = (\tan x + 1)/\sec x$ and the x-axis over the interval $(0, \pi/4)$.

53. For positive integers m and n that are not equal, prove the following identities.

a. $\int_0^{2\pi} \sin(mx)\sin(nx)\,dx = 0$

b. $\int_0^{2\pi} \sin(mx)\cos(nx)\,dx = 0$

c. $\int_0^{2\pi} \cos(mx)\cos(nx)\,dx = 0$

54. Use substitution to establish the following integration formula.

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

(**Hint:** Substitute $u = \sec x + \tan x$. Compare with Example 5d of Section 7.3.)

55–60 Use trigonometric substitution to evaluate the given indefinite or definite integral.

55. $\int \frac{2x^2}{\sqrt{4 - x^2}}\,dx$

56. $\int \frac{\sqrt{1 - x^2}}{x^2}\,dx$

57. $\int \frac{2}{(t^2 + 4)^{3/2}}\,dt$

58. $\int \frac{\sqrt{t^2 - 4}}{t^2}\,dt$

59. $\int_0^3 \frac{2x^2}{\sqrt{x^2 + 9}}\,dx$

60. $\int_1^2 \frac{dx}{\sqrt{16x^2 - 9}}$

61–64 Use the methods of this chapter to evaluate the given integral.

61. $\int \sqrt{20 + 16x - 4x^2}\,dx$

62. $\int \frac{\cos x}{4 \sin^2 x + 9}\,dx$

63. $\int \frac{e^x}{\sqrt{3 + 2e^x - e^{2x}}}\,dx$

64. $\int 4x \sinh^{-1} x\,dx$

65. Find the area of the region enclosed by the graphs of $y = \sqrt{x^2 + 1}$ and $y = 2$.

66. Rotate the region bounded by the graph of $y = \sqrt{x^2 - 4}/x^2$ and the x-axis, $2 \le x \le 4$, about the y-axis. Use the shell method to find the volume of the resulting solid.

67. Use the method of disks to determine the volume of the solid obtained by revolving the graph of $y = x^2/\sqrt[4]{4 - x^2}$, $0 \le x \le 1$, about the x-axis.

68. Find the surface area of the solid that results from rotating the graph of $y = x^2$ about the x-axis over the interval $[0, 2]$. (Round your answer to four decimal places.)

69. Find the surface area of the solid generated by rotating the graph of $y = \cos x$, $-\pi/2 \le x \le \pi/2$, about the x-axis.

70.* Find the length of the graph of $y = e^{-x}$, $0 \le x \le 2$. (Round your answer to four decimal places.)

71.* Find the surface area of the solid generated by rotating the graph of Exercise 70 about the x-axis. (Round your answer to four decimal places.)

72–73 Use substitution and a table of integrals to evaluate the given integral.

72. $\int \frac{e^x}{1 + \cos e^x}\,dx$

73. $\int \frac{\sin 2x}{2\sqrt{2 \cos x - \cos^2 x}}\,dx$

©HAWKES LEARNING

74–77 Use the Trapezoidal Rule with $n = 6$ to approximate the integral, and compare the result to the exact value of the integral by determining the absolute value of the error E_T.

74. $\int_1^4 x^{3/2}\,dx$

75. $\int_1^2 \frac{1}{x^2}\,dx$

76. $\int_0^4 \sqrt{x^2+1}\,dx$

77. $\int_0^{\pi/3} \tan x\,dx$

78–81. Use the error estimate for the Trapezoidal Rule to estimate $|E_T|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 74–77.

82–85. Use Simpson's Rule to approximate the integrals from Exercises 74–77 with $n = 6$. Determine the absolute value of the error E_S.

86–89. Use the error estimate for Simpson's Rule to estimate $|E_S|$ for $n = 6$, and compare the estimate with the actual error you found in Exercises 82–85.

90. Prove that if $f(x) = ax + b$ is a linear function on a closed interval $[a,b]$, then for any n, $T_n = \int_a^b f(x)\,dx$.

91–96 Identify the type of the improper integral and determine whether it is convergent or divergent. If it is convergent, find its value.

91. $\int_2^4 \frac{dx}{(x-2)^4}$

92. $\int_{-\infty}^0 x^2 e^x\,dx$

93. $\int_2^6 \frac{dx}{\sqrt{x-2}}$

94. $\int_0^\infty \frac{2}{(x+3)^{2/3}}\,dx$

95. $\int_{-\infty}^\infty \frac{2e^x}{e^{2x}+4}\,dx$

96. $\int_e^\infty \frac{dx}{x \ln x}$

97–98 Use the Direct Comparison Test to determine whether the integral converges.

97. $\int_0^\infty \frac{dx}{\sqrt{x^3+1}}$

98. $\int_1^\infty \frac{\ln x}{\sqrt{x}}\,dx$

99. Use substitution to turn the improper integral $\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ into a proper one and evaluate it.

100. Rotate about the x-axis the region bounded by the graph of $y = \sqrt{\ln x}/x^2$ and the x-axis over the interval $[1,\infty)$. Use the disk method to determine if the resulting unbounded solid has finite volume. If so, find the volume.

101.* Rotate about the x-axis the region bounded by the x-axis and the graph of $y = e^{-x}$ over the infinite interval $[0,\infty)$. Determine if the resulting infinite solid has finite volume or surface area. If so, find their values.

102. Prove that the improper integral $\int_2^\infty \frac{dx}{x(\ln x)^a}$ converges if and only if $a > 1$.

103–106 Find the Laplace transform. (See Exercises 88–93 in Section 7.7.)

103. $L\{te^{at}\}$

104.* $L\{t^2 e^{at}\}$

105.* $L\{t \sin kt\}$

106.* $L\{t \cos kt\}$

Concept Check

107–113 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

107. If $\lim_{x\to\infty} f(x) = L$ and $L \neq 0$, then $\int_0^\infty f(x)\,dx$ diverges.

108. If $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

109. Let $f(x)$ be defined on $[a,\infty)$ and S be the solid generated by rotating the graph of $f(x)$ about the x-axis. If the surface area of S is infinite, then the volume of S is also infinite.

110. If $f(x)$ is an odd function, then $\int_{-\infty}^\infty f(x)\,dx = 0$.

111. If $f(x)$ has a vertical asymptote at $x = 0$ and $a > 0$, then $\int_{-a}^a f(x)\,dx$ diverges.

112. Any rational function is integrable on any finite interval that doesn't include a zero of the denominator.

113. If an integrand contains the expression $\sqrt{a^2 \pm x^2}$, a trigonometric substitution must be used to evaluate the integral.

Chapter 7
Technology Exercises

114. Use a graphing utility to find the integral from Exercise 33. If the answer appears different from what you obtained by hand, prove that the answers are equivalent.

115. Write a program for a computer algebra system or programmable calculator that evaluates the trapezoidal approximation T_n for a given input function on a specified interval and positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal.

116. Use the program you wrote for Exercise 115 for the integral from Exercise 75.

©HAWKES LEARNING

117. Write a program for a computer algebra system or programmable calculator that evaluates the Simpson approximation S_n for a given integral and an even positive integer n. Find the smallest n that provides an answer to Exercise 74 that is correct to at least the first three digits after the decimal. Compare this with your answer for Exercise 115. What n ensures that the answer is correct to at least five decimal places?

118. Use the program you wrote for Exercise 117 for the integral from Exercise 75.

119–120 Use the programs you wrote for Exercises 115 and 117 to approximate the given nonelementary integral with $n = 50$. Which method (the Trapezoidal Rule or Simpson's Rule) do you expect to be more accurate? Use the built-in numerical integration command of your technology to verify your conjecture.

119. $\int_0^1 \sqrt{1+x^4}\,dx$ **120.** $\int_0^1 e^{e^x}\,dx$

©HAWKES LEARNING

Chapter 8
Review Exercises

1–4 Determine whether the differential equation is separable, linear, or autonomous; and find its order. (Note that more than one description may be applicable.)

1. $\dfrac{y'}{y+1} = y^3 - 2$

2. $xy\,dy = \sqrt{1-x^2}\,dx$

3. $xy' = 2y + x^4 e^x - y'$

4. $2y'' = y - 3y'$

5–6 Solve the differential equation with the given initial condition.

5. $\left(x^2+1\right)y' = x; \quad y(0) = 1$

6. $y' \sec x = \sin x; \quad y(0) = 0$

7–12 Solve the separable differential equation.

7. $xy' = 2y$

8. $y' = e^{-y}\cos x$

9. $x^2 y' = 2\sqrt{y}\,(x+1)$

10. $x^2\,du = u^2\,dx$

11. $x\,du = \left(u^2 - 1\right)dx$

12. $\dfrac{y'}{3} = e^{3x-y}$

13–16 Solve the given initial value problem.

13. $y' = \dfrac{9x^2 + 2x}{3y^2}; \quad y(0) = 2$

14. $x' = x\sin t; \quad x(\pi/2) = 5$

15. $\dfrac{dy}{dt} = 4t\sqrt{y-2}; \quad y(1) = 6$

16. $\dfrac{dy}{dx} = 2xy + y - 4x - 2; \quad y(0) = 0$

17–22 Solve the linear differential equation. (**Hint:** In some cases, x has to be the dependent variable in order for the equation to be linear.)

17. $y' - \dfrac{y}{x} = 0$

18. $xy' + y = x^2 \sin x$

19. $xy' + 6y = 2$

20. $y' - 4xy = 2x$

21. $\left(y^2+1\right)dx + 2xy\,dy = 4y\,dy$

22. $dx + x\cot y\,dy = \cos y\,dy$

23–24 Find a first-order linear differential equation in standard form that has the given general solution. (**Hint:** Identify the integrating factor and "reverse" the solution technique discussed in this chapter.)

23. $y = Ce^{x^3} - \dfrac{1}{3}$

24. $y = \dfrac{C}{\sqrt{x^2-1}} - 1$

25–28 Solve the given initial value problem.

25. $xy' + (x-1)y = x^3; \quad y(1) = 1$

26. $\dfrac{dy}{dx} - 2xy = 2xe^{x^2}; \quad y(0) = 2$

27. $\left(x^2+1\right)y' + xy = \dfrac{1}{x^2+1}; \quad y(0) = -3$

28. $(\cos t)\dfrac{dy}{dt} + (\sin t)y = 1; \quad y(0) = 1$

29–32 Match the differential equation with its slope field (labeled A–D). Classify each equilibrium solution as stable or unstable.

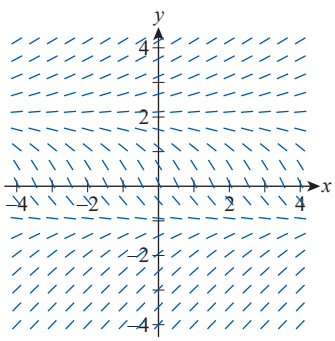
29. $y' = y^2 + 2y$

30. $y' = 9y - y^3$

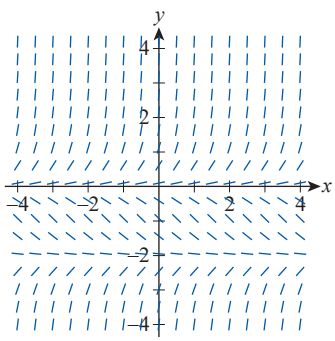
31. $y' = \dfrac{y^2 - y - 2}{y^2 + 1}$

32. $y' = y^4 - 8y$

A.

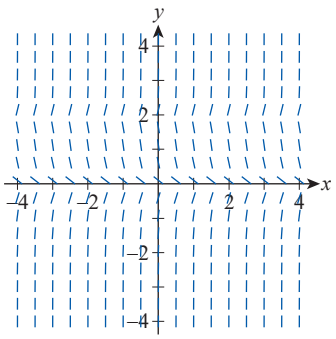


B.

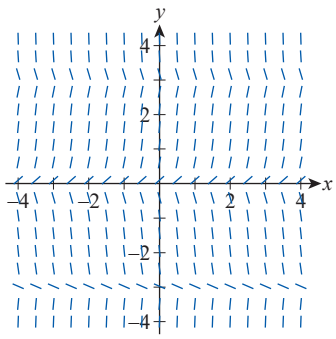


©HAWKES LEARNING

C.



D.

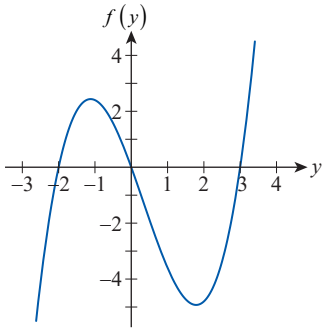


33–34 Graph by hand the slope field of the given differential equation. If applicable, find and classify each equilibrium solution as stable or unstable.

33. $y' = y^2 - 9$

34. $y' = y^3 + y^2 - 2y$

35. Create a rough sketch of the slope field of the differential equation $y' = f(y)$, where the graph of f is given below. Classify equilibria as stable or unstable.



36–37 For the initial value problem, **a.** use Euler's method with the indicated step sizes to approximate the given value of y and **b.** solve the IVP by conventional methods and compare your approximations with the exact value.

36. $y' = 2y + 1; \quad y(0) = 1;$
approximate $y(1)$ with (i) $h = 0.2$ (ii) $h = 0.1$

37. $y' = y - x; \quad y(0) = 2;$
approximate $y(2)$ with (i) $h = 0.5$ (ii) $h = 0.25$

38–43 Find the general solution of the second-order homogeneous linear equation.

38. $2y'' + 3y' - 2y = 0$

39. $y'' - 16y = 0$

40. $y'' - 2y' + 2y = 0$

41. $y'' - 5y' = 0$

42. $y'' + 9y = 0$

43. $4y'' - 4y' + y = 0$

44–47 Solve the given second-order initial value problem.

44. $9y'' - 6y' + y = 0; \quad y(0) = 3; \quad y'(0) = 2$

45. $y'' - y' - 6y = 0; \quad y(0) = 3; \quad y'(0) = -1$

46. $y'' + 2y' + 3y = 0; \quad y(0) = 0; \quad y'(0) = 2$

47. $2y'' - 3y' = 0; \quad y(2) = -5; \quad y'(2) = 3$

48–49 Solve the boundary value problem, if possible.

48. $2y'' + y = 0; \quad y(0) = 2; \quad y\left(\pi/\sqrt{2}\right) = -4$

49. $y'' + 4y = 0; \quad y(0) = 0; \quad y(\pi) = 1$

50. Find the general solution of the third-order equation $y''' - 3y'' - y' + 3y = 0$. (**Hint:** See Exercises 35–38 in Section 8.4.)

51. Find the general solution of the nonhomogeneous equation $2y'' - 7y' + 3y = \cos x$. As your initial guess, use $y_p = A\cos x + B\sin x$. (**Hint:** See Exercises 39–42 in Section 8.4.)

52–53 Determine the orthogonal trajectories of the family of curves, where a is an arbitrary nonzero constant.

52. $y = ax^4$

53. $y = \dfrac{ax}{\sqrt{x^2 + 1}}$

54. A 500-liter tank is filled with water holding 5 kilograms of salt in the solution. Through an inlet, a stronger solution with salt concentration of 0.05 kilograms per liter is being added at a rate of 16 liters per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the same rate. What is the amount of salt in the tank after 25 minutes?

55. Answer the question of Exercise 54 if the contents of the tank are drained out at a rate of 20 liters per minute.

©HAWKES LEARNING

56.* A container in a lab contains 14 gallons of pure distilled water. 10% and 25% acid solutions are pumped into the container through two respective inlets. The 10% solution is flowing in at a rate of 0.2 gallons per minute, while the 25% solution is being allowed in by the second inlet at a rate of 0.5 gallons per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the rate of 0.7 gallons per minute. How long does it take to form 14 gallons of 14% solution in this way?

57. A hailstone is melting so that its volume $V(t)$ decreases at a rate proportional to its surface area.

a. Assuming that the hailstone is nearly spherical, find a differential equation satisfied by $V(t)$.

b. If a hailstone of diameter 1 inch loses 20% of its volume in half an hour, predict how long it takes for it to completely melt away. (Consider it melted away when your model predicts less than 1 percent remaining).

58. If a vertical cylindrical tank of radius $\frac{1}{2}$ meters and height 4 meters is initially full of water but is draining through a circular orifice of diameter 2 centimeters that is on the bottom of the tank, what is the water level in the tank 2 minutes later? (**Hint:** See Exercise 59 in Section 8.1.)

59. Find the charge $q(t)$ of the 10^2-farad capacitor in an RC circuit if the impressed voltage on the circuit is $V(t) = t$ and the resistance is 25 ohms. Assume $q(0) = 0$. (**Hint:** See Exercise 62 in Section 8.1.)

60. Suppose that the impressed voltage in a simple RL circuit is $V(t) = 2t,\ I(0) = 0,$ the inductance is 0.1 henries, and the resistance 0.5 ohms. Find the electric current I at time $t = 4$ seconds. (**Hint:** See Example 5 in Section 8.2.)

61. A baking dish is removed from a 210 °C oven and left at 20 °C room temperature. Two and a half minutes later the dish's temperature is 155 °C. Find the bakeware's temperature 10 minutes after it was removed from the oven. (**Hint:** See Example 2 in Section 8.3.)

62. A snapping turtle population grows logistically with a carrying capacity of 200 turtles and constant of proportionality $k = 0.2$ per year.

a. Find the population size $P(t)$ as a function of time if initially 50 turtles are present in the habitat. (**Hint:** See Example 3 in Section 8.3.)

b. How long does it take for the population to reach 100 turtles?

63. Suppose that an object of mass 200 grams stretches a spring by 10 centimeters. If it is pulled upward to a position of 5 centimeters above equilibrium and released with a downward velocity of $1\ \text{m/s}$, find and graph the resulting displacement function, assuming that the surrounding medium offers resistance with a damping constant of $c = 0.5\ \text{kg/s}$. (**Hint:** See Example 5 in Section 8.4.)

Concept Check

64–70 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

64. The equation $(y')^2 + xy' - 3y = 0$ is a first-order differential equation.

65. The equation $y' = -y$ is not linear.

66. If $y_1(x)$ and $y_2(x)$ are solutions of a homogeneous linear differential equation, then so is $3y_1(x) - 2y_2(x)$.

67. Only autonomous equations have slope fields.

68. The logistic equation discussed in this text is autonomous.

69. The equations $y = 2e^{x/2}$ and $y = xe^{x/2}$ are linearly independent solutions of $4y'' - 4y' + y = 0$.

70. A second-order BVP with two boundary conditions always has a solution.

Chapter 8
Technology Exercises

71–72. Use a graphing utility to display the slope fields of the differential equations in Exercises 33 and 34. Compare the graphs to your original sketches.

73. Write a program for a computer algebra system that accepts a spring constant, a damping constant, and the mass of an oscillating object as inputs, and graphs the displacement function as output. Use it to check your answer for Exercise 63.

©HAWKES LEARNING

Chapter 9
Review Exercises

1–4 Sketch the curve defined by the parametric equations by eliminating the parameter.

1. $x = \dfrac{1}{36t}, \quad y = t^2$

2. $x = t + 5, \quad y = |t - 2|$

3. $x = \dfrac{3}{4t - 2}, \quad y = 2t - 2$

4. $x = 4\sin\theta, \quad y = \cos\theta + 1$

5–6 Construct parametric equations defining the graph of the given equation.

5. $y^2 = x^2 + 4$

6. $6x = 2 - y$

7–8 Find parametric equations to represent the graph described. (Answers will vary.)

7. Line, passing through $(14, 4)$ and $(-3, -8)$

8. Circle, center $(1, 1)$, radius 1

9–10 Find the equations of any horizontal or vertical tangent lines to the given curve.

9. $x = 2t + 1, \quad y = t^2 - 4$

10. $x = -2t^2, \quad y = \dfrac{3}{t - 2}, \quad t < 2$

11–14 Find the values of dy/dx and d^2y/dx^2 for the given curve at the indicated point.

11. $x = 2t^2 + 1, \quad y = \sqrt{t - 1}; \quad (9, 1)$

12. $x = e^t, \quad y = t^2 e^{-t}; \quad (1, 0)$

13. $x = \dfrac{1}{t}, \quad y = t^2 + t; \quad (1, 2)$

14. $x = \sin t, \quad y = \cos 2t; \quad \left(\dfrac{1}{2}, \dfrac{1}{2}\right)$

15–16 Find the value(s) of the parameter for any inflection point(s) of the given curve.

15. $x = t - t^3, \quad y = 3t + 1$

16. $x = 5t - 1, \quad y = t^3\left(2 - t^2\right)$

17–18 Find the area enclosed by the given curve.

17. $x = \sin 3t, \quad y = \cos 2t, \quad -\dfrac{\pi}{3} \le t \le \dfrac{\pi}{3}$

18. $x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \le t \le 2\pi$

19–20 Find the arc length of the given curve on the indicated interval.

19. $x = \dfrac{4}{3}t^{3/2}, \quad y = 4t, \quad 0 \le t \le 5$

20. $x = t\sin t + \cos t, \quad y = t\cos t - \sin t, \quad 0 \le t \le \pi$

21–23 Find the area of the surface generated by revolving the parametric curve about the indicated axis.

21. $x = \dfrac{t^3}{2}, \quad y = 2t + 1, \quad 0 \le t \le 1;$ about the y-axis

22. $x = \dfrac{1 - t}{2}, \quad y = \sqrt{t}, \quad 0 \le t \le 1;$ about the x-axis

23. $x = 3t - \dfrac{t^3}{3}, \quad y = \sqrt{3}\left(9 - t^2\right), \quad 0 \le t \le 3;$ about the y-axis

24–25 Convert the point from polar to Cartesian coordinates.

24. $\left(-3.45, \dfrac{\pi}{3}\right)$

25. $\left(7, \dfrac{7\pi}{6}\right)$

26–27 Convert the point from Cartesian to polar coordinates.

26. $\left(-\sqrt{3}, -1\right)$

27. $(10, 12)$

28–29 Rewrite the rectangular equation in polar form.

28. $x^2 + y^2 = 16a^2$

29. $x^2 + y^2 = 9ax$

30–31 Rewrite the polar equation in rectangular form.

30. $r = 4\cos\theta$

31. $r = \dfrac{16}{4\cos\theta + 4\sin\theta}$

32–33 Sketch a graph of the given polar equation.

32. $r = 4\sin 3\theta$

33. $r^2 = 25\cos 2\theta$

©HAWKES LEARNING

34–35 Find the slope of the line tangent to the given polar curve at the indicated point.

34. $r = 4\cos 3\theta; \quad \theta = \dfrac{\pi}{12}$

35. $r = \dfrac{1}{\theta^2 + 1}; \quad \theta = \dfrac{\pi}{2}$

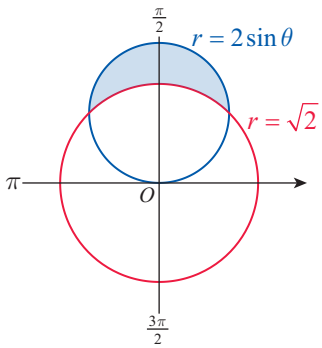
36–37 Find all points where the given polar curve has a horizontal or vertical tangent line.

36. $r = 2\sin\theta, \quad 0 \le \theta \le 2\pi$

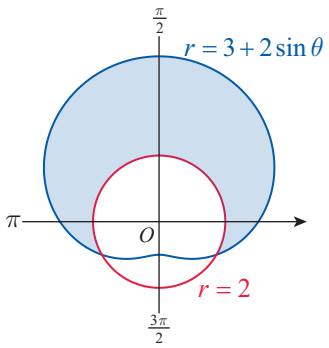
37. $r = 1 + \cos 2\theta, \quad 0 \le \theta \le \pi$

38–39 Find the area of the shaded region.

38.



39.



40–41 Find the area of the specified region.

40. A large loop of $r = 1 + 2\cos 2\theta$

41. The portion of the rose $r = 4\cos 2\theta$ outside the circle $r = 2$

42–45 Use polar coordinates to find the arc length of the curve.

42. The circle $r = 2\cos\theta$

43. The line segment $r = \sec\theta, -\pi/4 \le \theta \le \pi/4$

44. The spiral $r = 2\theta, 0 \le \theta \le 2\pi$

45. The cardioid $r = 2 + 2\cos\theta$

46. Find the area of the surface generated by revolving the curve $r = 2\cos\theta, 0 \le \theta \le \pi$ about $\theta = \pi/2$. **(Hint:** See Exercise 58 in Section 9.4)

47–52 Sketch the graph of the given conic section, and determine the coordinates of the foci and the equations of the directrix or asymptotes as appropriate.

47. $\dfrac{(x+1)^2}{9} + \dfrac{(y-2)^2}{16} = 1$

48. $x^2 + 9y^2 - 6x + 18y = -9$

49. $(y+1)^2 = -12(x+3)$

50. $x^2 - 8x + 2y + 14 = 0$

51. $\dfrac{(y+2)^2}{9} - \dfrac{(x-2)^2}{16} = 1$

52. $9x^2 - 4y^2 + 54x - 8y + 41 = 0$

53–63 Find the equation, in standard form, of the conic section with the given properties.

53. Ellipse, center at $(-1,4)$, major axis is vertical and of length 8, foci $\sqrt{7}$ units from the center

54. Ellipse, foci at $(1,2)$ and $(7,2)$, $e = \dfrac{1}{2}$

55. Ellipse, vertices at $\left(\dfrac{7}{2},-1\right)$ and $\left(\dfrac{1}{2},-1\right)$, $e = 0$

56. Ellipse, vertices at $(1,-8)$ and $(1,2)$, minor axis of length 6

57. Parabola, vertex at $(-2,3)$, directrix is the line $y = 2$

58. Parabola, vertex at $(5,-3)$, focus at $(5,1)$

59. Parabola, focus at $(3,-1)$, directrix is the line $x = 2$

60. Hyperbola, vertices at $(4,-1)$ and $(-2,-1)$, foci at $(5,-1)$ and $(-3,-1)$

61. Hyperbola, asymptotes of $y = \pm\frac{5}{2}(x+1) - 2$, vertices at $(-3,-2)$ and $(1,-2)$

62. Hyperbola, foci at $(-1,-2)$ and $(-1,8)$, asymptotes of $y = \pm\left(\frac{3}{4}x + \frac{3}{4}\right) + 3$

63. Hyperbola, asymptotes of $y = 3x - 4$, $y = 8 - 3x$, vertices at $(2,-1)$ and $(2,5)$

©HAWKES LEARNING

64–70 Identify the given conic section and find the equation for its directrix.

64. $r = \dfrac{8}{1+2\sin\theta}$

65. $r = \dfrac{5}{4-8\sin\theta}$

66. $r = \dfrac{3}{7+6\sin\theta}$

67. $r = \dfrac{6}{9-9\cos\theta}$

68. $r = \dfrac{7}{4+4\sin\theta}$

69. $r = \dfrac{4}{6-3\cos\theta}$

70. $r = \dfrac{7}{5+2\cos\theta}$

71–76 Construct a polar equation for the conic section with the focus at the origin and the given eccentricity and directrix.

71. Eccentricity: $e = 1$; directrix: $x = -3$

72. Eccentricity: $e = 4$; directrix: $y = 3$

73. Eccentricity: $e = \frac{1}{5}$; directrix: $y = -15$

74. Eccentricity: $e = \frac{1}{4}$; directrix: $x = 16$

75. Eccentricity: $e = 1$; directrix: $y = -7$

76. Eccentricity: $e = 9$; directrix: $x = \frac{1}{3}$

77. A motorcycle headlight is made by placing a strong light bulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 5y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

78.* Prove that the graph of $r = \cos\theta + \sin\theta$ is a circle.

79. Show that the two limaçons $r = 1 + \sin\theta$ and $r = 1 - \sin\theta$ are orthogonal, that is, they intersect at right angles. (Ignore their intersection at the pole.)

Concept Check

80–90 Determine whether each of the following statements is true or false. In case of a false statement, explain or provide a counterexample.

80. Two different sets of parametric equations may describe the same curve.

81. If $x = f(t)$ and $y = g(t)$ both pass through $(0,0)$, the parametric curve $x = f(t)$, $y = g(t)$ also passes through the origin.

82. The line $y = 2x$ can be parametrized as $x = t^2$, $y = 2t^2$.

83. If a parametric curve C is differentiable on (a,b) then its graph has a tangent line at any of its points.

84. Let C be the curve $x = t^5$, $y = t^4$. Since $\left.\dfrac{dy}{dt}\right|_{t=0} = 0$, the graph of C has a horizontal tangent at the origin.

85. The polar coordinates (r,θ) and $(-r,\theta-\pi)$ describe the same point.

86. If the polar coordinates (r_1,θ_1) and (r_2,θ_2) describe the same point, then $r_1 = \pm r_2$.

87. If $y = f(x)$ is an even function, then the graph of $r = f(\theta)$ is symmetric with respect to the polar axis.

88. It is impossible to find a polar equation for a straight line.

89. The distance between any point on a parabola from the focus is at least p.

90. If the asymptotes of a hyperbola are perpendicular and the hyperbola is centered at the origin with horizontally aligned foci, then the equation of the hyperbola can be written as $x^2 - y^2 = a^2$.

©HAWKES LEARNING

Chapter 9
Technology Exercises

91. Find the equation of the graph of $r = 1 - 2\cos\theta$ after a clockwise rotation by $\pi/4$ radians. Name the resulting curve and use a graphing utility to sketch it. (See Exercise 73 in Section 9.3.)

92–93 Use a graphing utility to sketch the given curve for various values of the parameter(s) and explore the effects on the shape of your graph.

92. $r = \theta\cos k\theta$

93. $x = \pm a\cos^{2/n} t, \quad y = \pm b\sin^{2/n} t, \quad a, b, n > 0$ (Lamé curves)

94–95 Use a graphing utility to sketch the curve and then find all horizontal and vertical tangent lines. Confirm your results by paper and pencil calculations.

94. $x = t^3 - t, \quad y = t^2 + 1, \quad -2 \le t \le 2$

95. $r = 2\sin 2\theta, \quad 0 \le \theta \le \pi/2$

96–97 Use a graphing utility to sketch the region enclosed by the given curve and find its area.

96. $x = t\sin t, \quad y = \sin 2t, \quad 0 \le t \le \pi$

97. Inner loop of $r = 2 - 3\cos\theta$

98–99 Use a graphing utility to approximate the arc length of the curve with the given parametrization.

98. $x = \sin 2t, \quad y = \sin t, \quad 0 \le t \le \pi$

99. $r = \cos(2\sin\theta), \quad 0 \le \theta \le 2\pi$

100–101 Use a graphing utility to approximate the surface area of the solid obtained by rotating the given curve about the indicated axis.

100. $(t^3 - 3t, t^2 - 2), \quad 0 \le t \le \sqrt{3}$; about the y-axis

101. $r = 3\sin 2\theta, \quad \pi/4 \le \theta \le \pi/2$; about the polar axis

©HAWKES LEARNING

Chapter 10
Review Exercises

1–2 Starting with $n = 0$, list the first five terms of the given sequence.

1. $a_n = \left(\dfrac{1-n}{n^2+1}\right)^n$

2. $a_n = \dfrac{2^{n+1}-n}{n!}$

3–4 Find the first five terms of the given recursively defined sequence.

3. $a_1 = 0, \quad a_n = 3a_{n-1} + 2$

4. $a_1 = 0, \quad a_2 = 1, \quad a_n = 2a_{n-2} - a_{n-1}$

5–6 Use the definition of the limit of a sequence to prove the limit statement.

5. $\lim\limits_{n\to\infty} \dfrac{2}{n^3+1} = 0$

6. $\lim\limits_{n\to\infty} \dfrac{2n+1}{n} = 2$

7–10 Find the limit of the sequence if it converges or prove that the sequence diverges.

7. $a_n = \dfrac{n^2-2}{3n+1}$

8. $a_n = \sqrt{n^2+1} - n$

9. $a_n = \left(1+\dfrac{2}{n}\right)^{3n}$

10. $a_n = 2\tan^{-1}\left(n^2\right)$

11–12 Use the Squeeze Theorem to prove that the given sequence converges.

11. $\left\{\dfrac{(-1)^n \cos n}{n^2}\right\}_{n=1}^{\infty}$

12. $\left\{\dfrac{\sin 2n}{2^n}\right\}_{n=1}^{\infty}$

13–14 Recognize the repeating decimal as a geometric series and write the decimal as a ratio of two integers.

13. $0.\overline{219}$

14. $0.2\overline{19}$

15. Use the Bounded Monotonic Sequence Theorem to prove that the recursive sequence $a_1 = \sqrt{3}, a_{n+1} = \sqrt{3a_n}$ converges, and find its limit.

16. Suppose that $\lim\limits_{n\to\infty} a_n = -\infty$ and $\{b_n\}$ is a sequence such that there is an N with $b_n \le a_n$ for all $n \ge N$. Prove that $\lim\limits_{n\to\infty} b_n = -\infty$.

17. Prove that if $\lim\limits_{n\to\infty} a_n = \infty$, then the sequence has a smallest term.

18. Let $\{a_n\}$ be a positive null sequence. Prove that $\lim\limits_{n\to\infty}\left(1/a_n\right) = \infty$. (See Exercises 92 and 95 in Section 10.1 for the definitions of a null sequence and a positive sequence, respectively.)

19. Suppose that the dosage of a certain medication is d milligrams once a day, to be taken at the same time of day.

a. Assuming that after 24 hours, half of the initial amount is still present in the bloodstream, find a recursive formula for the sequence $\{a_n\}$, the amount of medication present in the bloodstream right after the n^{th} dose is taken.

b. Find $\lim\limits_{n\to\infty} d_n$. (This is how much medication is in the patient's bloodstream, if it is taken over a long time period.)

20–21 Use partial sums to prove that the given series is convergent.

20. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$

21. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-n}$

22–25 Decide whether the series converges. If so, find its sum.

22. $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{3}{2}\right)^{-n}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{2+2n}{n^2+2n}$

25. $\displaystyle\sum_{n=2}^{\infty} \frac{3^n-1}{5^{n-2}}$

26. Referring back to Exercise 66 of Section 10.2, supposing that the positive series $\sum a_n$ is divergent, what can you say about $\sum\left(1/a_n\right)$? (See Exercise 70 in Section 10.2 for the definition of positive series.)

27–32 Use the Integral Test to determine whether the series converges or diverges.

27. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^2+4}$

28. $\displaystyle\sum_{n=0}^{\infty} \frac{4}{n^2+4}$

29. $\displaystyle\sum_{n=0}^{\infty} \frac{2n}{2^n}$

30. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{2+e^{1/n}}{n^3}$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln\left(\arctan n\right)}{n^2+1}$

33–34 Find the smallest possible n to approximate the sum of the series within the indicated error ε and provide the requested estimate.

33. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}; \quad \varepsilon = 0.04$

34. $\displaystyle\sum_{n=1}^{\infty} ne^{-n}; \quad \varepsilon = 0.03$

©HAWKES LEARNING

35–38 Use the Direct Comparison Test to determine whether the series converges or diverges.

35. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n-1}$

36. $\displaystyle\sum_{n=2}^{\infty}\frac{n^2+1}{n^3-1}$

37. $\displaystyle\sum_{n=0}^{\infty}\frac{2}{2^n+\sqrt{n}}$

38. $\displaystyle\sum_{n=0}^{\infty}\frac{\sin^4 n}{n^2+1}$

39–42 Use the Limit Comparison Test to determine whether the series converges or diverges.

39. $\displaystyle\sum_{n=0}^{\infty}\frac{n+1}{\sqrt{n^5+1}}$

40. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{1+\ln n}}{n^2}$

41. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n^3+n}}{n^{5/2}}$

42. $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^4+1}}$

43–50 Use the Ratio or Root Test, as appropriate, to determine whether the series converges or diverges.

43. $\displaystyle\sum_{n=1}^{\infty}\frac{n^7}{7^n}$

44. $\displaystyle\sum_{n=5}^{\infty}\frac{5^n}{(n-5)!}$

45. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n\cdot n!}$

46. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n(n!)}{(2n-1)!}$

47. $\displaystyle\sum_{n=0}^{\infty}\frac{3^n}{4^n+1}$

48. $\displaystyle\sum_{n=1}^{\infty}\frac{n^n}{2^{n+1}}$

49. $\displaystyle\sum_{n=1}^{\infty}\frac{(\ln n)^{2n}}{n^{n+1}}$

50. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2}+\frac{1}{2n}\right)^n$

51–54 Determine whether the alternating series converges and give a reason for your answer.

51. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n^2}$

52. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{e^n\sqrt{n}}$

53. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n-1}{5n+1}$

54. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n^5}$

55–56 Approximate the sum of the alternating series, accurate to at least the indicated number of decimal places. How many terms did you use? According to the n^{th}-remainder estimate for the error (see Section 10.6), how many terms guarantee the indicated accuracy? (Note that the n^{th}-remainder estimate is not necessarily "sharp".)

55. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4}$; accurate to 2 decimal places

56. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n2^n}$; accurate to 3 decimal places

57–62 Determine whether the given series converges absolutely, converges conditionally, or diverges.

57. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2+1}$

58. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n}{n^2+1}$

59. $\displaystyle\sum_{n=1}^{\infty}\frac{\sin n}{n^{3/2}}$

60. $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^n}{n^6}$

61. $\displaystyle\sum_{n=3}^{\infty}\left(\frac{2-n}{4n}\right)^n$

62. $\displaystyle\sum_{n=1}^{\infty}\frac{(-2)^{n+1}n}{4n-2}$

63–68 Determine the interval of convergence for the given power series.

63. $\displaystyle\sum_{n=0}^{\infty}2^n x^n$

64. $\displaystyle\sum_{n=0}^{\infty}\frac{(x-2)^n}{n^3+2}$

65. $\displaystyle\sum_{n=0}^{\infty}\frac{(3x)^n}{n!}$

66. $\displaystyle\sum_{n=1}^{\infty}\frac{n!(x-1)^n}{n^4}$

67. $\displaystyle\sum_{n=0}^{\infty}\frac{(x+1)^n}{(n+1)3^n}$

68. $\displaystyle\sum_{n=2}^{\infty}\frac{(2x-6)^n}{\sqrt{n}\ln n}$

69–70 Determine the interval of convergence for the series, and the limiting function of the series on that interval.

69. $\displaystyle\sum_{n=0}^{\infty}(2x-3)^n$

70. $\displaystyle\sum_{n=0}^{\infty}\frac{(3x-1)^n}{2^n}$

71–72 Find the power series expansion for the given function about 0. What is the radius of convergence? (**Hint:** Use the same approach as in Exercises 41–48 of Section 10.7.)

71. $f(x)=\dfrac{1}{1+3x}$

72. $f(x)=\dfrac{1}{2-4x^2}$

73. Find the power series representation of $f(x)=1/x$, centered at $a=2$. Differentiating twice, find the series for $g(x)=1/x^3$ around $a=2$. What is the radius of convergence? (**Hint:** Start by rewriting $1/x$ as $\dfrac{1}{2-(2-x)}$.)

74–81 Determine the Taylor series (or Maclaurin series if the center is not specified) of the given function about the indicated point. Find the radius of convergence. (**Hint:** Do not use the definition.)

74. $f(x)=e^{x^4/2}$

75. $f(x)=\cos(2x^3)$

76. $f(x)=\dfrac{1}{x+3};\quad a=2$

77. $f(x)=\ln x;\quad a=3$

78. $f(x)=\sin\dfrac{x}{2}$

79. $f(x)=\sin^2 x$
(**Hint:** Use a trigonometric indentity.)

80. $f(x)=x^2e^{3x^2}$

81. $f(x)=\dfrac{x}{x+2};\quad a=1$

©HAWKES LEARNING

82–83 Use the definition to find the first four nonzero terms of the Taylor series generated by the given function about the indicated point.

82. $f(x) = \cot x; \quad a = \pi/2$

83. $f(x) = \csc x; \quad a = \pi/4$

84–86 Find the first five nonzero terms of the Maclaurin series generated by the given function by using operations on familiar series (try not to use the definition).

84. $f(x) = xe^x - \sin x$ **85.** $f(x) = \dfrac{\cos x}{x+1}$

86. $f(x) = \cos(\sin x)$ (**Hint:** Substitute the series of $\sin x$ into that of $\cos x$.)

87–89 Use Taylor series to approximate the given function value or definite integral to within the indicated accuracy. How many (nonzero) terms did you need?

87. $\cos 1; \quad$ error $\le 10^{-6}$

88. $\displaystyle\int_0^1 e^{-x^3}\,dx; \quad$ error $\le 10^{-4}$

89. $\sqrt[4]{1.2}; \quad$ error $\le 10^{-5}$ (**Hint:** Use a binomial series.)

90. Use series expansion to verify the trigonometric limit $\displaystyle\lim_{x\to 0}\frac{1-\cos x}{x} = 0.$

91–92 Use Taylor series to find the indicated limit.

91. $\displaystyle\lim_{x\to 0}\frac{e^{-x}-1}{x}$ **92.** $\displaystyle\lim_{x\to 0}\frac{x\sin 2x}{1-e^{x^2}}$

93. Using an appropriate power series, find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\left(n/2^n\right)$. (**Hint:** Start with the familiar series expansion of $1/(1-x)$, and use termwise differentiation in its interval of convergence; then make an appropriate substitution.)

94. Using an appropriate Taylor series, verify that $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} = \ln 2.$

95–98 Find the first six nonzero terms of the binomial series expansion of the indicated function.

95. $f(x) = \dfrac{1}{\sqrt{x^2+1}}$ **96.** $f(x) = \dfrac{1}{\sqrt[3]{x+1}}$

97. $f(x) = \sqrt{2x^3+1}$ **98.** $f(x) = \left(1-\dfrac{x}{2}\right)^{4/3}$

99–100 Find a power series solution of the equation. In each case, use the initial conditions $y(0) = a$ and $y'(0) = b$.

99. $y'' + 4y = 0$ **100.** $y'' - \dfrac{2}{(1-x)}y' = 0$

101. Find a power series solution of the differential equation $y' + y = 0$ satisfying the initial condition $y(0) = 2$. Then solve the equation by traditional means and conclude that the solutions are equal.

102. Use the Maclaurin series you obtained in Exercise 86 to find $f^{(6)}(0)$.

103.* Prove that if every bounded monotonic sequence converges, then the Completeness Property of real numbers holds. (**Hint:** Let S be a set that is bounded above by M. Let $a_0 = M$ and for each $n > 0$, let s_n be the least positive integer such that $s_n M/2^n$ is an upper bound for S. Letting $a_n = s_n M/2^n$, proceed to show that a_n converges and its limit is the least upper bound of S. The existence of the greatest lower bound can be shown analogously. Note that this statement is the converse of Exercise 74 in Section 10.1, showing that the Bounded Monotonic Sequence Theorem is equivalent to the Completeness Property of $\mathbb{R}$.)

104. Prove that the alternating series $\displaystyle\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\ln(n^p)}{n}$ converges for all values of p.

105. Let $\displaystyle\sum_{n=1}^{\infty}a_n$ be a divergent series and assume that $\displaystyle\sum_{n=1}^{\infty}b_n$ is convergent. Prove that $\displaystyle\sum_{n=1}^{\infty}(a_n + b_n)$ is divergent.

106. Suppose that $\displaystyle\sum_{n=1}^{\infty}a_n$ is divergent and $k \ne 0$. Prove that $\displaystyle\sum_{n=1}^{\infty}ka_n$ is also divergent.

107. Decide whether the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{1+2+\cdots+n}$ converges or diverges. Prove your answer.

108. If a_n and b_n are positive sequences such that $\displaystyle\sum_{n=1}^{\infty}a_n$ is convergent and b_n is a null sequence, prove that $\displaystyle\sum_{n=1}^{\infty}a_n b_n$ is convergent.

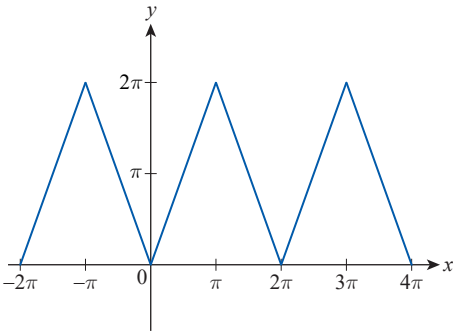
109. Let $a_n = \begin{cases} 1/n & \text{if } n \text{ is even} \\ 1/n^2 & \text{if } n \text{ is odd} \end{cases}$. Show that $a_n \to 0$, yet $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ diverges. Why does this not contradict Leibniz's Test?

©HAWKES LEARNING

110.* Use Taylor's formula to provide a proof of the Second Derivative Test as follows. Assuming that $f'(c)=0$, use Taylor's formula to conclude that $f(x)=f(c)+\frac{1}{2}f''(a)(x-c)^2$, for some a between x and c. Then examine the signs of $f(x)-f(c)$ and $f''(a)$. Next, assuming $f'(c)=f''(c)=0$ and $f'''(c)\neq 0$, argue that $f(c)$ is neither a relative maximum nor a minimum. (Assume initially that f is continuosly differentiable through at least the third order; then think about whether you can relax this condition.)

111. Find a second solution to Exercise 71 using long division.

112. Find the Fourier series expansion of the 2π-periodic extension of the function graphed below.



Concept Check

113–124 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

113. If a_n is monotonically decreasing, then $\lim_{n\to\infty} a_n = -\infty$.

114. If $\{a_n\}$ is convergent, then $\{a_n/n\}$ is a null sequence.

115. If $\{a_n/n\}$ is a null sequence, then $\{a_n\}$ is convergent.

116. If $\{a_n\}$ is convergent, then $\{a_{n+1}-a_n\}$ is a null sequence.

117. If $\{a_n\}$ is monotonically decreasing to zero, then $\sum_{n=1}^{\infty}(-1)^n a_n$ is absolutely convergent.

118. If $\{a_n\}$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

119. If $\sum_{n=1}^{\infty}|a_n|$ is divergent, then either $\sum_{n=1}^{\infty} a_n$ or $\sum_{n=1}^{\infty}(-a_n)$ is divergent.

120. If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty}|a_n|$ is divergent.

121. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

122. If the power series $\sum_{n=1}^{\infty} a_n x^n$ diverges at $x=c$, then it diverges at $x=-c$.

123. All power series converge at infinitely many x-values.

124. There is a power series whose convergence set is empty.

Chapter 10
Technology Exercises

125–127 Use a graphing utility to solve the problem.

125. We already know that the harmonic series diverges to infinity, and that it does so at a very slow pace. In this exercise, we will examine this series a bit further.

a. Find out how many terms are needed for the partial sum of the harmonic series to exceed 12.

b. What is the sum of the first 2 million terms? (Compare with Example 4 of Section 10.3. Notice that this calculation takes a bit of time even for today's powerful technology!)

126. The simple series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$ was shown by Gregory and Leibniz to converge to $\pi/4$ (hence its name, the *Gregory series*). However, it converges rather slowly. Find out how many terms of this series are necessary to approximate π accurate to two decimal places.

127. **a.** Graph $y=\sin x$ and its 11^{th}-order Maclaurin polynomial on the same screen, over the interval $[-4\pi, 4\pi]$. Visually estimate the subinterval over which you find the approximation acceptable.

b. Repeat part a. with the 21^{st}-order Maclaurin polynomial.

©HAWKES LEARNING

Chapter 11
Review Exercises

1–6 Find the distance from the point to the indicated plane.

1. $(-2,0,4)$; the yz-plane

2. $(3,-2,5)$; the plane $y = 1$

3. $(0,0,0)$; $4x + y - z - 3 = 0$

4. $(1,-2,1)$; $x - y + z - 5 = 0$

5. $\left(3,0,\frac{1}{2}\right)$; $y - 2z = x + 4$

6. $(-1,5,2)$; $x + 4 = 4y + 2z$

7–12 Describe the set of points represented by the given equation or inequality.

7. $x^2 + (y-2)^2 + z^2 > 1$

8. $x - 2z - 1 \geq y$

9. $x^2 + 4z^2 < 2$

10. $xyz^2 \neq 0$

11. $x^2 + 2y^2 - 4z^2 \leq 8$

12. $6x^2 + 2z^2 < 3y$

13–14 If $\mathbf{u} = \langle 2,5,-4 \rangle$ and $\mathbf{v} = \langle -1,4,-2 \rangle$, find the coordinates of the endpoint of the indicated vector with the given initial point.

13. $2\mathbf{u} - 3\mathbf{v}$ with initial point $(2,-7,3)$

14. $\frac{1}{2}\mathbf{v} - 4\mathbf{u}$ with initial point $(-1,4,-5)$

15. Find a vector $\mathbf{v}$ that solves the vector equation $\langle -6,2,0 \rangle - 2\mathbf{v} = \langle 1,0,-1 \rangle$.

16. Explain the difference between the following expressions: $(1,2,3)$ vs. $\langle 1,2,3 \rangle$ vs. $\{1, 2, 3\}$.

17. Determine whether the following points are collinear: $P(1,-3,0)$, $Q(2,5,1)$, and $R(4,21,3)$.

18–19 Find the component form and magnitude of the vector $\overrightarrow{PQ}$.

18. $P(2,-4,0)$, $Q(1,-3,-1)$

19. $P\left(\frac{\sqrt{3}}{2},-1,1\right)$, $Q\left(2\sqrt{3},-2,3\right)$

20–21 Use vectors to find the coordinates of the indicated point.

20. The point two-thirds of the way from $P(-6,0,3)$ to $Q(0,12,9)$

21. The point three-eighths of the way from $P(0,-4,8)$ to $Q(-16,0,2)$

22–23 Find the unit vector $\mathbf{u}$ pointing in the direction of the given vector $\mathbf{v}$.

22. $\mathbf{v} = \langle 2,4,6 \rangle$

23. $\mathbf{v} = \left\langle 1,-\frac{1}{4},\frac{1}{3} \right\rangle$

24–26 Find the dot product of $\mathbf{u}$ and $\mathbf{v}$.

24. $\mathbf{u} = 2\mathbf{i} - 5\mathbf{j}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{k}$

25. $\mathbf{u} = \frac{1}{3}\mathbf{i} + \pi\mathbf{j} - \frac{7}{2}\mathbf{k}$, $\mathbf{v} = -6\mathbf{i} - \frac{2}{\pi}\mathbf{j} + 8\mathbf{k}$

26. $|\mathbf{u}| = 3\sqrt{3}$, $|\mathbf{v}| = 2$, their angle is $30°$

27–28 Find the angle between the given vectors.

27. $\mathbf{u} = \langle 1,0,1 \rangle$, $\mathbf{v} = \langle 2,2,0 \rangle$

28. $\mathbf{u} = -\mathbf{j} - \sqrt{2}\,\mathbf{k}$, $\mathbf{v} = \mathbf{i} - \sqrt{3}\,\mathbf{k}$

29–30 Determine the value of the parameter so that the vectors are **a.** parallel, **b.** orthogonal.

29. $\langle s,4,2 \rangle$ and $\langle 1,2s,\sqrt{2} \rangle$

30. $\langle 2s,1,3 \rangle$ and $\langle 1,4s,3\sqrt{2} \rangle$

31. Find the angle between the planes $x + 4y + 3z = 2$ and $4z - 3x - y = 11$. (**Hint:** The angle between two planes is the same as that between their respective normal vectors.)

32–33 Find the direction angles of the given vector.

32. $\langle -1,3,4 \rangle$

33. $\langle \sqrt{3},2,1 \rangle$

34–37 Find the decomposition of $\mathbf{u}$ into a sum of two vectors, one parallel to, and the other perpendicular to $\mathbf{v}$.

34. $\mathbf{u} = \langle 1,3,0 \rangle$, $\mathbf{v} = \langle 2,0,-4 \rangle$

35. $\mathbf{u} = \left\langle \frac{1}{3},-3,1 \right\rangle$, $\mathbf{v} = \left\langle 2,-5,\frac{5}{3} \right\rangle$

36. $\mathbf{u} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

37. $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \frac{1}{2}\mathbf{k}$, $\mathbf{v} = \frac{1}{2}\mathbf{i} + \mathbf{j} - \mathbf{k}$

©HAWKES LEARNING

38–39 Find the work done by the force **F** as it moves an object from *P* to *Q*. (Suppose **F** is measured in newtons, and a unit distance is 1 meter.)

38. $\mathbf{F} = 2.1\,\mathbf{i} - 2.4\,\mathbf{j} - 5.7\,\mathbf{k}$ from $P(5,8,4)$ to $Q(-3,0,1.5)$

39. $\mathbf{F} = 3\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}$ from $P(0,-2,-17.3)$ to $Q(2,9.5,-11)$

40–41 Use the determinant formula to find the cross product.

40. $\langle 3,0,-2\rangle \times \langle 1,2,1\rangle$

41. $\langle 2,1,-1\rangle \times \left\langle \frac{1}{2},4,-2\right\rangle$

42–43 Find both unit vectors perpendicular to **u** and **v**.

42. $\mathbf{u} = \langle 0,-1,1\rangle, \quad \mathbf{v} = \langle 1,2,-1\rangle$

43. $\mathbf{u} = \left\langle -2,\frac{1}{2},1\right\rangle, \quad \mathbf{v} = \langle 0,2,4\rangle$

44–45 Construct a vector normal to the plane containing the indicated points.

44. $P(1,-1,1), \quad Q(0,2,2), \quad R(3,0,3)$

45. $P(-4,0,5), \quad Q\left(1,-\frac{1}{2},1\right), \quad R(2,3,0)$

46–47 Use cross products to check whether the points *P*, *Q*, and *R* are collinear.

46. $P(-1,0,1), \quad Q(1,-2,0), \quad R(1,1,3)$

47. $P(2,1,-2), \quad Q(3,2,-3), \quad R(-1,-2,1)$

48–49 Find the area of the triangle with the given vertices.

48. $A(0,1,2), \quad B\left(-\frac{1}{2},2,1\right), \quad C\left(\frac{3}{2},-3,4\right)$

49. $A(2,1,0), \quad B(3,-1,2), \quad C(-3,0,1)$

50–53 Let $\mathbf{u} = \langle 2,1,-3\rangle$, $\mathbf{v} = \langle 1,0,-4\rangle$, and $\mathbf{w} = \langle 1,-1,\frac{1}{2}\rangle$. If possible, find each of the following.

50. $\mathbf{u} \times (\mathbf{v} + \mathbf{w})$

51. $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$

52. $(\mathbf{u} \cdot \mathbf{v}) + \mathbf{w}$

53. $(\mathbf{u} + \mathbf{v}) \times \mathbf{w}$

54–55 Find the volume of the parallelepiped spanned by the indicated vectors.

54. $\mathbf{u} = \langle 1,1,1\rangle, \quad \mathbf{v} = \langle 2,0,3\rangle, \quad \mathbf{w} = \langle 1,-2,1\rangle$

55. $\mathbf{u} = \langle 2,1,1\rangle, \quad \mathbf{v} = \langle 1,2,1\rangle, \quad \mathbf{w} = \langle 1,1,2\rangle$

56–57 Find parametric equations for the line parallel to the indicated direction vector and passing through the given point.

56. Through the point $(3,4,-7)$ and parallel to $2\,\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$

57. Through the point $\left(\frac{5}{2},9,0\right)$ and parallel to $\left\langle 2,\frac{1}{2},-\frac{3}{4}\right\rangle$

58–59 Give a vector description of the line segment between the two given points.

58. The line segment between $(1.2,-2,5)$ and $(2.2,0,9)$

59. The line segment between $\left(4,-\frac{3}{2},7\right)$ and $(6,2,5)$

60–63 If possible, determine the point of intersection of the pair of lines.

60. $\mathbf{r}(t) = \langle 1+2t, 3-2t, 4-6t\rangle$ and $\mathbf{s}(u) = \langle 2u-1, u-1, -u\rangle$

61. $\mathbf{r}(t) = \left\langle 3-\frac{7}{2}t, 1-\frac{t}{2}, t-1\right\rangle$ and $\mathbf{s}(u) = \langle 1-5u, 2u-2, u\rangle$

62. $\mathbf{r}(t) = \langle 1+t, 3t, 2t-3\rangle$ and $\mathbf{s}(u) = \langle u-2, 4+u, u\rangle$

63. $\mathbf{r}(t) = \langle 2-2t, 1+3t, 4t\rangle$ and $\mathbf{s}(u) = \langle 2+u, 3-3u, 1+u\rangle$

64–65 Identify the plane containing the given point and having the indicated normal vector as a two-parameter set of points in $\mathbb{R}^3$.

64. The plane through the point $(3,-2,-1)$ with normal vector $\mathbf{n} = \langle 1,-2,-3\rangle$

65. The plane through the point $(4,-1,3)$ with normal vector $\mathbf{n} = \left\langle \frac{3}{2},-\frac{1}{3},2\right\rangle$

66–67 Find an equation for the plane containing the given points.

66. $A(1,0,2), \quad B(2,3,-2), \quad C(-2,1,3)$

67. $A\left(-\frac{1}{2},2,0\right), \quad B(4,1,1), \quad C(-1,-1,1)$

68–70 Find the parametric equations of the line as described.

68. The line through the point $(-3,1,2)$ that is perpendicular to the plane $x - 2y + z = 11$

69. The line through $\left(-2,2,\frac{1}{2}\right)$ that is parallel to the line $\mathbf{r}(t) = \langle t-2, 1+3t, 1-2t\rangle$

©HAWKES LEARNING

70. The line through $(-4,-5,-6)$ that is perpendicular to the vectors $3\mathbf{i}+\mathbf{k}$ and $\mathbf{j}-2\mathbf{k}$

71–72 Find an equation for the plane satisfying the given conditions.

71. The plane through the point $(0,-1,0)$ that forms an angle of $\pi/3$ radians with the positive y-axis

72. The plane through the point $(-5,2,1)$ that contains the line $x=1+2t,\ y=3-t,\ z=4t-1$

73–74 Find the point of intersection between the given line and plane.

73. The line $\mathbf{r}(t)=\langle 1+2t,5-4t,6t-1\rangle$ and the plane $2x+y-3z=1$

74. The line $\mathbf{r}(t)=\langle 1+t,5-2t,t\rangle$ and the plane $x+4y-2z=3$

75–76 Find parametric equations for the line formed by the intersection of the two given planes.

75. $2x+y-z=8,\quad 5x-y+3z=-1$

76. $3x+2y-z=0,\quad x-y+4z=5$

77. Find an equation for the set of points that are equidistant from the points $(-8,11,3)$ and $(10,-1,-9)$.

78–79 Find the shortest distance d between the given skew lines.

78. $\mathbf{r}(t)=\langle -2+2t,-t,1+2t\rangle$ and $\mathbf{s}(u)=\langle 3-2u,5+u,4+u\rangle$

79. $\mathbf{r}(t)=\langle 4+3t,-5,2-t\rangle$ and $\mathbf{s}(u)=\langle 3+u,1-u,7+2u\rangle$

80–87 Identify the surface defined by the equation and match it to the appropriate graph (labeled A–H).

80. $2x^2+y^2+2z^2=12$

81. $z^2+2y^2=4$

82. $8z^2+2y^2-4x^2=0$

83. $(x+2)^2+2y=x^2+z$

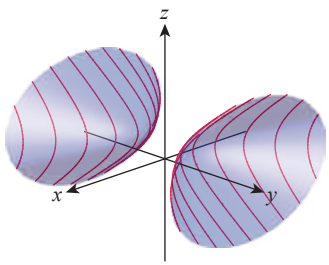
84. $8z^2-2y^2-4x=0$

85. $x^2+2z^2-4z-y^2+6=0$

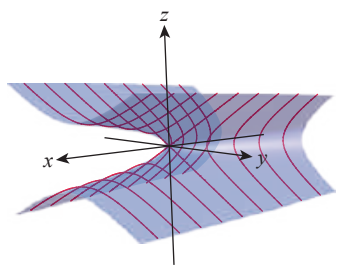
86. $3x^2+4y^2+6z=0$

87. $\dfrac{x^2}{2}+\dfrac{y^2}{4}-\dfrac{z^2}{3}=1$

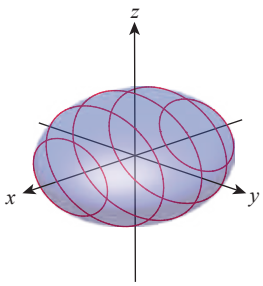
A.



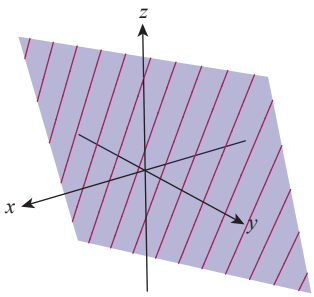
B.



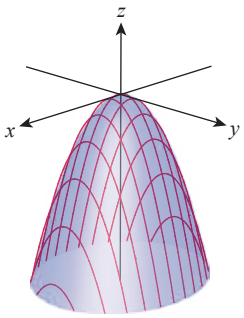
C.



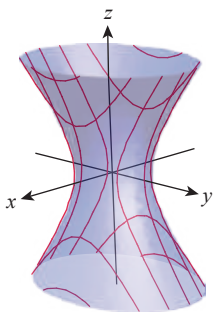
D.



E.

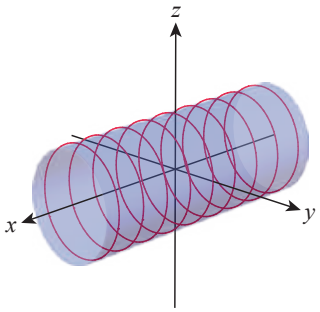


F.

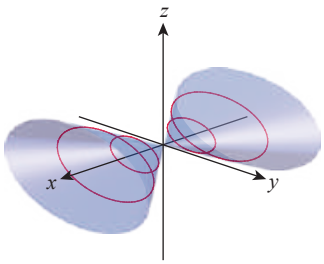


©HAWKES LEARNING

G.



H.



88–93 Use the equation to identify the quadric surface.

88. $225x^2 + 100y^2 - 36(z-2)^2 = 0$

89. $4(y+1)^2 - 16z^2 = x$

90. $\frac{x^2}{4} + x - \frac{y}{3} + z^2 + 1 = 0$

91. $x^2 - 2y^2 + 4y + 3z^2 = 12$

92. $2x^2 + 4y^2 + z^2 + 6z = 19$

93. $z^2 = 3x^2 + 2y^2 - 2z + 11$

94. An eight-pound weight is suspended from two ropes that form angles of 30° and 45°, respectively, with the horizontal direction. Find the tension forces $\mathbf{T}_1$ and $\mathbf{T}_2$.

95. A soccer player kicks the ball with an initial velocity that has a vertical component of 75 ft/s and a horizontal component of 15 ft/s. Ignoring air resistance and assuming the initial height is 0, determine the ball's velocity at time t, and sketch its position function.

96. A plane is flying 550 mph due east, so its velocity vector can be represented as $\langle 550, 0 \rangle$. It suddenly encounters a 65 mph tailwind that blows 45° east of north. Find the velocity vector and ground speed of the plane under these conditions.

97. A small plane flying at 140 mph due north encounters a 30 mph tailwind that blows from the direction of 30° west of north. At the same time, a 10 mph downdraft is affecting the plane's flight. Find the actual ground speed of the plane.

98. Find the acceleration of a 3.5 kg object if the following forces are acting on it: $\mathbf{F}_1 = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{F}_2 = \mathbf{j} + 3\mathbf{k}$, and $\mathbf{F}_3 = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ (units are in newtons).

99. A cable is pulling on a gate at an angle of 30° with a constant force of 120 pounds. Find the work done if the gate moves 18 feet to its closed position.

100. A 50-pound sled is being pulled up a 24° slope that is 80 feet long. If the rope makes a 21° angle with the surface of the slope, find **a.** the total work done and **b.** the force of tension in the rope (ignore friction and any acceleration of motion).

101. A mechanic tightens a bolt with a force of 80 newtons, applied at the end of a 20 cm wrench, at an angle of $\theta = 70°$. What is the magnitude of the torque applied to the bolt at the pivot point?

102. Find the magnitude of the force acted upon a 10 cm electric wire in a uniform magnetic field of $\mathbf{B} = 3.4$ T if it carries a current of 0.2 A and the angle between the wire and $\mathbf{B}$ is $\theta = 45°$. (**Hint:** See Exercise 61 of Section 11.4.)

103. Find the magnitude of the force that acts upon an electron moving in a uniform magnetic field of $\mathbf{B} = 0.003$ T at $\mathbf{v} = 2.8 \cdot 10^6$ m/s if the velocity vector and $\mathbf{B}$ form a 30° angle. (See Exercise 62 of Section 11.4.)

Concept Check

104–119 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

104. If vectors $\mathbf{u}$ and $\mathbf{v}$ have equal magnitude and are pointing in the same direction, but their starting points are different, then they are not equal as vectors.

105. For any nonzero vector $\mathbf{v}$ in the xy-plane there is an angle θ such that $\mathbf{v} = |\mathbf{v}|(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$.

106. Work is a vector quantity, pointing in the direction of the force doing the work.

107. If two vectors in three-dimensional space are equal, then their direction angles are equal.

108. The vector $\text{proj}_{\mathbf{v}}(\mathbf{u} \times \mathbf{v})$ is always the zero vector.

109. Torque always points in the direction of rotation.

©HAWKES LEARNING

110. If **u** and **v** are both vectors in the *xy*-plane, then it is impossible to find their cross product.

111. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u})$

112. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{w}\cdot(\mathbf{v}\times\mathbf{u})$

113. If **u**, **v**, and **w** are coplanar, then $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = 0$.

114. $\mathbf{u}\cdot[\mathbf{v}\times(\mathbf{w}+\mathbf{z})] = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) + \mathbf{u}\cdot(\mathbf{v}\times\mathbf{z})$

115. If **u** is orthogonal to both **v** and **w**, then **u** is orthogonal to $\mathbf{v}\times\mathbf{w}$.

116. If **u** is orthogonal to both **v** and **w**, then **u** is orthogonal to $\mathbf{v}\cdot\mathbf{w}$.

117. $(\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}+\mathbf{v}) = |\mathbf{u}|^2 - |\mathbf{v}|^2$

118. If **v** is a vector in the plane with normal vector $\mathbf{n} = \langle n_1, n_2, n_3\rangle$, then $\mathbf{v}\cdot\mathbf{n} = 0$.

119. If **v** is a vector in the plane that has normal vector $\mathbf{n} = \langle n_1, n_2, n_3\rangle$, then $\mathbf{v}\times\mathbf{n}$ is another vector in the said plane.

Chapter 11
Technology Exercises

120. Write a program for a computer algebra system that finds a unit vector pointing in the direction of $a\,\mathbf{u} + b\,\mathbf{v}$ for given vectors **u**, **v** and scalars *a* and *b*. Use it to find the unit vector pointing in the direction of $2\,\mathbf{u} - 3\,\mathbf{v}$ of Exercise 13.

121. Write a program for a computer algebra system that returns parametric equations for the line formed by the intersection of two given planes. (The program should accept the equations of the planes and return parametric equations for the line, displaying an appropriate message if the planes are parallel.) Use your program to check your answers for Exercises 75 and 76.

122. Write a short program for a computer algebra system that finds the distance between a point and a plane in three-dimensional space. Use it to revisit Exercises 1–6.

123. Write a program for your computer algebra system to determine the angle, in degrees, between two given three-dimensional vectors. Use it to check your answers for Exercises 27–28.

124–129. Use a graphing utility to check your answers for Exercises 88–93 by graphing the equations.

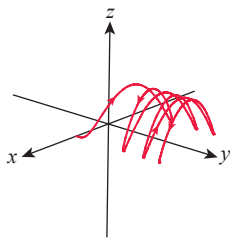
©HAWKES LEARNING

Chapter 12
Review Exercises

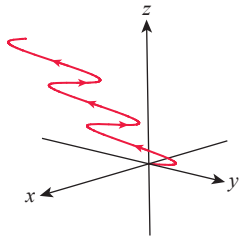
1–8 The following vector functions are defined on the interval $(0, 6\pi)$. Match each one with its graph (labeled A–H).

1. $\mathbf{r}(t) = \langle \cos t, 2\sin t, \sqrt{t} - 2 \rangle$
2. $\mathbf{r}(t) = \left\langle \frac{t}{2}\sin 3t, \frac{t}{2}\cos 3t, \frac{t}{2} \right\rangle$
3. $\mathbf{r}(t) = \langle 1.5\sin t, \sqrt{t}, 1.5\cos t \rangle$
4. $\mathbf{r}(t) = \left\langle \frac{t}{3}, 2\sin t, \frac{t}{3} \right\rangle$
5. $\mathbf{r}(t) = \langle 2\cos t, \sqrt{t}, 2\sin^2 t \rangle$
6. $\mathbf{r}(t) = \langle 0.1t\cos t, 0.2t\sin t, e^{0.1t} \rangle$
7. $\mathbf{r}(t) = \langle 2.5\sin 12t, 3.5\cos t, 3.5\sin t \rangle$
8. $\mathbf{r}(t) = \left\langle \frac{7\cos 5t}{2t}, \frac{7\sin 5t}{2t}, \frac{t}{5} \right\rangle$

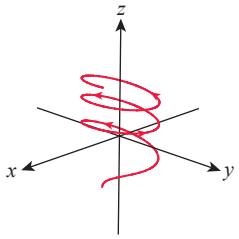
A.



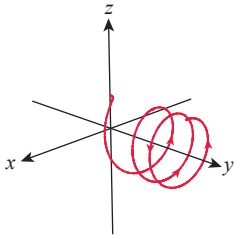
B.



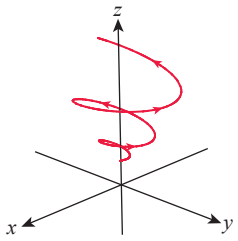
C.



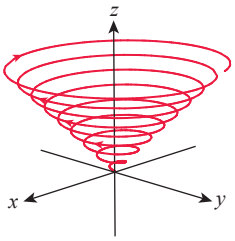
D.



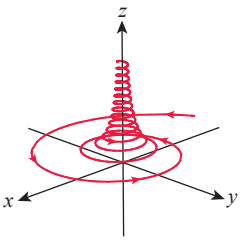
E.



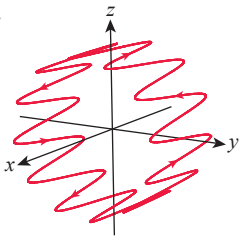
F.



G.



H.



9–10 Describe the intersection of the surfaces as a vector function. (Use the suggested parameter.)

9. The elliptic cylinder $5x^2 + 2y^2 = 10$ and the hyperbolic paraboloid $5x^2 - 2y^2 = z$ $(x = \sqrt{2}\cos t)$
10. The elliptic paraboloid $x^2 + 2y^2 = z$ and the parabolic cylinder $y = x^2$ $(x = t)$

11–12 Determine whether the indicated limit exists. If so, find it.

11. $\lim\limits_{t \to 1} \left\langle \ln t, \sec(t-1), \frac{t-1}{t^2-1} \right\rangle$
12. $\lim\limits_{t \to 0} \left(\frac{t}{t^2+1}\mathbf{i} + \arctan t\,\mathbf{j} + \frac{t^2}{|t|}\mathbf{k} \right)$

13–14 Find any discontinuities of the given vector function.

13. $\mathbf{r}(t) = \left\langle e^{t+1}, \frac{t+1}{t^2+1}, \frac{t+1}{t^2-1} \right\rangle$
14. $\mathbf{r}(t) = |t-1|\mathbf{i} + \tan t\,\mathbf{j} - \arctan t\,\mathbf{k}$

15–16 Find $\mathbf{r}'(t)$.

15. $\mathbf{r}(t) = \langle \sec t, \arcsin\sqrt{t}, \tan^2 t \rangle$
16. $\mathbf{r}(t) = \cos 2t\,\mathbf{i} - \frac{t+2}{\sqrt{t-1}}\mathbf{j} + \ln(t^3 - 3)\mathbf{k}$

17–20 Find the vector form of an equation for the line tangent to the curve at the specified value of t.

17. $\mathbf{r}(t) = \langle 2\sin 4t, \cos t, 1 - t^2 \rangle; \quad t = 0$
18. $\mathbf{r}(t) = \langle \tan^{-1} 2t, t^{3/2}, 3t - 2 \rangle; \quad t = \frac{1}{2}$

©HAWKES LEARNING

19. $\mathbf{r}(t) = t\,\mathbf{i} - \ln\left(2t^2+1\right)\mathbf{j} - \sqrt{3t}\,\mathbf{k}; \quad t = 3$

20. $\mathbf{r}(t) = e^{t^2+2}\,\mathbf{i} + e^{\sqrt{t+1}}\,\mathbf{j} + e^{\cos t}\,\mathbf{k}; \quad t = 0$

21–24 Perform the integration—definite or indefinite, as indicated.

21. $\displaystyle\int \left\langle e^{t+1}, \frac{t}{\sqrt{t^2+2}}, \sin 2t \right\rangle dt$

22. $\displaystyle\int \left(\sqrt{t}\,\mathbf{i} - \frac{1}{\sqrt{1-t^2}}\,\mathbf{j} - t\sqrt{1-t^2}\,\mathbf{k} \right) dt$

23. $\displaystyle\int_3^6 \left\langle \sqrt{t-2}, t, 3 \right\rangle dt$

24. $\displaystyle\int_{1/4}^1 \left(\ln\sqrt{t}\,\mathbf{i} + e^{4t}\,\mathbf{j} - \frac{1}{t^{3/2}}\,\mathbf{k} \right) dt$

25–26 Find the arc length of the curve over the given interval.

25. $\mathbf{r}(t) = \left\langle -3t^2, 6t, t^3 \right\rangle; \quad [0,2]$

26. $\mathbf{r}(t) = \left\langle 2\ln t, \frac{t^2}{4}, \sqrt{2}t \right\rangle; \quad \left[1, e^2\right]$

27–28 Reparametrize the given curve with respect to arc length.

27. $\mathbf{r}(t) = \langle 2-t, 1+4t, 3+2t \rangle$

28. $\mathbf{r}(t) = \langle 5\sin t, 12t, 5\cos t \rangle$

29–32 Find the unit tangent vector for the curve over the given interval.

29. $\mathbf{r}(t) = \left\langle \frac{t}{2}, \ln(\cos t), \frac{\sqrt{3}}{2}t \right\rangle; \quad 0 \le t < \frac{\pi}{2}$

30. $\mathbf{r}(t) = \left\langle t^2, \cos 2t, \sin 2t \right\rangle; \quad t \ge 0$

31. $\mathbf{r}(t) = \left\langle \ln t, t^2, t \right\rangle; \quad t > 0$

32. $\mathbf{r}(t) = \left\langle \arctan t, \ln\left(t^2+1\right), t \right\rangle; \quad t > 0$

33–36 Find the unit normal and binormal vectors for the curve.

33. $\mathbf{r}(t) = \langle 2t, 2\cos t, 2\sin t \rangle$

34. $\mathbf{r}(t) = \langle \sin 3t, \cos 3t, 3t \rangle$

35. $\mathbf{r}(t) = \left\langle e^t, e^{-t}, \sqrt{2}t \right\rangle$

36. $\mathbf{r}(t) = \left\langle 3t, -t, t^2 \right\rangle$

37–40 Using your work in Exercises 33–36, determine equations for the osculating, normal, and rectifying planes associated with the curve at the indicated point.

37. $\mathbf{r}(t) = \langle 2t, 2\cos t, 2\sin t \rangle; \quad t = 0$

38. $\mathbf{r}(t) = \langle \sin 3t, \cos 3t, 3t \rangle; \quad t = 0$

39. $\mathbf{r}(t) = \left\langle e^t, e^{-t}, \sqrt{2}t \right\rangle; \quad t = 1$

40. $\mathbf{r}(t) = \left\langle 3t, -t, t^2 \right\rangle; \quad t = 2$

41–44 Use Exercises 37–40 to find a parametric description of the surface.

41. The ribbon of width $\frac{1}{2}$ centered on the curve $\mathbf{r}(t) = \langle 2t, 2\cos t, 2\sin t \rangle$

42. The ribbon of width 1 centered on the curve $\mathbf{r}(t) = \langle \sin 3t, \cos 3t, 3t \rangle$

43. The circular tube of radius 2 centered on the curve $\mathbf{r}(t) = \left\langle e^t, e^{-t}, \sqrt{2}t \right\rangle$

44. The circular tube of radius 1 centered on the curve $\mathbf{r}(t) = \left\langle 3t, -t, t^2 \right\rangle$

45–48 Find the curvature and torsion functions for the given curve. (Suppose $t > 0$.)

45. $\mathbf{r}(t) = \left\langle \cos t, \sin t, \frac{t}{\sqrt{2}} \right\rangle$

46. $\mathbf{r}(t) = \left\langle \frac{1}{t^2}, \frac{1}{t}, 1 \right\rangle$

47. $\mathbf{r}(t) = \left\langle t, \frac{1}{t}, \ln t \right\rangle$

48. $\mathbf{r}(t) = \left\langle t^2, t, \frac{1}{t} \right\rangle$

49. Find a formula for the curvature of the ellipse $\mathbf{r}(t) = \langle a\cos t, b\sin t, 0 \rangle$ $(a > b)$ at the endpoints of its **a.** major axis and **b.** minor axis.

50. Use Exercise 50 of Section 12.3 to provide a second solution for Exercise 49.

51–52 Use Exercise 50 of Section 12.3 to find the point(s) of maximum curvature for the curve.

51. $f(x) = \frac{1}{x}$ **52.** $f(x) = x^4$

53–54 Find the osculating circle of the graph of the equation at the indicated point. (See Exercises 60–68 in Section 12.3.)

53. $y = x^3; \quad x = -1$ **54.** $y = \tan x; \quad x = \frac{\pi}{4}$

55–58 Find the tangential and normal components of acceleration for the given position function.

55. $\mathbf{r}(t) = \langle 4t, \cos 2t, \sin 2t \rangle$

©HAWKES LEARNING

56. $\mathbf{r}(t) = \left\langle \frac{1}{t}, 1, t^2 \right\rangle; \quad t > 0$

57. $\mathbf{r}(t) = \langle t\cos t, t\sin t, 1 \rangle; \quad 0 \le t \le \frac{\pi}{4}$

58. $\mathbf{r}(t) = \langle 1, 1 - \sin t, t - \cos t \rangle$

59–60 Describe in terms of $\mathbf{u}$ and $\mathbf{u}_\perp$ the velocity and acceleration vectors of a particle with the given position function.

59. $\mathbf{r}(t) = \left\langle t\cos\frac{t}{2}, t\sin\frac{t}{2} \right\rangle$

60. $\mathbf{r}(t) = \langle t^2\cos 2t, t^2\sin 2t \rangle$

61. A projectile is launched from the ground with an initial speed of 129.33 feet per second at an angle of elevation of 60° from horizontal. Use the vector functions $\mathbf{v}(t)$ and $\mathbf{r}(t)$ to find the maximum altitude reached by the projectile as well as its range. (Ignore air resistance.)

62. The acceleration function of a moving object in $\mathbb{R}^3$ is $\mathbf{a}(t) = \langle t/2, 0, -t \rangle$. Find the velocity and position functions of this object if it starts at the point $\mathbf{r}(0) = \langle 3, -1, 5 \rangle$ with initial velocity $\mathbf{v}(0) = \langle 0, 2, 1 \rangle$.

63. Suppose a shell is shot from an artillery cannon with a muzzle velocity of 3000 ft/s and leaves the cannon 4 ft above the ground, at a 30° angle of elevation. What is the range of the cannon under these conditions? (By "range," we mean the horizontal distance between the base of the launching pad and the point of impact. Assume level terrain and ignore air resistance.)

64. A projectile is launched from the point $(0, 8)$ of a coordinate system where unit length is measured in feet. The launching speed of the projectile is 192 feet per second and it is launched in the positive direction at an angle of elevation of 30°.

a. Find a vector function that models the projectile's path.

b. Find the maximum height attained by the projectile, its range, and the speed of impact.

(As customary, ignore air resistance.)

65. The banking of each of the curves on the historical oval track in Monza, Italy, is 30°, with a radius of 320 m. When used for racing, those banked curves were navigated by daring race drivers at speeds in excess of 150 mph (240 km/h). Find the vertical component of acceleration of a race car in the curve under the above conditions (assume a constant speed of exactly 150 mph).

66.* Using the result of Exercise 89 of Section 12.3, find the speed that the banking is able to support on the track of the previous exercise. In reality, much higher speeds were achieved in the curves of the track. Explain how that was possible.

67. A dive bomber, approaching its target at an angle of depression of 60° and at a speed of 600 kilometers per hour, releases its payload at an altitude of 500 meters. Use the units of meters and seconds to find **a.** a vector function modeling the path of the bomb, **b.** the horizontal distance traveled by the bomb after its release, and **c.** its speed of impact. (Convert your answer to part c. to kilometers per hour. You can suppose that the flight is taking place in the positive x-direction. Ignore air resistance.)

68. If a particle's acceleration along a path $\mathbf{r}(t)$ at time $t = t_0$ is $\mathbf{a}(t_0) = \langle 1, -2, -1 \rangle$, and its velocity $\mathbf{v}(t_0) = \langle 9, 21, -12 \rangle$, decide whether the particle's speed is increasing or decreasing at $t = t_0$.

69. Suppose a particle is moving along a path $\mathbf{r}(t)$ which is on the sphere of radius R, centered at the origin. Show that the velocity vector of the particle is tangent to the sphere at all times.

70.* Show that the moving particle of Exercise 69 satisfies $\mathbf{r}(t) \cdot \mathbf{r}''(t) = -|\mathbf{v}(t)|^2$.

71.* Suppose that an object in the three-dimensional coordinate system is moving along a path $\mathbf{r}(t)$ so that it is acted upon by a single, so-called central force; that is, if $\mathbf{F}(t)$ denotes the acting resultant force on the object, then $\mathbf{F}(t) = k\,\mathbf{r}(t)$, for some constant $k > 0$. Show that in this case, the motion is planar, that is, $\mathbf{r}(t)$ is contained in a plane.

72.* Show that if the vector function $\mathbf{r}(t)$ is *bounded*, that is, if $|\mathbf{r}(t)| \le M$ for some positive constant M, then

$$\left| \int_a^b \mathbf{r}(t)\,dt \right| \le M(b - a).$$

73. Find the curvature at $t = b$ for the Cornu spiral of Section 12.2, Exercise 44. What can you say about the relationship between arc length and curvature for this particular curve?

74. Show that the graph of the polar equation $r^2 = \cos^2\theta - \sin^2\theta$ has the property that its curvature is directly proportional to r.

75. Find the orbital speed of a satellite that is in a circular orbit, 300 kilometers above Earth's surface. Express your answer in kilometers per hour. (**Hint:** See Exercise 8 of Section 12.4.)

©HAWKES LEARNING

76. Ganymede, Jupiter's largest moon (discovered by Galileo Galilei in 1610), has an orbital period of approximately 7.15455296 earth days; its *periapsis* (distance from Jupiter upon closest approach) is approximately 1,069,008 kilometers, while its *apoapsis* (its greatest distance from Jupiter) is about 1,071,792 km. Use these data to estimate the mass of Jupiter. (**Hint:** See Exercise 10 of Section 12.4.)

77. Given that a day on Jupiter lasts a mere 9 hours and 55.5 minutes, use Exercise 76 along with Kepler's Third Law to estimate the necessary height above Jupiter's surface for a stationary satellite. (**Hint:** See Exercise 12 in Section 12.4. Approximate Jupiter's radius by 69,911 kilometers.)

78. The aphelion of Jupiter's orbit is 5.458104 AU (astronomical units), while its perihelion is 4.950429 AU. Use these along with Earth's orbital data to estimate the period of Jupiter in Earth years.

79. Use Exercise 78 to find the speeds of Jupiter when it is **a.** closest to and **b.** farthest from the sun. Express your answer in kilometers per second, then convert it to miles per hour. (**Hint:** See Exercise 23 in Section 12.4.)

Concept Check

80–89 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

80. $d\mathbf{T}/ds$ is perpendicular to $\mathbf{T}$.

81. If the graph of $\mathbf{r}(t)=\langle f(t),g(t),h(t)\rangle$ is a straight line, then f, g, and h are linear polynomials.

82. The torsion of a curve satisfies the formula

$$\tau=-\left(\frac{1}{|\mathbf{v}(t)|}\right)\frac{d\mathbf{B}}{dt}\cdot\mathbf{N}.$$

83. The tangential component of acceleration satisfies $a_T=\mathbf{T}\cdot\mathbf{a}$.

84. The normal component of acceleration satisfies $a_N=\mathbf{T}\times\mathbf{a}$.

85. If $f''(x)=0$, then the curvature of the graph of $y=f(x)$ is constant.

86. If $\mathbf{r}(t)$ is a space curve with $\kappa=0$, then $\mathbf{r}(t)$ can only "bend" in the direction of the unit binormal vector $\mathbf{B}$.

87. The magnitudes of the curvature and torsion of a space curve depend on the parametrization (i.e., "how fast the curve is being traced out,") unless the curve is parametrized with respect to arc length.

88. The maximum curvature of an ellipse occurs at the endpoints of its major axis.

89. $\dfrac{d}{dt}\left[\mathbf{r}(t)\times\mathbf{r}'(t)\right]=\mathbf{r}(t)\times\mathbf{r}''(t)$

Chapter 12
Technology Exercises

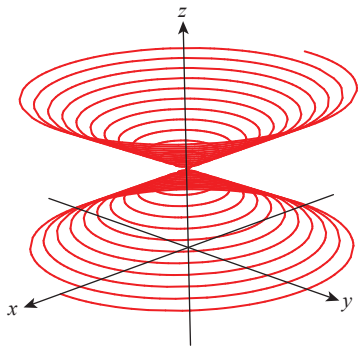
90. **a.** Use a graphing utility to graph and explore the following curve.

$$\mathbf{r}(t)=\left\langle\frac{\sin 15t}{2},(3+\cos 15t)\sin t,(3+\cos 15t)\cos t\right\rangle$$

(Such curves are nicknamed "slinky curves." Can you see why?)

b. It is possible to create "slinky curves" where the spiral is wound around a helix. Find a formula for such a curve. (**Hint:** A good starting point is appropriately modifying the formula in part a. Answers will vary.)

91. Assuming $|x|,|y|\le 10$, z is nonnegative, and $0\le t\le 10\pi$, use a graphing utility to find a formula for a vector function whose graph is as close as possible to the one displayed in the figure below. (Answers may vary.)



92. Write a program for a computer algebra system or programmable calculator that returns the equations for the osculating, normal, and rectifying planes associated with a given space curve at a specified point. Use your program to check the answers you have given to Exercises 37–40.

©HAWKES LEARNING

Chapter 13
Review Exercises

1–6 Match the function with its graph (labeled A–F).

1. $f(x,y)=\dfrac{-4}{2x^2+y^2+1}$

2. $f(x,y)=5\sqrt{1-\dfrac{x^2}{8}-\dfrac{y^2}{4}}$

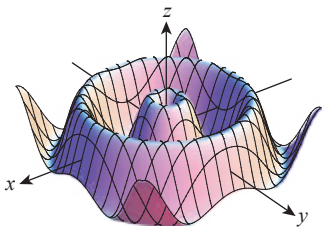
3. $f(x,y)=\dfrac{(x-2y)^2}{5}$

4. $f(x,y)=2.5\sin\sqrt{x^2+y^2}$

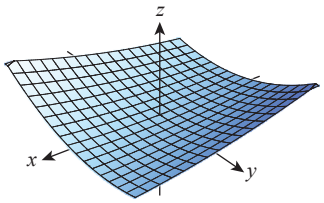
5. $f(x,y)=x^2-y^2$

6. $f(x,y)=\cos(2x-y)$

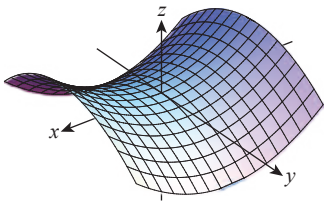
A.



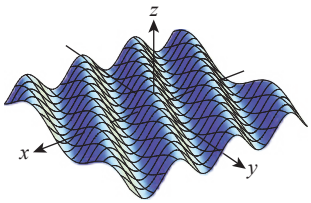
B.



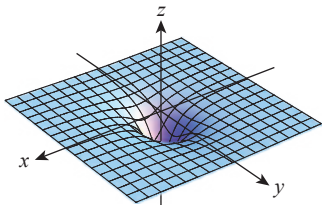
C.



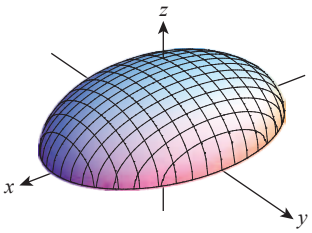
D.



E.

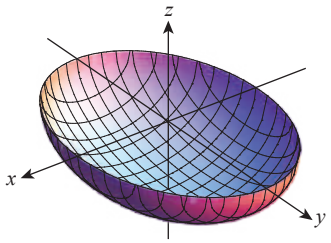


F.

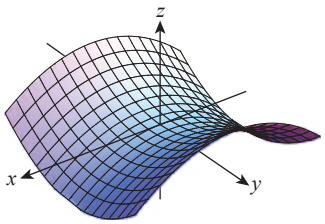


7–12 Match the graph with its contour map (labeled A–F).

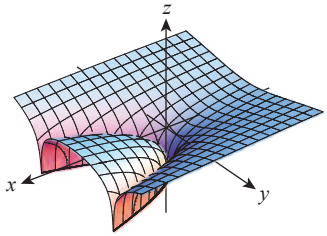
7.



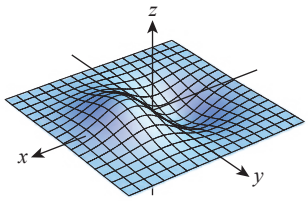
8.



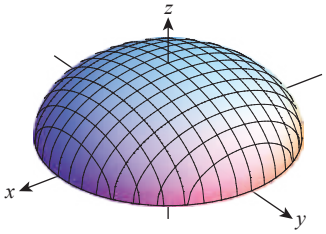
9.



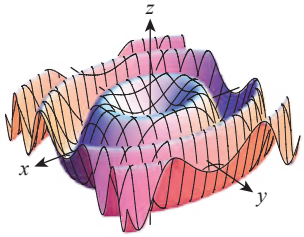
10.



11.

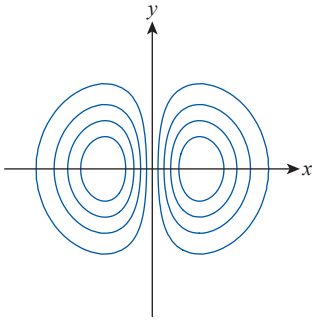


12.

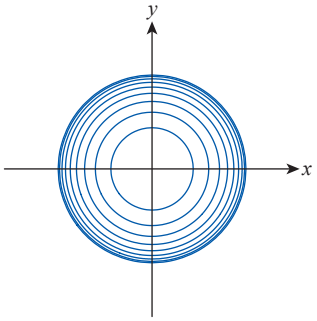


©HAWKES LEARNING

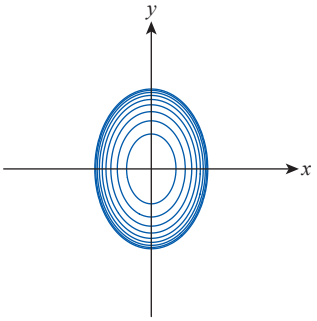
A.



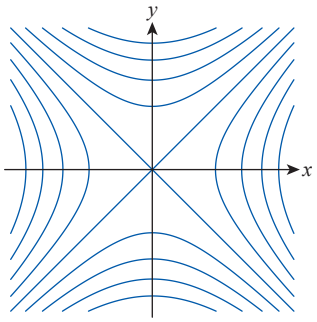
B.



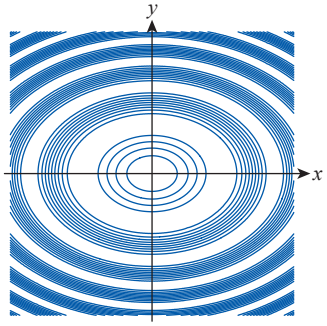
C.



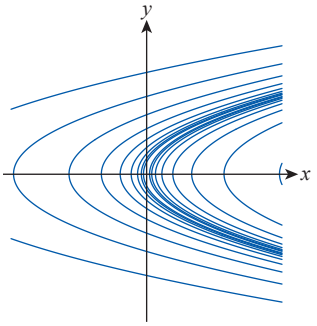
D.



E.



F.



13–18 Determine whether the limit exists. If so, find it.

13. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3 - xy^2}{x+y}$

14. $\lim\limits_{(x,y)\to(1,1)} \dfrac{\ln(xy)}{\sin(\ln(xy))}$

15. $\lim\limits_{(x,y)\to(0,1)} \dfrac{\sqrt{2x+y} - \sqrt{x+y}}{2x}$

16. $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\sin\dfrac{\pi}{x^2+y^2}$

17. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin y}{xy^2}$

18. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2 - y^4}{x^2+y^4}$

19. Find two different paths in the xy-plane toward $(0,0)$ along which the limiting values of the function $f(x,y) = \dfrac{2xy^2}{x^3+y^3}$ are different. (Answers will vary.)

20. Use an ε-δ argument to show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^2+y^2} = 0$.

21–24 Find all discontinuities (if any) of the given function. Classify them as removable or nonremovable.

21. $f(x,y) = \ln(x^4+y^4)$

22. $g(x,y) = (6 - 2x^2 - 3y^2)^{-1}$

23. $h(x,y) = \arctan\dfrac{2}{(x^2-1)(y^2-4)}$

24. $k(x,y) = \dfrac{1-\cos(x^2+y^2)}{x^2+y^2}$

25–26 Determine z_x and z_y at the indicated point; then find the equations for the corresponding tangent lines that are parallel to, respectively, the xz-plane and the yz-plane.

25. $z = x^3y^2 + x^2y; \quad (-1,1)$

26. $z = (x^2y - 2)(y + 4x^2); \quad (1,2)$

27–28 Find all first-order partial derivatives of the function.

27. $f(x,y,z) = y^2 \sin^{-1}(z\sqrt{x})$

28. $g(x,y,z) = \dfrac{ze^{xz}}{\sqrt{x+y^2}}$

29–30 Use implicit differentiation to determine z_x and z_y.

29. $2y^3 - x^2z - xyz^3 = 0$

30. $(z-y)^3 + 2\ln(xy) = z$

31–32 Verify the equality of the mixed partials f_{xy} and f_{yx}.

31. $f(x,y) = (x^3+y)^2 \ln(xy)$

32. $f(x,y) = \tan^{-1}\sqrt{x^2+y^2}$

33–34 Verify that the third-order mixed partials f_{xyz}, f_{yzx}, and f_{zxy} are equal.

33. $f(x,y,z) = 2x^2z^2 + xyz - 4zy^3$

34. $f(x,y,z) = xy\ln(z-2x)^2$

©HAWKES LEARNING

35–36 Use the definition of the partial derivative to find $\partial f/\partial x$ and $\partial f/\partial y$ for the given function.

35. $f(x,y)=\sqrt{x}\,y^2$ **36.** $f(x,y)=\dfrac{x^3+y^2}{x}$

37–38 If possible, find a function $f(x,y)$ that has the indicated partial derivatives. If such a function doesn't exist, explain why.

37. $f_x(x,y)=y^3+\dfrac{1}{x},\quad f_y(x,y)=3xy^2$

38. $f_x(x,y)=yx^2-\cos y,\quad f_y(x,y)=\dfrac{x^3}{3}-x\sin y$

39–40 Determine whether the function is differentiable at the origin. Give a reason for your answer.

39. $f(x,y)=x\sin^2 y+xy^2$

40. $f(x,y)=\sqrt{x^2+y^2}\arctan(xy)$

41. Show that the piecewise–defined function

$$f(x,y)=\begin{cases}\dfrac{2xy}{\sqrt{x^6+y^6}} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$

is not differentiable at the origin, even though both $f_x(0,0)$ and $f_y(0,0)$ exist.

42. Use the definition to show that the function $f(x,y)=2x-3y^2$ is differentiable.

43–46 Use the Chain Rule to determine dy/dx.

43. $y=2uv^3;\quad u=x^2-2,\quad v=3x$

44. $y=u^2e^{u^2v};\quad u=\sqrt{x},\quad v=x^2$

45. $y=u^2v\cos w;\quad u=\sqrt{x},\quad v=x^3,\quad w=x^2$

46. $y=uv^2+\sqrt{w};\quad u=\ln x,\quad v=2x,\quad w=x^2$

47–49 Use the Chain Rule to find $f'(t)$ at the given point.

47. $f(x,y)=(x-2y)^2$ along the curve $x=t\sin t$, $y=2t\cos t$, at $t=\pi/2$

48. $f(x,y)=\ln(xy)$ along the curve $x=t^2$, $y=t-1$, at $t=2$

49. $f(x,y,z)=xyz$ along the space curve $x=t\cos t$, $y=t\sin t$, $z=2t$, at $t=\pi$

50–54 Use the Chain Rule to determine the partial derivatives of the given function f. (Answers may be left in terms of the intermediate and independent variables.)

50. $f(u,v)=3u^2-v\sqrt{u};\quad u(x,y)=x^2,$
$v(x,y)=xy^2$

51. $f(u,v)=\ln(u^2+v^2);\quad u(x,y)=x\sqrt{y},$
$v(x,y)=x^2y$

52. $f(u,v,w)=u\cos v-\sin w;$
$u(x,y)=2x+y,\quad v(x,y)=y-x^2,$
$w(x,y)=xy$

53. $f(u,v,w)=u(v-w^2);\quad u(x,y)=2x^3,$
$v(x,y)=x-y^2,\quad w(x,y)=3x+y$

54. $f(u,v,w)=uve^w;\quad u(x,y,z)=yz^2,$
$v(x,y,z)=(x+y)^2,\quad w(x,y,z)=x-4z$

55–56 Find dy/dx where y is given implicitly by the given equation. (**Hint:** Rewrite the equation in the form $F(x,y)=0$, and use the appropriate formula from Section 13.4.)

55. $x^2(2-xy)=xy^3$ **56.** $(x^3-y^2)^2=4x^2y$

57–58 Find z_x and z_y where z is defined implicitly by the given equation. (**Hint:** Use the appropriate formula from Section 13.4.)

57. $xy^2+yz^3=x^3z^2$ **58.** $ze^x+2x\sin y+xyz^3=1$

59. Suppose $z=f(x,y)$ has continuous second partials, $x=r\cos\theta$ and $y=r\sin\theta$. Use the Chain Rule to prove the following.

$$\frac{\partial^2 f}{\partial r^2}=\frac{\partial^2 f}{\partial x^2}\cos^2\theta+2\frac{\partial^2 f}{\partial x\partial y}\sin\theta\cos\theta+\frac{\partial^2 f}{\partial y^2}\sin^2\theta$$

60. The height of a right rectangular pyramid is increasing at a rate of 0.2 in./min, while the edge of its square base is decreasing at 0.1 in./min. Find the rate of change of its volume and surface area when the edge of the base is 15 in. long and the height is 20 in.

61–64 Find the gradient of the function at the indicated point.

61. $f(x,y)=3x^2y-2\sqrt{x};\quad (1,3)$

62. $f(x,y)=\dfrac{\cos(x-y)}{y^2};\quad (1,1)$

63. $f(x,y,z)=x^2e^{yz};\quad (-1,0,2)$

64. $f(x,y,z)=\dfrac{\tan^{-1}(x+y)}{z};\quad (3,-2,4)$

©HAWKES LEARNING

65–66 Use the definition to find the directional derivative of f at the given point in the indicated direction.

65. $f(x,y,z)=\dfrac{xy^2}{z};\quad (2,4,1);\quad \mathbf{v}=\langle 3,1,2\rangle$

66. $f(x,y)=x+2\sqrt{y};\quad (1,1);\quad \mathbf{v}=\langle 1,2\rangle$

67–70 Find the derivative of the function at the given point in the indicated direction.

67. $f(x,y)=\ln\sqrt{x^2+y};\quad (1,0);\quad \mathbf{v}=\langle -2,14\rangle$

68. $f(x,y)=x^2+3xy-2y^2;\quad (1,2);$ in the direction toward $(-2,3)$

69. $f(x,y,z)=z\sqrt{\dfrac{xz+y}{x}};\quad (1,-1,2);$ in the direction toward $(4,-6,3)$

70. $f(x,y,z)=xy^2z^3;\quad (1,1,1);\quad \mathbf{v}=\langle 3,2,1\rangle$

71–72 Find the direction and value of the greatest rate of increase for the function at the given point.

71. $f(x,y,z)=\dfrac{y+z^2}{x+1};\quad (0,1,-1)$

72. $f(x,y)=4x^2+xy+\dfrac{x}{y};\quad (0,1)$

73. Suppose that the temperature in three-dimensional space around a heat source that is located at the origin can be described by the function

$$T(x,y,z)=450e^{-\left(2x^2+y^2+z^2\right)/2}.$$

a. Find the direction and magnitude of the temperature's maximum rate of decrease at the point $(1,1,1)$.

b. Find the rate of change of temperature at the point $(1,1,1)$ in the direction of the point $(1,2,3)$.

74–75 Find the direction of no change for the function at the given point.

74. $f(x,y)=\dfrac{2xy}{x^2-y^2};\quad (2,-1)$

75. $f(x,y)=x\ln\left(y-x^2\right);\quad (1,2)$

76–79 In Exercises 76–77, use the technique seen in this chapter to find an equation for the tangent line to the graph of the equation at the indicated point. In Exercises 78–79, generalize to obtain the respective tangent plane.

76. $2x^2+xy-3y^2+13=0;\quad (2,3)$

77. $x^2(y+2)=4x^3;\quad (1,2)$

78. $2y^2-x^2=2z^2;\quad (4,3,1)$

79. $4z-3x^2=3y^2;\quad (4,2,15)$

80–81 Find a function with the given gradient. If it is not possible, explain why. (Answers will vary.)

80. $\nabla f=\left\langle \dfrac{1}{\sqrt{x}}+y^2,2xy\right\rangle$ **81.** $\nabla f=\langle xy+1,x^2\rangle$

82–85 Determine the tangent plane and the normal line at the indicated point of the given surface.

82. $5x^2-y^2-2z=0;\quad (1,3,-2)$

83. $(2x-1)\left(5x+y^2\right)=z^2;\quad (1,2,3)$

84. $\dfrac{x}{\sqrt{y^2+x}}-z=0;\quad \left(3,1,\dfrac{3}{2}\right)$

85. $\ln\left(x^2+y^2+z^2\right)=0;\quad \left(\dfrac{\sqrt{3}}{2},\dfrac{1}{2},0\right)$

86–87 Find parametric equations for the line tangent to the intersection of the two surfaces at the indicated point.

86. $x^2+4y^2=6-z;\quad 2z=x^2+y^2;\quad (1,1,1)$

87. $x^3-2y^3+xy^2-z^2=5xy;\quad 2y^2-x^2=z;\quad (-1,1,1)$

88–89 Find **a.** the differential df at an arbitrary point (x,y) and **b.** the linearization of f at the indicated point.

88. $f(x,y)=(x-1)^3+y^2-x;\quad (2,1)$

89. $f(x,y)=\sin\left(x^2-y^2\right);\quad (-1,1)$

90–91 Find the linear approximation of the function at the indicated point.

90. $f(x,y,z)=x\tan^{-1}(yz);\quad (2,1,1)$

91. $f(w,x,y,z)=\dfrac{x^2y-wz^2}{xyw};\quad (1,2,-1,4)$

92. Suppose the base lengths of the rectangular pyramid of Exercise 60 were measured within a relative error of 2.5%, and its height within an error of 2%. Use differentials to find an upper bound on the relative error with which the pyramid's volume is known.

93–96 Find any extrema of the given function. Use the Second Partial Derivative Test (if necessary) to classify critical points.

93. $f(x,y)=x^2+y^2-4x+2y+6$

©HAWKES LEARNING

94. $f(x,y) = 4y - y^2 - 8x - x^2$

95. $f(x,y) = x^3 + y^3 - 4y^2 - x^2$

96. $f(x,y) = 2y^4 - y^2 + x^2 + x$

97–98 Find the absolute extrema of the given function on the indicated region.

97. $f(x,y) = x^2 - xy + 1$;
$D = \{(x,y) \mid -2 \le x \le 2, 0 \le y \le 2\}$

98. $f(x,y) = 4xy - 3x - y + 2$; D: The triangle with vertices $(0,0)$, $(5,0)$, and $(0,4)$

99. Determine m and b such that the sum of the squares of vertical distances from the line $y = mx + b$ to the points $(1,2)$, $(4,2)$, $(6,6)$, and $(8,4)$ is minimal.

100. Find the minimum distance between the point $(2,-1,1)$ and the plane $z = 4x - y - 2$.

101. Find the minimum distance from the origin to the surface $z = 2xy - 1$.

102. A rectangular box is placed in the three-dimensional coordinate system with one vertex at the origin and the three edges containing it lying along the positive coordinate axes. If the opposite vertex lies in the plane $x + 4y + 2z = 6$, what is the greatest possible volume for such a box?

103. Suppose the sum of three positive numbers is 150, what is the greatest possible value for the sum of their squares?

104–107 Find any extreme values of the function subject to the given constraint.

104. $f(x,y) = x^2 y$; constraint: $\dfrac{x^2}{2} + \dfrac{y^2}{4} = 1$

105. $f(x,y) = x^2 + y^2$; constraint: $xy = 4$

106. $f(x,y) = x^2 - y^2$; constraint: $3x + 4y = 2$

107. $f(x,y) = x^2 + xy - y^2$; constraint: $x = y^2$

108–109 Use Lagrange multipliers to find the coordinates of the point Q on the given surface that is closest to the point P.

108. $3z - 2x + y + 6 = 0$; $P(1,0,1)$

109. $z = x + y^2$; $P(1,0,0)$

110–111 Find the absolute extreme value(s) of the function subject to the given constraints.

110. $f(x,y,z) = x - 2y + z$;
constraints: $x^2 + z^2 = 1$ and $x + y + z = 2$

111. $f(x,y,z) = 2(x^2 + y^2 + z^2)$;
constraints: $x + y + z = 2$ and $2x + z = 1$

112. A small company profits \$10 per unit on products A and C, while the profit on product B is \$20 per unit. Let x, y, and z denote hundreds of units produced from each product.

a. Find the profit function $P(x,y,z)$.

b. Assuming that the manufacturing process is under the constraint $x^2 + 10y^2 + 2z^2 \le 60$, find the maximum profit for the company under these conditions.

113. Find the points closest to and farthest from the origin on the rotated ellipse $2x^2 + xy + 2y^2 = 4$.

114. Find the highest and lowest points on the curve of intersection of the two surfaces $x^2 + 2y^2 + z^2 = 10$ and $x + 4y + z = 8$.

Concept Check

115–126 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

115. Any vertical line (i.e., one that is parallel to the z-axis) can intersect the graph of a two-variable function $f(x,y)$ at most once.

116. If a function has three or more independent variables, then it has no graph.

117. If $f(x,y)$ is continuous at (a,b) and $f(a,b) = L$, then $\lim\limits_{(x,y)\to(a,b)} f(x,y) = L$.

118. If $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$ and $a \in \mathbb{R}$, then $\lim\limits_{x\to 0} f(x,ax) = 0$.

119. If $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$, then for any $\mathbb{R} > 0$ there exists $R \in \mathbb{R}$ such that $|f(x,y)| < \varepsilon$ whenever $0 < x^2 + y^2 < R$.

120. If $\lim\limits_{(x,y)\to(a,b)} f(x,y) = L$, then $f(x,y) \to L$ as (x,y) approaches (a,b) along any curve in the xy-plane.

121. If $f(x,y)$ is continuous at (a,b), then both $f_x(a,b)$ and $f_y(a,b)$ exist.

©HAWKES LEARNING

122. If $f(x,y)=\dfrac{g(x)}{h(y)}$, then

$$f_x(x,y)+f_y(x,y)=\frac{g'(x)h(y)-g(x)h'(y)}{h^2(y)}.$$

123. If $f_x(a,b)$ and $f_y(a,b)$ both exist, then $f(x,y)$ is differentiable at (a,b).

124. If $f_x(x,y)$ and $f_y(x,y)$ are continuous at (a,b), then $f(x,y)$ is differentiable at (a,b).

125. If f has a relative maximum or minimum at (a,b), then all directional derivatives $D_{\mathbf{u}}f(a,b)$ are 0.

126. If f has a relative maximum or minimum at (a,b), then the tangent plane to its graph at (a,b) is horizontal.

Chapter 13
Technology Exercises

127–130. Write a program on your computer algebra system or programmable calculator that accepts the equation of a surface and a point on the surface, and generates the corresponding tangent plane and normal line to the surface. Test your program by checking the answers you obtained for Exercises 82–85.

131–134. Write a program on your computer algebra system or programmable calculator that accepts a two-variable function $f(x,y)$ and then locates and classifies its critical points by utilizing the Second Partial Derivative Test. Use your program to check the answers you obtained for Exercises 93–96.

135. Recall the Ideal Gas Law, $pV = nRT$ from Section 3.4, Exercise 97. Suppose $\frac{1}{2}$ mole of an ideal gas is present in an expandable container. Assuming the volume of the container can expand up to 1 m^3, and that it can withstand a maximum pressure of 120 kPa along with temperatures of up to 500 K, use a computer algebra system to generate a contour map that illustrates how the volume depends on pressure when temperature is kept at constant levels. (Note that the resulting contour lines are called *isotherms*. **Hint:** Start by expressing T as a function of p and V from the Ideal Gas Law.)

136. Mimic Exercise 135 to generate a few contour lines of the pressure function under the same conditions (these curves are called *isobars*).

©HAWKES LEARNING

Chapter 14
Review Exercises

1. Estimate $\iint_R x^2 y\,dA$ on the square $R=[0,3]\times[0,3]$ by using the Riemann sum approximation corresponding to $n=m=3$, with each sample point $\left(x_{ij}^*, y_{ij}^*\right)$ chosen to be the center point of the respective subsquare, $1\le i\le 3$ and $1\le j\le 3$. Then use repeated integration to find the true value of the integral.

2. Use symmetry and the properties of double integrals to evaluate $\iint_R (2-x)\,dA$ over the rectangle $R=\{(x,y)\,|\,-1\le x\le 1, -2\le y\le 3\}$. (Do not use repeated integration.)

3–4 Suppose $\iint_{R_1} f(x,y)\,dA=6$, $\iint_{R_2} f(x,y)\,dA=-1$, $\iint_{R_1} g(x,y)\,dA=2$, and $\iint_{R_1\cup R_2} g(x,y)\,dA=8$ over the disjoint rectangular regions R_1 and R_2. Use the properties of double integrals to evaluate the given integral.

3. $\iint_{R_1\cup R_2} \left[2f(x,y)+3g(x,y)\right]dA$

4. $\iint_{R_2} \left[3f(x,y)-g(x,y)\right]dA$

5–6 The given iterated integral represents the volume of a well-known solid. Use a formula from geometry to evaluate the integral.

5. $\int_0^3 \int_0^{1-x/3} (6-2x-6y)\,dy\,dx$

6. $\int_0^2 \int_0^{\sqrt{4-y^2}} \sqrt{4-x^2-y^2}\,dx\,dy$

7–10 Evaluate the iterated integral.

7. $\int_0^{\pi/2} \int_0^2 x\cos y\,dx\,dy$

8. $\int_0^1 \int_0^1 \frac{y}{x^2+1}\,dy\,dx$

9. $\int_0^6 \int_{x/3}^{8-x} (xy-1)\,dy\,dx$

10. $\int_0^1 \int_y^{\sqrt{y}} xe^y\,dx\,dy$

11–12 Rework the indicated problem by reversing the order of integration and verify that the answer does not change.

11. Exercise 9

12. Exercise 10

13–14 Evaluate the given double integral over the indicated region. Choose the most convenient order of integration.

13. $\iint_R (x^2-y^2)\,dA$; R is the triangle with vertices $(0,0)$, $(2,4)$, and $(6,0)$.

14. $\iint_R 2xy\,dA$; R is the region bounded by $y=3x-x^2$ and $y=x-3$.

15–16 Evaluate the integral by reversing the order of integration. (Note that integrating in the given order would be impossible.)

15. $\int_0^1 \int_y^1 \sqrt[4]{1+x^2}\,dx\,dy$

16. $\int_0^1 \int_{\sqrt{y}}^1 e^{-x^3}\,dx\,dy$

17. By integrating the function $f(x,y)=1$, find the area of the region R bounded by the circle $x^2+y^2=1$ and the ellipse $4x^2+y^2=4$.

18. Use the Domination Property to show that if $f(x,y)$ is integrable on the bounded region R, then

$$\left|\iint_R f(x,y)\,dA\right| \le \iint_R |f(x,y)|\,dA.$$

19–20 Find the average value of $f(x,y)$ over the region bounded by the graphs of the given equations.

19. $f(x,y)=x(x+y)$; $y=x^2$, $2y=12-x^2$

20. $f(x,y)=4x^3y$; $4y=x^3$, $y=x$, $y\ge 0$

21–22 Find the center of mass of the plane region of varying density that is bounded by the graphs of the given equations.

21. $y=3-\frac{x}{2}$, $x=0$, $y=0$; $\rho(x,y)=x+y^2$

22. $y=x$, $y=\sqrt{2-x}$, $y=0$; $\rho(x,y)=x\sqrt{y}$

23–26 Determine the second moments I_x, I_y, and I_0 for the thin plate of constant density ρ modeled by the planar region R. Then find its corresponding radii of gyration.

23. R: The triangle bounded by the graph of $y=4-2x$ and the coordinate axes

©HAWKES LEARNING

24. R: The square connecting the four points $(\pm a, 0)$ and $(0, \pm a)$

25. R: The first-quadrant region bounded by the graph of $y = 4 - x^2$ and the coordinate axes

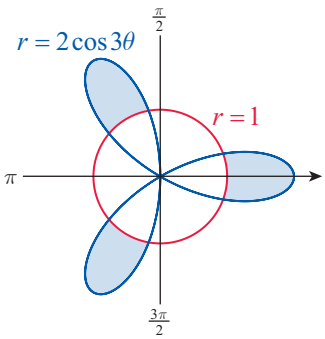
26.* R: The region bounded by the graph of $y = \dfrac{1-x^2}{1+x^2}$ and the x-axis

27. Suppose a thin plate of constant density ρ is modeled by the region R that is outside the square of diagonal length 2 centered at the origin, but inside the unit circle, also centered at the origin. Use the Principle of Superposition to find the moment of inertia of R about the origin.

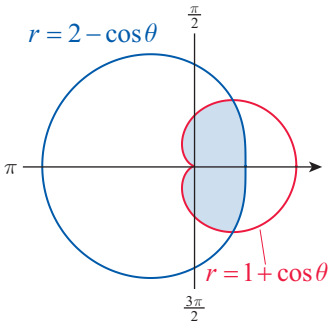
28. Suppose the thin plate of Exercise 25 has nonconstant density $\rho(x, y) = x$. Determine the second moments I_x, I_y, and I_0 and the corresponding radii of gyration.

29–30 Use a double integral in polar coordinates to find the area of the shaded region.

29.



30.



31–32 Use a double integral in polar coordinates to find the area of the given region R.

31. R: The region common to the circle $r = \frac{1}{2}$ and the cardioid $r = 1 - \cos\theta$

32. R: The region outside the circle $r = 1$ and inside the limaçon $r = \frac{1}{2} + \cos\theta$

33–34 Evaluate the double integral by changing to polar coordinates.

33. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{dy\,dx}{\sqrt{x^2+y^2+2}}$

34. $\displaystyle\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} e^{\frac{x^2+y^2}{2}}\,dy\,dx$

35–36 Convert the integral into a Cartesian double integral and evaluate it.

35. $\displaystyle\int_0^{\pi/3} \int_0^{\sec\theta} r^2(\cos\theta - \sin\theta)\,dr\,d\theta$

36. $\displaystyle\int_{\pi/4}^{3\pi/4} \int_0^{4\csc\theta} r^4 \cos^2\theta \sin\theta\,dr\,d\theta$

37–38 Make your choice between the Cartesian and polar coordinate systems and evaluate the double integral.

37. $\displaystyle\iint_R \ln\sqrt{x^2+y^2}\,dA$; R: $1 \le x^2 + y^2 \le e$

38. $\displaystyle\iint_R 14(x^2+y^2)^3\,dA$; R is the region bounded by $y = 2x$, $y = 2$, and the y-axis.

39–42 Use double integration in polar coordinates on an appropriate region to find the volume of the solid S bounded by the given surfaces.

39. S: The solid bounded by the xy-plane and the paraboloid $z = x^2 + y^2 - 9$

40. S: The solid bounded by the xy-plane, the plane $z = 3x + 4y + 1$, and the cylinder $x^2 + y^2 = 16$

41. S: The solid common to the paraboloids $z = 18 - x^2 - y^2$ and $z = x^2 + y^2$

42. S: The solid common to the ellipsoid $8x^2 + 8y^2 + z^2 = 36$ and the cone $z = \sqrt{x^2 + y^2}$

43–44 Evaluate the triple integral on the rectangular box S.

43. $\displaystyle\iiint_S (2x + yz)\,dV$, where $S = [-2, 2] \times [1, 4] \times [0, 5]$

44. $\displaystyle\iiint_S xz^2\,dV$, where $S = [1, 4] \times [-1, 1] \times [2, 3]$

45–46 Evaluate the iterated integral.

45. $\displaystyle\int_0^1 \int_0^{1-y} \int_0^{3-3x-3y} (2x - y)\,dz\,dx\,dy$

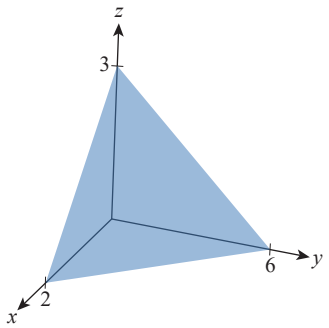
46. $\displaystyle\int_0^1 \int_0^{4-4x} \int_0^{2-\frac{y}{2}-2x} 5xy\,dz\,dy\,dx$

©HAWKES LEARNING

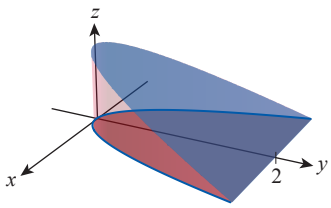
47–51 Use a triple integral to find the volume of the solid S.

47. S: The cylinder bounded by $y = \sqrt{1-x}$, the coordinate planes, and $z = 4$. Use the following orders of integration: **a.** $dy\,dx\,dz$ **b.** $dz\,dy\,dx$

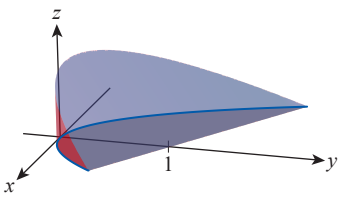
48. S: The tetrahedron bounded by the coordinate planes and $3x + y + 2z = 6$. Use the following orders of integration: **a.** $dx\,dy\,dz$ **b.** $dy\,dz\,dx$



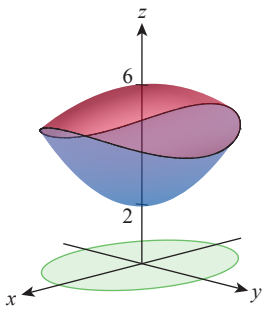
49. S: The solid bounded by the parabolic cylinder $y = 2x^2$ and the planes $z = 2 - y$ and $z = 0$. Use the following orders of integration:
a. $dz\,dy\,dx$ **b.** $dx\,dy\,dz$



50. S: The solid bounded by the parabolic cylinder $y = x^2/2$ and the planes $2x + 4y + z = 4$ and $z = 0$. Choose the most convenient order of integration.



51. S: The solid bounded below by the surface $z = 3x^2 + y^2 + 2$ and above by the surface $z = 6 - x^2 - y^2$. Choose the most convenient order of integration.



52–53 Revisit the indicated exercise, integrating in the given order. Verify that the answer does not change. (**Hint:** Sketching the solid of integration may be helpful.)

52. Exercise 45, order of integration: $dx\,dz\,dy$

53. Exercise 46, order of integration: $dy\,dz\,dx$

54–55 Use a triple integral to find the center of mass of the solid S with indicated density.

54. S: The solid of Exercise 49, with constant density

55. S: The tetrahedron bounded by the coordinate planes and $2x + y + z = 3$, its density at each point being proportional to the distance from the yz-plane

56–57 Find the center of mass, second moments, and radii of gyration of the given solid S. Assume S is made of a substance with constant density ρ.

56. S: The first-octant region of the paraboloid $z = 4 - x^2 - y^2$

57. S: The solid bounded by $z = 6 - x - 3y$ and the coordinate planes

58. Write a short paragraph about when and why you would choose one particular coordinate system over the other two from among the Cartesian, cylindrical, and spherical systems.

59–60 Find a set of cylindrical coordinates for the point given in Cartesian coordinates.

59. $\left(\sqrt{2}, -\sqrt{2}, -\sqrt{2}\right)$ **60.** $\left(-1, -\sqrt{3}, \sqrt{3}\right)$

61–62 Find a set of spherical coordinates for the point given in Cartesian coordinates.

61. $\left(\sqrt{2}, \sqrt{2}, -2\right)$ **62.** $\left(\sqrt{3}, -3, -2\right)$

63–64 Find the Cartesian coordinates of the point given in cylindrical coordinates.

63. $\left(\sqrt{2}, -\frac{\pi}{4}, -\frac{\pi}{4}\right)$ **64.** $\left(-\pi, \frac{\pi}{2}, 1\right)$

65–66 Find the Cartesian coordinates of the point given in spherical coordinates.

65. $\left(4\sqrt{2}, -\frac{\pi}{4}, \frac{5\pi}{6}\right)$ **66.** $\left(4, \frac{\pi}{3}, \frac{3\pi}{4}\right)$

©HAWKES LEARNING

67–68 Write the equation in cylindrical coordinates.

67. $x^2+(y-2)^2=z+4$ **68.** $2z(x^2+y^2)=y$

69–70 Write the equation in spherical coordinates.

69. $(x-1)^2+y^2+z^2=1$ **70.** $y=-\sqrt{3}x$

71–74 Change the given cylindrical or spherical equation into a rectangular one.

71. $r^2=1+z$ **72.** $r\csc\theta=3z$

73. $\varphi=\dfrac{2\pi}{3}$ **74.** $\sec\varphi+\rho=0$

75–76 Use cylindrical coordinates to determine the volume of the solid S.

75. S: The solid bounded by the paraboloid $z=9-(x^2+y^2)$ and the xy-plane

76. S: The solid bounded by the cylinder $x^2+(y-\frac{1}{2})^2=\frac{1}{4}$, the cone $z=1-\sqrt{x^2+y^2}$, and the xy-plane

77–78 Use the cylindrical coordinate system to find the mass and the center of mass of the solid S with the given density function.

77. S: The solid of Exercise 75; $\rho(x,y,z)=9-z$

78. S: The solid of Exercise 76; $\rho(x,y,z)=z$

79–80 Use a triple integral in spherical coordinates to determine the volume of the solid S.

79. S: The solid bounded by the cone $z=\sqrt{\frac{1}{3}(x^2+y^2)}$ and the sphere $x^2+y^2+z^2=9$

80. S: The solid bounded by the sphere $x^2+y^2+z^2=1$ and the planes $y=x$ and $x=\sqrt{3}y,\ x>0$

81. Find the coordinates of the center of mass of the solid of Exercise 79, assuming constant density.

82. Find the mass of the solid of Exercise 80, assuming its density at any of its points is proportional to the distance from the origin.

83–84 Use a change of variables to evaluate the double integral on the given region R.

83. $\iint_R \dfrac{x+2y}{4x+3y}\,dA$; R is the region bounded by the coordinate axes and $x+2y=3$.

84. $\iint_R \dfrac{2x+5y}{x+4y}\,dA$; R is the region bounded by the coordinate axes and $2x+5y=6$.

85–86 Find a linear transformation $T(u,v)$ that maps a rectangular region onto the given parallelogram P.

85. P is bounded by $2y=x$, $2y=x+2$, $y=4x$, and $y=4x-12$.

86. P is bounded by $y=2x+1$, $y=2x+3$, $y=3-x$, and $y=-x-\frac{1}{2}$.

87. Evaluate $\iint_P (7x^2-14y)\,dA$ on the parallelogram P of Exercise 85.

88. Evaluate $\iint_P 3x^2y\,dA$ on the parallelogram P of Exercise 86.

89–90 Use a change of variables to evaluate the given integral on the solid R.

89. Evaluate $\iiint_R \dfrac{3y-x+2z}{2}\,dV$, where R is the solid bounded by the planes $x=0$, $x=4$, $z=0$, $z=2$, $3y=x$, and $3y=x+3$.

90. Evaluate $\iiint_R \left(\dfrac{y-3x}{3}+\dfrac{5z-2y}{4}\right)dV$, where R is the solid bounded by the planes $z=0$, $z=3$, $y=3x$, $y=3x+4$, and $z=2y$, $z=2y-6$.

91. Suppose $f(x,y,z)$ is defined on the box $R=[-a,a]\times[-b,b]\times[-c,c]$. Use the Fundamental Theorem of Calculus to prove the following formula.

$$\begin{aligned}\iiint_R f_{xyz}\,dV = {} & f(a,b,c)-f(a,b,-c)\\ & -f(a,-b,c)+f(a,-b,-c)\\ & -f(-a,b,c)+f(-a,b,-c)\\ & +f(-a,-b,c)-f(-a,-b,-c)\end{aligned}$$

Concept Check

92–98 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

92. If $a>0$ and $f(x)$ is continuous, then $\int_o^a\int_o^y f(x,y)\,dx\,dy=\int_0^a\int_0^x f(x,y)\,dy\,dx$.

©HAWKES LEARNING

93. If $f(x,y) \neq 0$ on the bounded region R, then its average value on R cannot be zero.

94. The value of a double integral should be interpreted as the volume of the solid bounded by the graph of the integrand and one of the coordinate planes.

95. In order to find $\iint_R e^{\frac{x^2+y^2}{2}} dA$ on the rectangular region $R = \{(x,y) \mid |x| \leq a, |y| \leq b, a,b > 0\}$, the use of Cartesian coordinates is recommended.

96. In the cylindrical coordinate system, r can be negative.

97. In the spherical coordinate system, ρ can be negative.

98. If the binomial $x^2 + y^2$ is present in the integrand (or in the limits) of a triple integral, you should use cylindrical coordinates.

Chapter 14
Technology Exercises

99. Use a computer algebra system to find the center of mass of the solid of Exercise 80.

100. Write a program on your computer algebra system that performs a change from x- and y-coordinates to u- and v-coordinates and evaluates a given integral in the new coordinate system. Test your program by checking the answers you obtained for Exercises 89–90.

101. Use a computer algebra system to create the graph of the solid seen in Example 3 of Section 14.5.

©HAWKES LEARNING

25. Repeat Exercise 23 for the right circular cone of radius R and height h.

26. If S is a surface such as in Exercise 22, use the Divergence Theorem to show that the volume of the solid D bounded by S can also be found as

$$V(D) = \iint_S x\,dy\,dz = \iint_S y\,dz\,dx = \iint_S z\,dx\,dy.$$

27. Let Q be a single point charge at the origin, as in our discussion that follows Example 2. Show that $\nabla \cdot \mathbf{E} = 0$ for the electric field $\mathbf{E}$ created by Q.

28. Mimic our discussion following Example 2 to prove the following, slightly more general version of Gauss' Law.

If S is a closed surface satisfying the hypotheses of the Divergence Theorem, k is a constant and $\mathbf{F} = \dfrac{k\mathbf{r}}{|\mathbf{r}|^3}$ is an inverse square field, then the flux of $\mathbf{F}$ over S is

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \begin{cases} 4k\pi & \text{if } S \text{ encloses the origin.} \\ 0 & \text{otherwise} \end{cases}$$

29.* Recall the uniformly charged sphere of radius R and total charge Q from Exercises 55–57 of Section 15.6. In this problem, you will provide a "second solution" to the aforementioned Exercise 57 as follows. Use Gauss' Law to show that the electric field $\mathbf{E}$ due to the sphere is

$$\mathbf{E} = \begin{cases} \dfrac{\varepsilon Q \mathbf{e}_r}{r^2} & \text{if } R < r \\ 0 & \text{if } r < R. \end{cases}$$

(Recall $\mathbf{e}_r$ from Exercise 54 of Section 15.1.
Hint: Note that $\mathbf{E}$ is a central force as in Exercise 29 of Section 15.7. Using the notation $\mathbf{E}(\mathbf{r}) = E(r)\mathbf{e}_r$, show that the flux across a sphere of radius r is $4\pi r^2 E(r)$ and use Gauss' Law. Finally, note that if $r < R$, the total charge enclosed by any sphere of radius r is zero, and hence so is $\mathbf{E}$.)

30. Use the Divergence Theorem to provide a second solution to Exercise 28 of Section 15.7. (**Hint:** See Exercise 39 of Section 15.4.)

31.* Prove that if $\mathbf{F}$ has continuous partial derivatives, then its divergence at a point P can be obtained from the formula

$$\nabla\cdot\mathbf{F}(P) = \lim_{R\to 0} \frac{3}{4\pi R^3} \iint_{S_R(P)} \mathbf{F}\cdot\mathbf{n}\,d\sigma,$$

where $S_R(P)$ is the sphere of radius R centered at P. (**Hint:** Use the Divergence Theorem; then argue that the divergence function "assumes its average value" somewhere in the closed ball bounded by $S_R(P)$. For a refresher on average value, see Section 5.2.)

32–35 Prove the identity, assuming the scalar-valued functions f and g have continuous partial derivatives at least through the second order, and S and D satisfy the hypotheses of the Divergence Theorem. Note that $\nabla^2 f$ stands for $f_{xx} + f_{yy} + f_{zz}$, i.e., $\nabla^2 f = 0$ means that f satisfies Laplace's equation (see Section 13.3). Recall also that $D_{\mathbf{n}} f$ stands for the directional derivative of f in the direction of $\mathbf{n}$. (For a review of directional derivatives, see Section 13.5.)

32.* $\displaystyle\iint_S D_{\mathbf{n}} f\,d\sigma = \iiint_D \nabla^2 f\,dV$

33.* $\displaystyle\iint_S f D_{\mathbf{n}} g\,d\sigma = \iiint_D \left(f\nabla^2 g + \nabla f\cdot\nabla g\right) dV$
(This is called *Green's first identity*. **Hint:** See Exercise 44 in Section 15.4.)

34.* $\displaystyle\iint_S f D_{\mathbf{n}} f\,d\sigma = \iiint_D |\nabla f|^2\,dV$, if f satisfies Laplace's equation on D. (**Hint:** You may give a direct proof, or use Exercise 33.)

35.* $\displaystyle\iint_S \left(f D_{\mathbf{n}} g - g D_{\mathbf{n}} f\right) d\sigma = \iiint_D \left(f\nabla^2 g - g\nabla^2 f\right) dV$ (This is called *Green's second identity*. **Hint:** Use Green's first identity from Exercise 33 twice.)

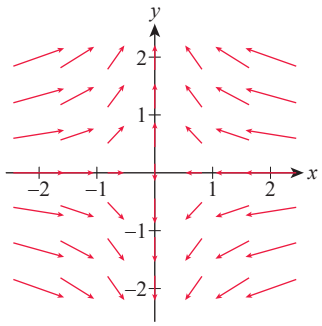
Chapter 15
Review Exercises

1–4 Match the given planar vector field $\mathbf{F}(x,y)$ with its graph (labeled A–D).

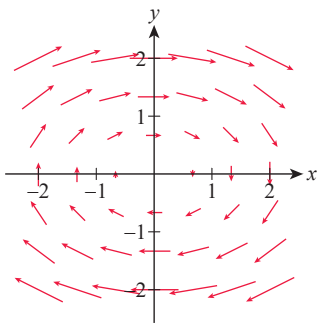
1. $\mathbf{F}(x,y) = \langle 1, y\rangle$ **2.** $\mathbf{F}(x,y) = \left\langle \dfrac{x}{2}, y\right\rangle$ **3.** $\mathbf{F}(x,y) = \langle 2y, -x\rangle$ **4.** $\mathbf{F}(x,y) = \left\langle -x, \dfrac{y}{|\langle x,y\rangle|}\right\rangle$

©HAWKES LEARNING

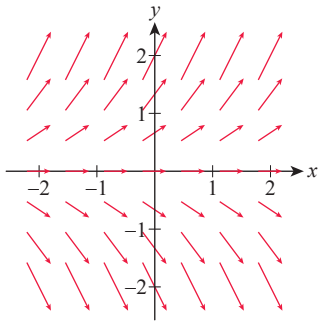
A.



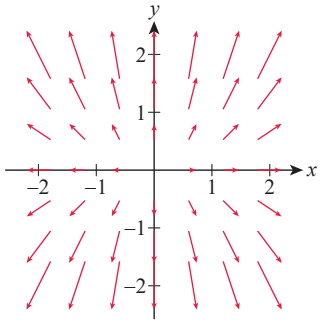
B.



C.



D.



5–6 Sketch the vector field **F** by hand, using a few representative vectors.

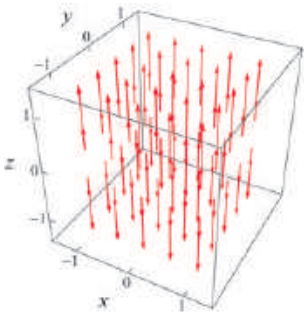
5. $\mathbf{F}(x,y)=\left\langle \sqrt{|x|},y\right\rangle$

6. $\mathbf{F}(x,y)=\left\langle \sqrt{x^2+y^2},\sqrt{x^2+y^2}\right\rangle$

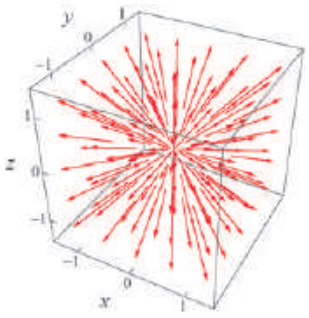
7–10 Match the given three-dimensional vector field with its graph (labeled A–D).

7. $\mathbf{F}(x,y,z)=\langle x,y,0\rangle$ **8.** $\mathbf{F}(x,y,z)=\langle 0,0,z\rangle$ **9.** $\mathbf{F}(x,y,z)=\langle 0,-z,y\rangle$ **10.** $\mathbf{F}(x,y,z)=\dfrac{\langle x,y,z\rangle}{2\left|\langle x,y,z\rangle\right|}$

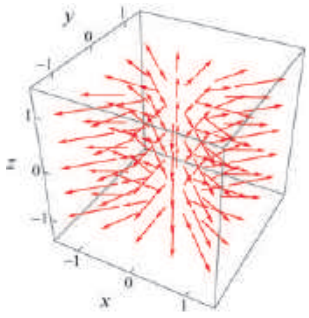
A.



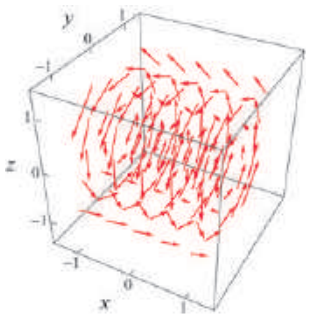
B.



C.



D.



©HAWKES LEARNING

11–12 Determine ∇f for the given scalar field $f(x,y,z)$.

11. $f(x,y,z) = 2x^2y + xz^3$

12. $f(x,y,z) = \ln\sqrt{x^2z^4 + 4y^2}$

13–16 Decide whether the given vector field is conservative.

13. $\mathbf{F}(x,y) = \left\langle \dfrac{y^2}{(x+y^2)^2}, \dfrac{-2xy}{(x+y^2)^2} \right\rangle$

14. $\mathbf{F}(x,y) = \left\langle -x^2y, \dfrac{x}{y} \right\rangle$

15. $\mathbf{F}(x,y,z) = \left\langle 3x^2yz + \dfrac{1}{yz}, x^3z - \dfrac{x}{y^2z}, x^3y - \dfrac{x}{yz^2} \right\rangle$

16. $\mathbf{F}(x,y,z) = \left\langle \dfrac{3x^2y}{z}, \dfrac{x^3}{z}, \dfrac{x^3y}{z^2} \right\rangle$

17–18 Find a potential function for the given conservative vector field.

17. $\mathbf{F}(x,y) = \langle 12x^2 - 4xy, 3y^2 - 2x^2 \rangle$

18. $\mathbf{F}(x,y) = \langle y^2\cos(xy), xy\cos(xy) + \sin(xy) \rangle$

19–22 Evaluate the line integral.

19. $\int_C x^2y\,ds$, where C is the semicircle $y = \sqrt{4-x^2}$, traversed counterclockwise

20. $\int_C \left(x - \dfrac{y}{2}\right) ds$, where C is the arc of the unit circle centered at the origin connecting $(1,0)$ with $(0,1)$, followed by the line segment to the point $(2,2)$

21. $\int_C (xz + 3y)\,ds$, where C is the line segment joining $(2,0,3)$ and $(1,3,0)$

22. $\int_C 2z(x-y)\,ds$, where C is the helix $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, t \rangle$, for $0 \le t \le 6\pi$

23. Determine the center of mass of the V-shaped object of constant density connecting the point $(-1,-2,4)$ with the origin, and then the origin with the point $(1,8,4)$.

24. Suppose that in Exercise 23, the density of the object at any point is proportional to the distance from the xy-plane. Find its mass and center of mass in this case.

25. Determine the center of mass of the object with constant density that consists of a line segment connecting the point $\left(2,-1,-\sqrt{2}\right)$ with the point $(1,0,0)$, followed by the helix $\mathbf{r}(t) = \langle \cos \pi t, \sin \pi t, t \rangle$, $0 \le t \le 1$.

26.* Find the mass and center of mass of the object in Exercise 25 if the density at any point is proportional to the distance from the xy-plane.

27–28 Determine the work done by the force field $\mathbf{F}$ in moving a particle along the indicated curve C.

27. $\mathbf{F}(x,y,z) = \langle 2y^2, -xz, 3y \rangle$, where C is defined by $\mathbf{r}(t) = \langle t, 2t^2, 3t^3 \rangle$, $0 \le t \le 2$

28. $\mathbf{F}(x,y,z) = \langle -y, x, 3z \rangle$, where C is the helix defined by $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, t \rangle$, $0 \le t \le 4\pi$

29–30 A fluid's velocity field is given by $\mathbf{F}(x,y,z)$. Determine the fluid's flow along the indicated curve C.

29. $\mathbf{F}(x,y,z) = \langle 2xyz, x^2z, x^2y \rangle$, where C is the line segment connecting the points $(0,2,-1)$ and $(1,4,5)$

30. $\mathbf{F}(x,y,z) = \langle y^2, 2xy, 4z \rangle$, where C is the curve defined by $\mathbf{r}(t) = \langle \sin t, \cos t, t/4 \rangle$, $0 \le t \le 3\pi$

31. Suppose the semicircle parametrized by $\mathbf{r}(t) = \langle 3\cos t, 0, 3\sin t \rangle$, $0 \le t \le \pi$ has charge density $q(x,y,z) = \dfrac{2+z}{10^6}$ C/m. Find the electrostatic potential at the point $(0,4,0)$. (See Exercise 50 in Section 15.2.)

32–35 Use the Fundamental Theorem for Line Integrals to evaluate the integral.

32. $\int_C 2(x+y^2)\,dx + 4xy\,dy$, where C is a piecewise smooth path from $(0,0)$ to $(3,2)$

33. $\int_C e^{xy}(xy+1)\,dx + x^2e^{xy}\,dy$, where C is a piecewise smooth path from $(0,-1)$ to $(1,1)$

34. $\int_C \left(yz^2 + e^x\sin y\right)dx + \left(xz^2 + e^x\cos y\right)dy + 2xyz\,dz$, where C is a piecewise smooth path from $(0,-\pi/2,2)$ to $(0,0,0)$

35. $\int_C \left(\dfrac{2x}{x^2+y^2} - 2y^3\right)dx + \left(\dfrac{2y}{x^2+y^2} - 6xy^2 - z\sin y\right)dy + \cos y\,dz$, where C is a piecewise smooth path from $(2,3,0)$ to $(1,0,2)$

36–37 Show that the line integral is not path independent by finding two different values for the integral along two different paths connecting A and B. (Answers may vary.)

36. $\int_C xy\,dx - 5y\,dy$; $A(0,0)$, $B(1,1)$

37. $\int_C 2x\,dx + x\,dy + yz\,dz$; $A(2,0,0)$, $B(-2,0,0)$

©HAWKES LEARNING

38–39 Show that the force field $\mathbf{F}$ is conservative, and use this fact to determine the work done by $\mathbf{F}$ in moving an object from A to B.

38. $\mathbf{F}(x,y)=\langle \cos y + y\sin x, -\cos x - x\sin y\rangle$; $A(0,1)$, $B(-1,-1)$

39. $\mathbf{F}(x,y,z)=\left\langle \ln(y^2), \dfrac{2x}{y}+z^2, 2yz\right\rangle$; $A(1,1,1)$, $B\left(2,\sqrt{e},2\right)$

40. Use the Fundamental Theorem for Line Integrals to verify your answer for Exercise 29.

41. Use the Fundamental Theorem for Line Integrals to verify your answer for Exercise 30.

42. Recall the vector field $\mathbf{F}(x,y)=\left\langle \dfrac{-y}{|\langle x,y\rangle|^2}, \dfrac{x}{|\langle x,y\rangle|^2}\right\rangle$ from Exercise 35 of Section 15.3. Show that $\mathbf{F}$ is conservative on any region disjoint from the origin.

43. Show that the vector field $\mathbf{F}(x,y)=\left\langle \dfrac{-y}{|\langle x,y\rangle|^p}, \dfrac{x}{|\langle x,y\rangle|^p}\right\rangle$ is not conservative if $p\neq 2$.

44–47 Find the divergence and curl of the vector field $\mathbf{F}$.

44. $\mathbf{F}(x,y)=\langle x^2+4xy, 2x^2y\rangle$

45. $\mathbf{F}(x,y)=\left\langle \dfrac{x}{y^2+1}, y(x^2-x)\right\rangle$

46. $\mathbf{F}(x,y,z)=\langle 2x(z^2-1), \ln(x^2+y^2), x^2yz\rangle$

47. $\mathbf{F}(x,y,z)=\langle x\sin(yz), y^2z, x(y+z)\rangle$

48. Consider the function $f(\mathbf{r})=\dfrac{1}{r^4}$, where, as usual, $r=|\mathbf{r}|$. Prove: $\nabla f=-\dfrac{4\mathbf{r}}{r^6}$.

49–50 Verify Green's Theorem by demonstrating the equality $\oint_C P\,dx+Q\,dy=\iint_R\left(\dfrac{\partial Q}{\partial x}-\dfrac{\partial P}{\partial y}\right)dA$ for the given $\mathbf{F}(x,y)=\langle P(x,y),Q(x,y)\rangle$ and region R enclosed by the piecewise smooth closed curve C.

49. $P(x,y)=2x+y^2$, $Q(x,y)=xy$, where R is the triangle with vertices $(-1,0)$, $(1,0)$, and $(0,1)$

50. $P(x,y)=x-y$, $Q(x,y)=2y$, where R is the region bounded by the x-axis and the upper semicircle $y=\sqrt{1-x^2}$

51–52 Use Green's Theorem to evaluate the indicated line integral.

51. $\oint_C y^2\,dx+2xy\,dy$, where C is the boundary of the region enclosed by the graphs of $y=\sqrt{x}$, $x=1$, and the x-axis, oriented counterclockwise

52. $\oint_C \dfrac{x^3y^2}{2}dx-x^2y\,dy$, where C is the boundary of the region bounded by the coordinate axes and the line $y=2-x$, oriented clockwise

53–54 Use Green's Theorem to find the work done as a particle moves counterclockwise around the closed path C in the force field $\mathbf{F}$.

53. $\mathbf{F}(x,y)=\langle -3x^2y, (2x-y)y\rangle$, where C is the boundary of the triangle with vertices $(0,-1)$, $(0,1)$, and $(2,0)$

54. $\mathbf{F}(x,y)=\langle x^2+2y, \cos y+3x\rangle$, where C is the circle $x^2+(y-1)^2=1$ (**Hint:** Use polar coordinates.)

55–56 Evaluate the outward flux of the vector field $\mathbf{F}(x,y)$ across the given path C.

55. $\mathbf{F}(x,y)=\langle 2xy, y\rangle$, C is the boundary of the upper half disk bounded by $y=\sqrt{4-x^2}$ and the x-axis, oriented counterclockwise (**Hint:** Use polar coordinates.)

56. $\mathbf{F}(x,y)=\langle 4x(x+y), x-y\rangle$, C is the boundary of the region between the graphs of $y=x^2$ and $x=y^2$, oriented counterclockwise

57. Use the line integral method (Exercise 84, Section 15.4) to find the area of the heptagon with vertices $(1,0)$, $(3.5,0.5)$, $(4,2)$, $(3,4)$, $(1,5)$, $(0,3)$, and $(0,1)$.

58. Let f_1 and f_2 be differentiable single-variable functions, and C a piecewise smooth, simple closed curve. Prove $\int_C f_1(x)\,dx+f_2(y)\,dy=0$. (**Hint:** Use Green's Theorem.)

©HAWKES LEARNING

59–64 Match the parametric surface with its graph (labeled A–F).

59. $\mathbf{r}(s,t)=\left\langle \dfrac{\cos t}{s}, \dfrac{\sin t}{s}, s \right\rangle$,
$0 \le s \le 4, \quad 0 \le t \le 2\pi$

60. $\mathbf{r}(s,t)=\left\langle s\sin t, s\cos t, \dfrac{3\sqrt{s}}{2} \right\rangle$,
$0 \le s \le 2\pi, \quad 0 \le t \le 2\pi$

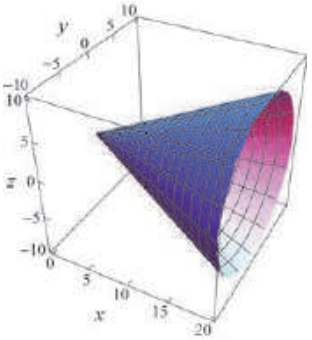
61. $\mathbf{r}(s,t)=\langle 4s, 2s\cos t, 2s\sin t \rangle$,
$0 \le s \le 5, \quad 0 \le t \le 2\pi$

62. $\mathbf{r}(s,t)=\left\langle \dfrac{3s}{2}\cos t, \dfrac{3s}{2}\sin t, -s^2 \right\rangle$,
$0 \le s \le 3, \quad 0 \le t \le 2\pi$

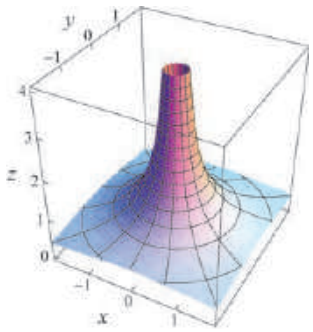
63. $\mathbf{r}(s,t)=\langle s\sin t\cos s, s\sin t\sin s, 2s \rangle$,
$0 \le s \le 9, \quad 0 \le t \le 2\pi$

64. $\mathbf{r}(s,t)=\left\langle s\sin t, s\cos t, \dfrac{3}{1+s^2} \right\rangle$,
$0 \le s \le 2\pi, \quad 0 \le t \le 2\pi$

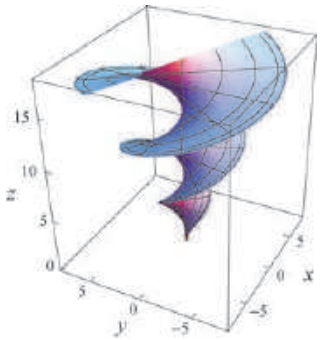
A.



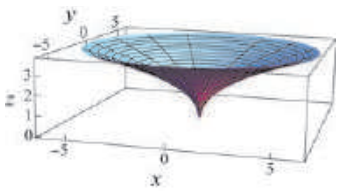
B.



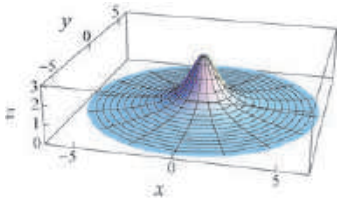
C.



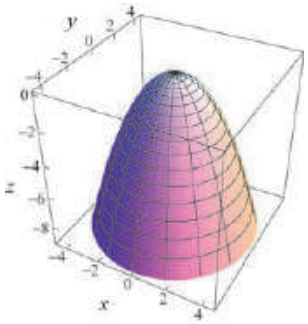
D.



E.



F.



65–66 Describe the surface with a vector function of two parameters.

65. The graph of $z = 1/y^2$, $1 \le y \le 4$, revolved about the y-axis

66. The graph of $z = \tan x$, $0 \le x \le \pi/4$, revolved about the x-axis

67–68 Identify the surface by examining its grid curves.

67. $\mathbf{r}(s,t)=\langle 2s\sin t, s\cos t, s \rangle$,
$-\infty < s < \infty, \quad 0 \le t \le 2\pi$

68. $\mathbf{r}(s,t)=\langle \sin t, (2+\cos t)\sin s, (2+\cos t)\cos s \rangle$,
$0 \le s \le 2\pi, \quad 0 \le t \le 2\pi$

69–70 Obtain a parametrization for the indicated surface. (Answers will vary.)

69. $z = 2x^2 + y^2$

70. $z = \sqrt{9 - 9x^2 - \dfrac{9}{4}y^2}$

71–72 Find an equation of the plane tangent to the surface at the indicated point.

71. $\mathbf{r}(s,t)=\langle st, t-s, 2t^2 \rangle$; $\mathbf{r}(1,3)$

72. $\mathbf{r}(s,t)=\langle s\cos t, s\sin t, s^4 \rangle$; $\mathbf{r}\left(1, \dfrac{\pi}{4}\right)$

©HAWKES LEARNING

73–76 Find the area of the surface S. (Use polar coordinates wherever they simplify your calculations.)

73. S is the surface common to the interior of the cylinder $x^2 + y^2 = 9$ and the plane $2z + 2y + x = 0$.

74. S is the surface $\mathbf{r}(s,t) = \langle s\cos t, s\sin t, 2s\rangle$ between the planes $z = 2$ and $z = 4$.

75. S is the surface $\mathbf{r}(s,t) = \left\langle s\cos t, s\sin t, \dfrac{s^2}{2}\right\rangle$ between the planes $z = 1$ and $z = 4$.

76. S is the graph of $4z = y^2 - x^2$ above the unit disk centered at the origin.

77–78 A surface with variable mass density is given. Determine the mass of the surface.

77. $\mathbf{r}(s,t) = \langle \cos t, \sin t, s\rangle$, $0 \le s \le 1$, $0 \le t \le \dfrac{\pi}{2}$; mass density is $\rho(x,y,z) = z^2$

78. $\mathbf{r}(s,t) = \langle 2\cos t, 2\sin t, s\rangle$, $0 \le s \le 2$, $0 \le t \le \pi$; mass density is $\rho(x,y,z) = 8 - z^3$

79. Determine the center of mass of the surface of Exercise 77.

80. Determine the center of mass of the surface of Exercise 78.

81–84 Determine the indicated flux of the vector field $\mathbf{F}$ across the surface S. Unless otherwise specified, the surfaces are oriented with outward-pointing normal vectors.

81. The flux of $\mathbf{F} = \langle 1, 2y, z\rangle$ out of the solid R bounded by $z = \dfrac{x^2 + y^2}{2}$ and the plane $z = 2$

82. The flux of $\mathbf{F} = \langle 1, -z, y\rangle$ across the first-octant portion of the hemisphere of radius 2 centered at the origin

83. The flux of $\mathbf{F} = \langle xy, 1, z\rangle$ across the first-octant portion of the cylinder $x^2 + y^2 = 4$, $0 \le z \le 6$

84. The flux of $\mathbf{F} = \langle 2x, y, -z\rangle$ out of the solid R bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$

85. Consider the thin conical surface of constant mass density, with both height and radius equaling R. Show that the second moment about its axis of rotation is $I = mR^2/2$, where m is the mass of the shell.

86–87 Verify Stokes' Theorem by showing that the integrals $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$ and $\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma$ are equal.

86. $\mathbf{F}(x,y,z) = \langle -4x - y, -2z, y\rangle$, where S is the upper hemisphere of radius 3 centered at the origin

87. $\mathbf{F}(x,y,z) = \langle 2y, -x, z^2\rangle$, where S is the surface $z = 4 - x^2 - y^2$, $z > 0$

88–91 Use Stokes' Theorem to evaluate the indicated line integral.

88. $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$, where $\mathbf{F} = \langle 2x^2, -z, y^2\rangle$ and C is the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $2x + \dfrac{y}{2} + z = 6$, with positive orientation when viewed from above

89. $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$, where $\mathbf{F} = \langle -2y^2, z^2, -x^2\rangle$ and C is the intersection of the cylinder $x^2 + y^2 = 2x$ and the plane $z = \dfrac{x}{2}$, with positive orientation when viewed from above

90. $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$, where $\mathbf{F} = \langle -y, -2z, -3x\rangle$ and C is the intersection of $\dfrac{z}{2} = x^2 + y^2$ and $z = 12 - x^2 - y^2$, with positive orientation when viewed from above

91. $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$, where $\mathbf{F} = \langle -xz, 2z, x - y^2\rangle$ and C is the intersection of $z = x^2 + y^2$ and $z = 4x + 5$, with negative orientation when viewed from above

92–95 Use Stokes' Theorem to evaluate the surface integral.

92. $\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma$, where $\mathbf{F} = \langle -2z, x, 3xy\rangle$ and S is the hemisphere $z = \sqrt{4 - x^2 - y^2}$, oriented with an upward-pointing unit normal vector field

93. $\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma$, where $\mathbf{F} = \langle 3y, z, -2x\rangle$ and S is the portion of the paraboloid $z = x^2 + y^2$ with $0 \le z \le 4$, oriented with a downward-pointing unit normal vector field

94. $\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma$, where $\mathbf{F} = \langle z^2, 2x^2, x^2 - y\rangle$ and S is the triangle determined by the first-octant portion of the plane $x + 2y + 3z = 6$, oriented with an upward-pointing unit normal vector field

©HAWKES LEARNING

95. $\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma$, where $\mathbf{F} = \left\langle x - \frac{yz^2}{4}, 3x^2z, z - y^2 \right\rangle$ and S is the cone frustum $z = 2\sqrt{x^2 + y^2}$ with $2 \le z \le 6$, oriented with an inward-pointing unit normal vector field

96–97 Verify the Divergence Theorem by showing the equality of the integrals $\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$ and $\iiint_D \nabla \cdot \mathbf{F}\, dV$ for the given vector field $\mathbf{F}$ on the solid D.

96. $\mathbf{F}(x, y, z) = \left\langle 3z, \frac{y}{4}, -2x \right\rangle$, where D is the ball of radius 3 centered at the origin

97. $\mathbf{F}(x, y, z) = \langle 3xy, -x, 2z \rangle$, where D is the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane

98–103 Use the Divergence Theorem to find the flux of the vector field $\mathbf{F}$ over the surface of the given solid D. Consider cylindrical or spherical coordinates where appropriate.

98. $\mathbf{F}(x, y, z) = \langle -x^2y, yz, 5z \rangle$, where D is the solid bounded by the parabolic cylinder $z = 4 - y^2$, the xy-plane, and the planes $x = 0$ and $x = 2$

99. $\mathbf{F}(x, y, z) = \langle 5x(z - y), z^2 + \cos x, y \sin x \rangle$, where D is the solid cylinder bounded by $x^2 + y^2 = 9$, the xy-plane, and the plane $z = 3$

100. $\mathbf{F}(x, y, z) = \left\langle 2xy, y - z, \frac{x^2y}{3} \right\rangle$, where D is the tetrahedron with vertices at the origin, $(6, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 4)$

101. $\mathbf{F}(x, y, z) = \langle 2x + z, x - y^3, 2z \rangle$, where D is the solid inside the cylinder $x^2 + y^2 = 4$, bounded by the xy-plane and $z = 3 - y$

102. $\mathbf{F}(x, y, z) = \langle -xy^2, 3yz^2, y^3 \rangle$, where D is the solid cylindrical shell $2 \le x^2 + y^2 \le 4$ between the planes $z = 1$ and $z = 6$

103. $\mathbf{F}(x, y, z) = \left\langle \frac{yz^2}{2}, \frac{y^3}{8}, x(z - x) \right\rangle$, where D is the portion of the solid cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 2$ and $z = 4$

104. Write a paragraph discussing the relationships and analogies between the Fundamental Theorem of Calculus, the Fundamental Theorem for Line Integrals, Green's and Stokes' Theorems, and the Divergence Theorem.

Concept Check

105–112 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

105. If a vector field $\mathbf{F}$ is constant, then $\nabla \times \mathbf{F} = \mathbf{0}$.

106. If a vector field $\mathbf{F}$ is constant, then $\nabla \cdot \mathbf{F} = 0$.

107. A vector field is conservative if and only if its curl is zero.

108. The Fundamental Theorem for Line Integrals can only be used if the underlying vector field is conservative.

109. If a force field $\mathbf{F}$ is conservative, then the work done by $\mathbf{F}$ on a particle moving along a smooth path is zero.

110. Green's Theorem and Stokes' Theorem are unrelated.

111. When a charged particle moves along a piecewise smooth closed curve in an electric force field, the total work done by the force field is zero.

112. If the vector field $\mathbf{F}$ has continuous partials in an open neighborhood of the closed, piecewise smooth surface S, then $\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, d\sigma = 0$.

Chapter 15
Technology Exercises

113–116 Use a graphing utility to find the mass and center of mass of the solid with the given density.

113. The solid bounded by the cone $z = \sqrt{4 - x^2 - y^2}$ and the xy-plane, with constant density ρ

114. The solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane, with constant density ρ

115. The solid of Exercise 114, with its density at any point being proportional to the distance from the xy-plane

116. The solid of Exercise 114, with its density at any point being proportional to the distance from the z-axis

©HAWKES LEARNING