Chapter 8
Review Exercises

1–4 Determine whether the differential equation is separable, linear, or autonomous; and find its order. (Note that more than one description may be applicable.)

1. $\dfrac{y'}{y+1} = y^3 - 2$

2. $xy\,dy = \sqrt{1-x^2}\,dx$

3. $xy' = 2y + x^4e^x - y'$

4. $2y'' = y - 3y'$

5–6 Solve the differential equation with the given initial condition.

5. $\left(x^2+1\right)y' = x; \quad y(0) = 1$

6. $y'\sec x = \sin x; \quad y(0) = 0$

7–12 Solve the separable differential equation.

7. $xy' = 2y$

8. $y' = e^{-y}\cos x$

9. $x^2y' = 2\sqrt{y}\,(x+1)$

10. $x^2\,du = u^2\,dx$

11. $x\,du = \left(u^2-1\right)dx$

12. $\dfrac{y'}{3} = e^{3x-y}$

13–16 Solve the given initial value problem.

13. $y' = \dfrac{9x^2+2x}{3y^2}; \quad y(0) = 2$

14. $x' = x\sin t; \quad x(\pi/2) = 5$

15. $\dfrac{dy}{dt} = 4t\sqrt{y-2}; \quad y(1) = 6$

16. $\dfrac{dy}{dx} = 2xy + y - 4x - 2; \quad y(0) = 0$

17–22 Solve the linear differential equation. (**Hint:** In some cases, *x* has to be the dependent variable in order for the equation to be linear.)

17. $y' - \dfrac{y}{x} = 0$

18. $xy' + y = x^2\sin x$

19. $xy' + 6y = 2$

20. $y' - 4xy = 2x$

21. $\left(y^2+1\right)dx + 2xy\,dy = 4y\,dy$

22. $dx + x\cot y\,dy = \cos y\,dy$

23–24 Find a first-order linear differential equation in standard form that has the given general solution. (**Hint:** Identify the integrating factor and "reverse" the solution technique discussed in this chapter.)

23. $y = Ce^{x^3} - \dfrac{1}{3}$

24. $y = \dfrac{C}{\sqrt{x^2-1}} - 1$

25–28 Solve the given initial value problem.

25. $xy' + (x-1)y = x^3; \quad y(1) = 1$

26. $\dfrac{dy}{dx} - 2xy = 2xe^{x^2}; \quad y(0) = 2$

27. $\left(x^2+1\right)y' + xy = \dfrac{1}{x^2+1}; \quad y(0) = -3$

28. $(\cos t)\dfrac{dy}{dt} + (\sin t)y = 1; \quad y(0) = 1$

29–32 Match the differential equation with its slope field (labeled A–D). Classify each equilibrium solution as stable or unstable.

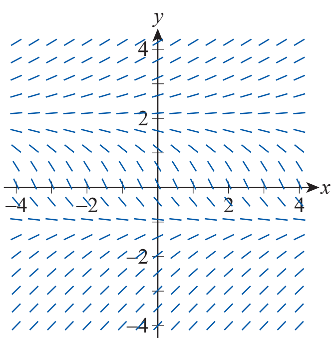
29. $y' = y^2 + 2y$

30. $y' = 9y - y^3$

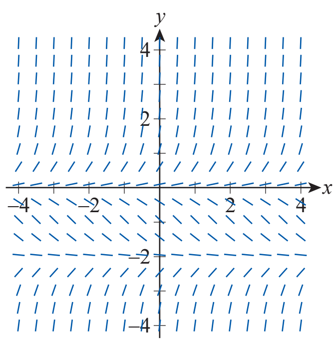
31. $y' = \dfrac{y^2-y-2}{y^2+1}$

32. $y' = y^4 - 8y$

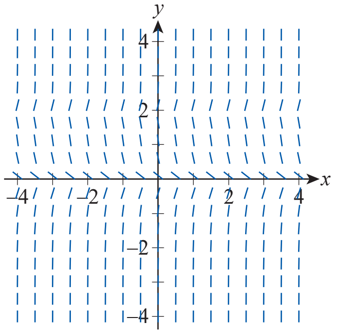
A.



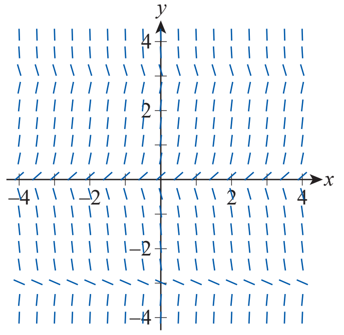
B.



C.



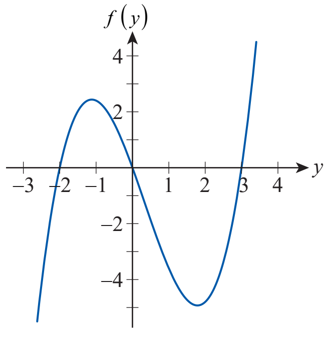
D.



33–34 Graph by hand the slope field of the given differential equation. If applicable, find and classify each equilibrium solution as stable or unstable.

33. $y' = y^2 - 9$ **34.** $y' = y^3 + y^2 - 2y$

35. Create a rough sketch of the slope field of the differential equation $y' = f(y)$, where the graph of f is given below. Classify equilibria as stable or unstable.



36–37 For the initial value problem, **a.** use Euler's method with the indicated step sizes to approximate the given value of y and **b.** solve the IVP by conventional methods and compare your approximations with the exact value.

36. $y' = 2y + 1;\quad y(0) = 1;$
approximate $y(1)$ with (i) $h = 0.2$ (ii) $h = 0.1$

37. $y' = y - x;\quad y(0) = 2;$
approximate $y(2)$ with (i) $h = 0.5$ (ii) $h = 0.25$

38–43 Find the general solution of the second-order homogeneous linear equation.

38. $2y'' + 3y' - 2y = 0$ **39.** $y'' - 16y = 0$

40. $y'' - 2y' + 2y = 0$ **41.** $y'' - 5y' = 0$

42. $y'' + 9y = 0$ **43.** $4y'' - 4y' + y = 0$

44–47 Solve the given second-order initial value problem.

44. $9y'' - 6y' + y = 0;\quad y(0) = 3;\quad y'(0) = 2$

45. $y'' - y' - 6y = 0;\quad y(0) = 3;\quad y'(0) = -1$

46. $y'' + 2y' + 3y = 0;\quad y(0) = 0;\quad y'(0) = 2$

47. $2y'' - 3y' = 0;\quad y(2) = -5;\quad y'(2) = 3$

48–49 Solve the boundary value problem, if possible.

48. $2y'' + y = 0;\quad y(0) = 2;\quad y\left(\pi/\sqrt{2}\right) = -4$

49. $y'' + 4y = 0;\quad y(0) = 0;\quad y(\pi) = 1$

50. Find the general solution of the third-order equation $y''' - 3y'' - y' + 3y = 0$. (**Hint:** See Exercises 35–38 in Section 8.4.)

51. Find the general solution of the nonhomogeneous equation $2y'' - 7y' + 3y = \cos x$. As your initial guess, use $y_p = A\cos x + B\sin x$. (**Hint:** See Exercises 39–42 in Section 8.4.)

52–53 Determine the orthogonal trajectories of the family of curves, where a is an arbitrary nonzero constant.

52. $y = ax^4$ **53.** $y = \dfrac{ax}{\sqrt{x^2+1}}$

54. A 500-liter tank is filled with water holding 5 kilograms of salt in the solution. Through an inlet, a stronger solution with salt concentration of 0.05 kilograms per liter is being added at a rate of 16 liters per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the same rate. What is the amount of salt in the tank after 25 minutes?

55. Answer the question of Exercise 54 if the contents of the tank are drained out at a rate of 20 liters per minute.

56.* A container in a lab contains 14 gallons of pure distilled water. 10% and 25% acid solutions are pumped into the container through two respective inlets. The 10% solution is flowing in at a rate of 0.2 gallons per minute, while the 25% solution is being allowed in by the second inlet at a rate of 0.5 gallons per minute. The contents of the tank are continuously and thoroughly mixed and drained out at the rate of 0.7 gallons per minute. How long does it take to form 14 gallons of 14% solution in this way?

57. A hailstone is melting so that its volume $V(t)$ decreases at a rate proportional to its surface area.

a. Assuming that the hailstone is nearly spherical, find a differential equation satisfied by $V(t)$.

b. If a hailstone of diameter 1 inch loses 20% of its volume in half an hour, predict how long it takes for it to completely melt away. (Consider it melted away when your model predicts less than 1 percent remaining).

58. If a vertical cylindrical tank of radius $\frac{1}{2}$ meters and height 4 meters is initially full of water but is draining through a circular orifice of diameter 2 centimeters that is on the bottom of the tank, what is the water level in the tank 2 minutes later? (**Hint:** See Exercise 59 in Section 8.1.)

59. Find the charge $q(t)$ of the 10^2-farad capacitor in an RC circuit if the impressed voltage on the circuit is $V(t) = t$ and the resistance is 25 ohms. Assume $q(0) = 0$. (**Hint:** See Exercise 62 in Section 8.1.)

60. Suppose that the impressed voltage in a simple RL circuit is $V(t) = 2t$, $I(0) = 0$, the inductance is 0.1 henries, and the resistance 0.5 ohms. Find the electric current I at time $t = 4$ seconds. (**Hint:** See Example 5 in Section 8.2.)

61. A baking dish is removed from a 210 °C oven and left at 20 °C room temperature. Two and a half minutes later the dish's temperature is 155 °C. Find the bakeware's temperature 10 minutes after it was removed from the oven. (**Hint:** See Example 2 in Section 8.3.)

62. A snapping turtle population grows logistically with a carrying capacity of 200 turtles and constant of proportionality $k = 0.2$ per year.

a. Find the population size $P(t)$ as a function of time if initially 50 turtles are present in the habitat. (**Hint:** See Example 3 in Section 8.3.)

b. How long does it take for the population to reach 100 turtles?

63. Suppose that an object of mass 200 grams stretches a spring by 10 centimeters. If it is pulled upward to a position of 5 centimeters above equilibrium and released with a downward velocity of 1 m/s, find and graph the resulting displacement function, assuming that the surrounding medium offers resistance with a damping constant of $c = 0.5\text{ kg/s}$. (**Hint:** See Example 5 in Section 8.4.)

Concept Check

64–70 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

64. The equation $(y')^2 + xy' - 3y = 0$ is a first-order differential equation.

65. The equation $y' = -y$ is not linear.

66. If $y_1(x)$ and $y_2(x)$ are solutions of a homogeneous linear differential equation, then so is $3y_1(x) - 2y_2(x)$.

67. Only autonomous equations have slope fields.

68. The logistic equation discussed in this text is autonomous.

69. The equations $y = 2e^{x/2}$ and $y = xe^{x/2}$ are linearly independent solutions of $4y'' - 4y' + y = 0$.

70. A second-order BVP with two boundary conditions always has a solution.

Chapter 8 Technology Exercises

71–72. Use a graphing utility to display the slope fields of the differential equations in Exercises 33 and 34. Compare the graphs to your original sketches.

73. Write a program for a computer algebra system that accepts a spring constant, a damping constant, and the mass of an oscillating object as inputs, and graphs the displacement function as output. Use it to check your answer for Exercise 63.