Chapter 5
Review Exercises

1–2 Use $(O_4 + U_4)/2$ to estimate the area under the graph of the function and above the x–axis on the given interval.

1. $f(x) = \dfrac{x^2}{2}$ on $[0,2]$

2. $f(x) = \sin x$ on $\left[0, \dfrac{\pi}{2}\right]$

3–6 Write the given sum using sigma notation.

3. $\dfrac{1}{2} - \dfrac{1}{9} + \dfrac{1}{28} - \dfrac{1}{65} + \cdots - \dfrac{1}{1{,}000{,}001}$

4. $a_1 + a_5 + a_9 + a_{13} + \cdots + a_{97}$

5. $f\left(\dfrac{2}{n^2}\right) + f\left(\dfrac{4}{n^2}\right) + f\left(\dfrac{6}{n^2}\right) + \cdots + f\left(\dfrac{100}{n^2}\right)$

6. $g(t_{-2}^*)\Delta t + g(t_{-1}^*)\Delta t + g(t_0^*)\Delta t + \cdots + g(t_{2n}^*)\Delta t$

7–8 Assuming that $\sum_{i=0}^{n} a_i = 50$ and $\sum_{i=0}^{n} b_i = 80$, find the sum.

7. $\sum_{i=0}^{n} (a_i + 2b_i + 2)$

8. $\sum_{i=0}^{n} \left(\dfrac{a_i}{5} - \dfrac{b_i}{4}\right)$

9–10 Use summation formulas to find the value of the sum.

9. $\sum_{i=1}^{10} (3i^3 - 1)$

10. $\sum_{j=1}^{n} \dfrac{(2j+1)(j-2)}{2}$

11. Find the value of the sum $\sum_{i=1}^{n} \left[\dfrac{1}{i^2} - \dfrac{1}{(i+1)^2}\right]$.

(**Hint:** Write out the first few terms as well as the last few terms.)

12–13 Evaluate the geometric sum using the formula proven in Exercise 53 of Section 5.1.

12. $\sum_{i=0}^{10} \dfrac{3}{2^i}$

13. $\sum_{j=0}^{6} (-1)^j (0.3)^j$

14–15 Evaluate the given double sum.

14. $\sum_{i=3}^{11} \sum_{j=1}^{4} (2i - j)$

15. $\sum_{i=1}^{n} \sum_{j=1}^{m} i^2 j$

16–18 Find the area under the graph of $f(x)$ and above the x–axis on the given interval, by taking the limit of the associated Riemann sums.

16. $f(x) = x^2 + 1$ on $[0,2]$

17. $f(x) = x^3$ on $[0,1]$

18. $f(x) = \sqrt{x}$ on $[0,4]$

(**Hint:** Choose $x_i^* = \dfrac{4(i-1)^2}{n^2}$.)

19. Identify the region whose area is the limit given by $\lim_{n\to\infty} \dfrac{2}{n} \sum_{i=1}^{n} \left[\dfrac{4i}{n} - \left(\dfrac{2i}{n}\right)^2\right]$. Then use summation formulas to evaluate the limit.

20–21 Prove that the given function is not integrable on the interval $[0,1]$.

20. $f(x) = \dfrac{1}{x^2}$

21. $g(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$

22–25 Suppose that f is an even function, g is odd, both are integrable on $[-a,a]$, and we know that $\int_0^a f(x)\,dx = 2$, while $\int_0^a g(x)\,dx = 0.5$ $(a > 0)$. If possible, find the integral.

22. $\int_{-a}^{a} [5f(x) + 4g(x)]\,dx$

23. $\int_{-a}^{a} [f(x)]^2 g(x)\,dx$

24. $\int_{-a}^{a} f(x)[g(x)]^2\,dx$

25. $\int_{-a}^{0} [f(x) + g(x)]\,dx$

26–27 Find the average value of $f(x)$ over the given interval and identify all points in the domain where $f(x)$ assumes its average value.

26. $f(x) = 4x - x^2$ on $[0,4]$

27. $f(x) = |x-2| - 1$ on $[1,5]$

28. Use the Fundamental Theorem of Calculus to evaluate the limit

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\frac{i}{n}}-\frac{i}{n}\right)$$

by recognizing it as a Riemann sum of a function over an interval.

29–30 Use Part I of the Fundamental Theorem of Calculus to find the derivative of the given function.

29. $F(x)=\int_0^x \sqrt{1+t^2}\,dt$ **30.** $G(x)=\int_0^{x^2} e^{t^2}\,dt$

31–38 Use Part II of the Fundamental Theorem of Calculus to evaluate the definite integral.

31. $\int_1^2 \left(2x^4+3x^2-2\right)dx$ **32.** $\int_0^2 (3x+2)(5-x)\,dx$

33. $\int_1^4 \left(\frac{1}{t}-\frac{2}{t^2}+1\right)dt$ **34.** $\int_1^9 \frac{x^2-2\sqrt{x}+2}{x}\,dx$

35. $\int_0^1 \frac{2}{\sqrt{1-x^2}}\,dx$

36. $\int_{\pi/4}^{3\pi/4} \left(2\csc^2 x-\cos x\right)dx$

37. $\int_2^3 \frac{x+2}{x-1}\,dx$ **38.** $\int_0^1 \frac{2x^2-1}{x^2+1}\,dx$

39–40 Find the area of the region between the graph of the given function and the x-axis on the indicated interval.

39. $y=\frac{1}{2x^2}$ on $[1,10]$

40. $y=2\sqrt{x}-x^2$ on $[0,2]$

41. Find a formula for $f(x)$ if $\int_0^{x^3} f(t)\,dt=\sin\left(x^3\right)$.

42. The velocity function of a particle moving along the x–axis is $v(t)=3t-t^2$ units per second. If it started at the origin, find **a.** the position of the particle at $t=5$ seconds and **b.** the total distance traveled by the particle in the time interval $[0,5]$.

43–54 Use an appropriate substitution (when necessary) to evaluate the indefinite integral.

43. $\int \frac{-2}{\sqrt{1-x^2}}\,dx$ **44.** $\int \frac{-2x}{\sqrt{1-x^2}}\,dx$

45. $\int \sec x(\sec x+\tan x)\,dx$

46. $\int \sec^2 x\tan x\,dx$

47. $\int 6x^2\left(2x^3-7\right)^9 dx$ **48.** $\int x^4\sqrt{x^5-3}\,dx$

49. $\int \frac{4x}{\left(x^2+1\right)^2}\,dx$ **50.** $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$

51. $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$ **52.** $\int \frac{1}{x\ln\left(x^2\right)}\,dx$

53. $\int \frac{e^{2x}}{e^x+1}\,dx$ **54.** $\int \frac{1}{x^2}\sin\left(\frac{x+1}{x}\right)dx$

55–56 Find $y(x)$ that satisfies the given conditions.

55. $\frac{dy}{dx}=\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)^2};\quad y(9)=1$

56. $y''(x)=1-\sin x;\quad y'(0)=2;\quad y(0)=0$

57. A particle is moving along the x-axis in the positive direction with a velocity function of $v(t)=\frac{t}{t^2+1}$ units per second. If it started at the point $(1,0)$, what is the particle's position at $t=4$ seconds?

58–63 Evaluate the definite integral.

58. $\int_0^1 3x^5\left(x^6-1\right)^{12} dx$

59. $\int_0^4 (x+1)\sqrt{x^2+2x}\,dx$

60. $\int_1^2 \frac{x^3}{x^4+1}\,dx$ **61.** $\int_{-1/2}^0 \frac{2^t}{\sqrt{1-4^t}}\,dt$

62. $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx$ **63.** $\int_1^4 \frac{dx}{\sqrt{x}\left(\sqrt{x}+1\right)^2}$

64–69 Find the area of the region bounded by the graphs of the given equations. (If convenient or necessary, integrate with respect to y rather than x.)

64. $y=x^3-4x,\quad 3y=15x,\quad x\ge 0$

65. $y=1-x^2,\quad y=1-x^6,\quad x\ge 0$

66. $y=2\sqrt[4]{x},\quad y=4-2x,\quad y=0$

67. $y=\ln x,\quad (e-1)y=x-1$

68. $y=\frac{1}{1+x^2},\quad 2y=1$

69. $y=\sin x,\quad y=\sin x\cos x,\quad 0\le x\le \pi$

70. Consider the region bounded by the graph of $y=1/x$ and the x-axis over the interval $[1,a]$ $(a>1)$. Find the vertical line $x=c$ that bisects the region in two subregions of equal area.

71. Consider the function $f(x) = 1/x^2$ defined on some interval $[a,b]$. Partition $[a,b]$ and in each subinterval $[x_{i-1}, x_i]$ choose the sample point $x_i^* = \sqrt{x_{i-1}x_i}$ (the geometric mean of the endpoints). Show that

$$\frac{1}{\left(x_i^*\right)^2}\Delta x_i = \frac{1}{x_{i-1}} - \frac{1}{x_i}$$

and use this observation to prove the following formula.

$$\int_a^b \frac{1}{x^2}\,dx = \frac{1}{a} - \frac{1}{b}$$

72. Prove that if the conditions of Part I of the Fundamental Theorem of Calculus are satisfied and $F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$, where $g(x)$ and $h(x)$ are differentiable, then $F'(x) = f(h(x))h'(x) - f(g(x))g'(x)$. (**Hint:** See Example 3 of Section 5.3.)

73. Prove that if f is a linear function, then its definite integral on an interval $[a,b]$ is the average of its left and right Riemann sums, that is,

$$\int_a^b f(x)\,dx = \frac{L_n + R_n}{2}.$$

What is your expectation regarding the integral and the average above if f is concave up? Concave down?

Concept Check

74–81 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

74. If $n_1 < n_2$, then the Riemann sum R_{n_2} is always a better approximation of the integral than R_{n_1}.

75. If f is piecewise continuous on a closed interval, then the limit of its Riemann sums always exists.

76. When applying the Fundamental Theorem of Calculus, we must choose the antiderivative with $C = 0$.

77. $\int \frac{1}{e^x}\,dx = \ln\left(e^x\right) + C = x + C$

78. The definite integral of the velocity function of a moving object on $[t_1, t_2]$ is equal to the total distance traveled by the object from time $t = t_1$ to $t = t_2$.

79. $\int_a^b f(x)\,dx > 0$ if and only if $f(x) > 0$ on $[a,b]$.

80. $\int \sec x\,dx = \sec x \tan x + C$

81. $\int_{-1}^{1} \frac{1}{x^3}\,dx = \left.\frac{-1}{2x^2}\right]_{-1}^{1} = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

Chapter 5
Technology Exercises

82. Use the summation feature of a graphing utility to verify your answers for Exercises 9–15.

83. Write a program for a graphing calculator or computer algebra system that calculates the n^{th} Riemann sum for a given function on a given interval, using subintervals of equal width and sample points of your choice. Use your program to verify your answers for Exercises 16–18.

84. Use a graphing utility to evaluate the limit of Exercise 28. What do you find? (Answer will vary depending on the capabilities of the particular software used.)