Chapter 11
Review Exercises

1–6 Find the distance from the point to the indicated plane.

1. $(-2,0,4)$; the yz-plane
2. $(3,-2,5)$; the plane $y=1$
3. $(0,0,0)$; $4x+y-z-3=0$
4. $(1,-2,1)$; $x-y+z-5=0$
5. $\left(3,0,\frac{1}{2}\right)$; $y-2z=x+4$
6. $(-1,5,2)$; $x+4=4y+2z$

7–12 Describe the set of points represented by the given equation or inequality.

7. $x^2+(y-2)^2+z^2>1$
8. $x-2z-1\ge y$
9. $x^2+4z^2<2$
10. $xyz^2\ne 0$
11. $x^2+2y^2-4z^2\le 8$
12. $6x^2+2z^2<3y$

13–14 If $\mathbf{u}=\langle 2,5,-4\rangle$ and $\mathbf{v}=\langle -1,4,-2\rangle$, find the coordinates of the endpoint of the indicated vector with the given initial point.

13. $2\mathbf{u}-3\mathbf{v}$ with initial point $(2,-7,3)$
14. $\frac{1}{2}\mathbf{v}-4\mathbf{u}$ with initial point $(-1,4,-5)$
15. Find a vector $\mathbf{v}$ that solves the vector equation $\langle -6,2,0\rangle-2\mathbf{v}=\langle 1,0,-1\rangle$.
16. Explain the difference between the following expressions: $(1,2,3)$ vs. $\langle 1,2,3\rangle$ vs. $\{1,2,3\}$.
17. Determine whether the following points are collinear: $P(1,-3,0)$, $Q(2,5,1)$, and $R(4,21,3)$.

18–19 Find the component form and magnitude of the vector $\overrightarrow{PQ}$.

18. $P(2,-4,0)$, $Q(1,-3,-1)$
19. $P\left(\frac{\sqrt{3}}{2},-1,1\right)$, $Q\left(2\sqrt{3},-2,3\right)$

20–21 Use vectors to find the coordinates of the indicated point.

20. The point two-thirds of the way from $P(-6,0,3)$ to $Q(0,12,9)$
21. The point three-eighths of the way from $P(0,-4,8)$ to $Q(-16,0,2)$

22–23 Find the unit vector $\mathbf{u}$ pointing in the direction of the given vector $\mathbf{v}$.

22. $\mathbf{v}=\langle 2,4,6\rangle$
23. $\mathbf{v}=\left\langle 1,-\frac{1}{4},\frac{1}{3}\right\rangle$

24–26 Find the dot product of $\mathbf{u}$ and $\mathbf{v}$.

24. $\mathbf{u}=2\mathbf{i}-5\mathbf{j}$, $\mathbf{v}=3\mathbf{i}-\mathbf{k}$
25. $\mathbf{u}=\frac{1}{3}\mathbf{i}+\pi\mathbf{j}-\frac{7}{2}\mathbf{k}$, $\mathbf{v}=-6\mathbf{i}-\frac{2}{\pi}\mathbf{j}+8\mathbf{k}$
26. $|\mathbf{u}|=3\sqrt{3}$, $|\mathbf{v}|=2$, their angle is $30°$

27–28 Find the angle between the given vectors.

27. $\mathbf{u}=\langle 1,0,1\rangle$, $\mathbf{v}=\langle 2,2,0\rangle$
28. $\mathbf{u}=-\mathbf{j}-\sqrt{2}\,\mathbf{k}$, $\mathbf{v}=\mathbf{i}-\sqrt{3}\,\mathbf{k}$

29–30 Determine the value of the parameter so that the vectors are **a.** parallel, **b.** orthogonal.

29. $\langle s,4,2\rangle$ and $\langle 1,2s,\sqrt{2}\rangle$
30. $\langle 2s,1,3\rangle$ and $\langle 1,4s,3\sqrt{2}\rangle$
31. Find the angle between the planes $x+4y+3z=2$ and $4z-3x-y=11$. (**Hint:** The angle between two planes is the same as that between their respective normal vectors.)

32–33 Find the direction angles of the given vector.

32. $\langle -1,3,4\rangle$
33. $\langle \sqrt{3},2,1\rangle$

34–37 Find the decomposition of $\mathbf{u}$ into a sum of two vectors, one parallel to, and the other perpendicular to $\mathbf{v}$.

34. $\mathbf{u}=\langle 1,3,0\rangle$, $\mathbf{v}=\langle 2,0,-4\rangle$
35. $\mathbf{u}=\left\langle \frac{1}{3},-3,1\right\rangle$, $\mathbf{v}=\left\langle 2,-5,\frac{5}{3}\right\rangle$
36. $\mathbf{u}=3\mathbf{i}-\mathbf{j}+\mathbf{k}$, $\mathbf{v}=\mathbf{i}+2\mathbf{j}-4\mathbf{k}$
37. $\mathbf{u}=\mathbf{i}-2\mathbf{j}+\frac{1}{2}\mathbf{k}$, $\mathbf{v}=\frac{1}{2}\mathbf{i}+\mathbf{j}-\mathbf{k}$

38–39 Find the work done by the force **F** as it moves an object from *P* to *Q*. (Suppose **F** is measured in newtons, and a unit distance is 1 meter.)

38. $\mathbf{F} = 2.1\,\mathbf{i} - 2.4\,\mathbf{j} - 5.7\,\mathbf{k}$ from $P(5,8,4)$ to $Q(-3,0,1.5)$

39. $\mathbf{F} = 3\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}$ from $P(0,-2,-17.3)$ to $Q(2,9.5,-11)$

40–41 Use the determinant formula to find the cross product.

40. $\langle 3,0,-2\rangle \times \langle 1,2,1\rangle$

41. $\langle 2,1,-1\rangle \times \left\langle \frac{1}{2},4,-2\right\rangle$

42–43 Find both unit vectors perpendicular to **u** and **v**.

42. $\mathbf{u} = \langle 0,-1,1\rangle, \quad \mathbf{v} = \langle 1,2,-1\rangle$

43. $\mathbf{u} = \left\langle -2,\frac{1}{2},1\right\rangle, \quad \mathbf{v} = \langle 0,2,4\rangle$

44–45 Construct a vector normal to the plane containing the indicated points.

44. $P(1,-1,1), \quad Q(0,2,2), \quad R(3,0,3)$

45. $P(-4,0,5), \quad Q\left(1,-\frac{1}{2},1\right), \quad R(2,3,0)$

46–47 Use cross products to check whether the points *P*, *Q*, and *R* are collinear.

46. $P(-1,0,1), \quad Q(1,-2,0), \quad R(1,1,3)$

47. $P(2,1,-2), \quad Q(3,2,-3), \quad R(-1,-2,1)$

48–49 Find the area of the triangle with the given vertices.

48. $A(0,1,2), \quad B\left(-\frac{1}{2},2,1\right), \quad C\left(\frac{3}{2},-3,4\right)$

49. $A(2,1,0), \quad B(3,-1,2), \quad C(-3,0,1)$

50–53 Let $\mathbf{u} = \langle 2,1,-3\rangle$, $\mathbf{v} = \langle 1,0,-4\rangle$, and $\mathbf{w} = \langle 1,-1,\frac{1}{2}\rangle$. If possible, find each of the following.

50. $\mathbf{u} \times (\mathbf{v} + \mathbf{w})$

51. $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$

52. $(\mathbf{u} \cdot \mathbf{v}) + \mathbf{w}$

53. $(\mathbf{u} + \mathbf{v}) \times \mathbf{w}$

54–55 Find the volume of the parallelepiped spanned by the indicated vectors.

54. $\mathbf{u} = \langle 1,1,1\rangle, \quad \mathbf{v} = \langle 2,0,3\rangle, \quad \mathbf{w} = \langle 1,-2,1\rangle$

55. $\mathbf{u} = \langle 2,1,1\rangle, \quad \mathbf{v} = \langle 1,2,1\rangle, \quad \mathbf{w} = \langle 1,1,2\rangle$

56–57 Find parametric equations for the line parallel to the indicated direction vector and passing through the given point.

56. Through the point $(3,4,-7)$ and parallel to $2\,\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$

57. Through the point $\left(\frac{5}{2},9,0\right)$ and parallel to $\left\langle 2,\frac{1}{2},-\frac{3}{4}\right\rangle$

58–59 Give a vector description of the line segment between the two given points.

58. The line segment between $(1.2,-2,5)$ and $(2.2,0,9)$

59. The line segment between $\left(4,-\frac{3}{2},7\right)$ and $(6,2,5)$

60–63 If possible, determine the point of intersection of the pair of lines.

60. $\mathbf{r}(t) = \langle 1+2t, 3-2t, 4-6t\rangle$ and $\mathbf{s}(u) = \langle 2u-1, u-1, -u\rangle$

61. $\mathbf{r}(t) = \left\langle 3-\frac{7}{2}t, 1-\frac{t}{2}, t-1\right\rangle$ and $\mathbf{s}(u) = \langle 1-5u, 2u-2, u\rangle$

62. $\mathbf{r}(t) = \langle 1+t, 3t, 2t-3\rangle$ and $\mathbf{s}(u) = \langle u-2, 4+u, u\rangle$

63. $\mathbf{r}(t) = \langle 2-2t, 1+3t, 4t\rangle$ and $\mathbf{s}(u) = \langle 2+u, 3-3u, 1+u\rangle$

64–65 Identify the plane containing the given point and having the indicated normal vector as a two-parameter set of points in $\mathbb{R}^3$.

64. The plane through the point $(3,-2,-1)$ with normal vector $\mathbf{n} = \langle 1,-2,-3\rangle$

65. The plane through the point $(4,-1,3)$ with normal vector $\mathbf{n} = \left\langle \frac{3}{2},-\frac{1}{3},2\right\rangle$

66–67 Find an equation for the plane containing the given points.

66. $A(1,0,2), \quad B(2,3,-2), \quad C(-2,1,3)$

67. $A\left(-\frac{1}{2},2,0\right), \quad B(4,1,1), \quad C(-1,-1,1)$

68–70 Find the parametric equations of the line as described.

68. The line through the point $(-3,1,2)$ that is perpendicular to the plane $x - 2y + z = 11$

69. The line through $\left(-2,2,\frac{1}{2}\right)$ that is parallel to the line $\mathbf{r}(t) = \langle t-2, 1+3t, 1-2t\rangle$

70. The line through $(-4,-5,-6)$ that is perpendicular to the vectors $3\mathbf{i}+\mathbf{k}$ and $\mathbf{j}-2\mathbf{k}$

71–72 Find an equation for the plane satisfying the given conditions.

71. The plane through the point $(0,-1,0)$ that forms an angle of $\pi/3$ radians with the positive y-axis

72. The plane through the point $(-5,2,1)$ that contains the line $x = 1 + 2t$, $y = 3 - t$, $z = 4t - 1$

73–74 Find the point of intersection between the given line and plane.

73. The line $\mathbf{r}(t) = \langle 1+2t, 5-4t, 6t-1 \rangle$ and the plane $2x + y - 3z = 1$

74. The line $\mathbf{r}(t) = \langle 1+t, 5-2t, t \rangle$ and the plane $x + 4y - 2z = 3$

75–76 Find parametric equations for the line formed by the intersection of the two given planes.

75. $2x + y - z = 8, \quad 5x - y + 3z = -1$

76. $3x + 2y - z = 0, \quad x - y + 4z = 5$

77. Find an equation for the set of points that are equidistant from the points $(-8,11,3)$ and $(10,-1,-9)$.

78–79 Find the shortest distance d between the given skew lines.

78. $\mathbf{r}(t) = \langle -2+2t, -t, 1+2t \rangle$ and $\mathbf{s}(u) = \langle 3-2u, 5+u, 4+u \rangle$

79. $\mathbf{r}(t) = \langle 4+3t, -5, 2-t \rangle$ and $\mathbf{s}(u) = \langle 3+u, 1-u, 7+2u \rangle$

80–87 Identify the surface defined by the equation and match it to the appropriate graph (labeled A–H).

80. $2x^2 + y^2 + 2z^2 = 12$

81. $z^2 + 2y^2 = 4$

82. $8z^2 + 2y^2 - 4x^2 = 0$

83. $(x+2)^2 + 2y = x^2 + z$

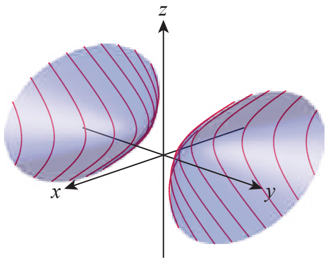
84. $8z^2 - 2y^2 - 4x = 0$

85. $x^2 + 2z^2 - 4z - y^2 + 6 = 0$

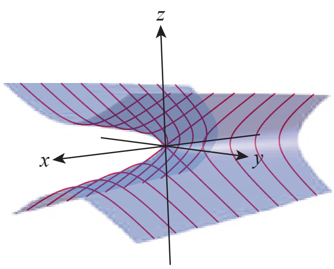
86. $3x^2 + 4y^2 + 6z = 0$

87. $\dfrac{x^2}{2} + \dfrac{y^2}{4} - \dfrac{z^2}{3} = 1$

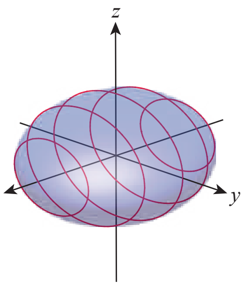
A.



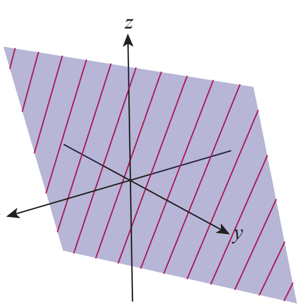
B.



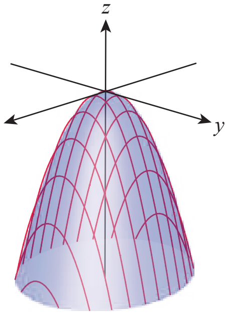
C.



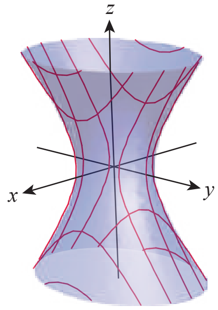
D.



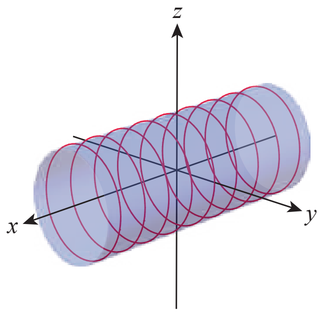
E.



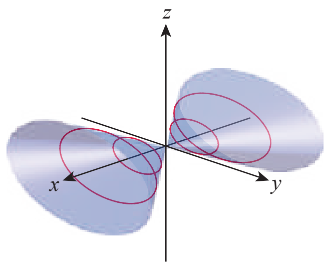
F.



G.



H.



88–93 Use the equation to identify the quadric surface.

88. $225x^2 + 100y^2 - 36(z-2)^2 = 0$

89. $4(y+1)^2 - 16z^2 = x$

90. $\frac{x^2}{4} + x - \frac{y}{3} + z^2 + 1 = 0$

91. $x^2 - 2y^2 + 4y + 3z^2 = 12$

92. $2x^2 + 4y^2 + z^2 + 6z = 19$

93. $z^2 = 3x^2 + 2y^2 - 2z + 11$

94. An eight-pound weight is suspended from two ropes that form angles of 30° and 45°, respectively, with the horizontal direction. Find the tension forces $\mathbf{T}_1$ and $\mathbf{T}_2$.

95. A soccer player kicks the ball with an initial velocity that has a vertical component of 75 ft/s and a horizontal component of 15 ft/s. Ignoring air resistance and assuming the initial height is 0, determine the ball's velocity at time *t*, and sketch its position function.

96. A plane is flying 550 mph due east, so its velocity vector can be represented as $\langle 550, 0 \rangle$. It suddenly encounters a 65 mph tailwind that blows 45° east of north. Find the velocity vector and ground speed of the plane under these conditions.

97. A small plane flying at 140 mph due north encounters a 30 mph tailwind that blows from the direction of 30° west of north. At the same time, a 10 mph downdraft is affecting the plane's flight. Find the actual ground speed of the plane.

98. Find the acceleration of a 3.5 kg object if the following forces are acting on it: $\mathbf{F}_1 = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{F}_2 = \mathbf{j} + 3\mathbf{k}$, and $\mathbf{F}_3 = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ (units are in newtons).

99. A cable is pulling on a gate at an angle of 30° with a constant force of 120 pounds. Find the work done if the gate moves 18 feet to its closed position.

100. A 50-pound sled is being pulled up a 24° slope that is 80 feet long. If the rope makes a 21° angle with the surface of the slope, find **a.** the total work done and **b.** the force of tension in the rope (ignore friction and any acceleration of motion).

101. A mechanic tightens a bolt with a force of 80 newtons, applied at the end of a 20 cm wrench, at an angle of $\theta = 70°$. What is the magnitude of the torque applied to the bolt at the pivot point?

102. Find the magnitude of the force acted upon a 10 cm electric wire in a uniform magnetic field of $\mathbf{B} = 3.4$ T if it carries a current of 0.2 A and the angle between the wire and $\mathbf{B}$ is $\theta = 45°$. (**Hint:** See Exercise 61 of Section 11.4.)

103. Find the magnitude of the force that acts upon an electron moving in a uniform magnetic field of $\mathbf{B} = 0.003$ T at $\mathbf{v} = 2.8 \cdot 10^6$ m/s if the velocity vector and $\mathbf{B}$ form a 30° angle. (See Exercise 62 of Section 11.4.)

Concept Check

104–119 Determine whether the given statement is true or false. In case of a false statement, explain or provide a counterexample.

104. If vectors **u** and **v** have equal magnitude and are pointing in the same direction, but their starting points are different, then they are not equal as vectors.

105. For any nonzero vector **v** in the *xy*-plane there is an angle θ such that $\mathbf{v} = |\mathbf{v}|(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$.

106. Work is a vector quantity, pointing in the direction of the force doing the work.

107. If two vectors in three-dimensional space are equal, then their direction angles are equal.

108. The vector $\text{proj}_{\mathbf{v}}(\mathbf{u} \times \mathbf{v})$ is always the zero vector.

109. Torque always points in the direction of rotation.

110. If **u** and **v** are both vectors in the *xy*-plane, then it is impossible to find their cross product.

111. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})=\mathbf{v}\cdot(\mathbf{w}\times\mathbf{u})$

112. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})=\mathbf{w}\cdot(\mathbf{v}\times\mathbf{u})$

113. If **u**, **v**, and **w** are coplanar, then $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})=0$.

114. $\mathbf{u}\cdot[\mathbf{v}\times(\mathbf{w}+\mathbf{z})]=\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})+\mathbf{u}\cdot(\mathbf{v}\times\mathbf{z})$

115. If **u** is orthogonal to both **v** and **w**, then **u** is orthogonal to $\mathbf{v}\times\mathbf{w}$.

116. If **u** is orthogonal to both **v** and **w**, then **u** is orthogonal to $\mathbf{v}\cdot\mathbf{w}$.

117. $(\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}+\mathbf{v})=|\mathbf{u}|^2-|\mathbf{v}|^2$

118. If **v** is a vector in the plane with normal vector $\mathbf{n}=\langle n_1,n_2,n_3\rangle$, then $\mathbf{v}\cdot\mathbf{n}=0$.

119. If **v** is a vector in the plane that has normal vector $\mathbf{n}=\langle n_1,n_2,n_3\rangle$, then $\mathbf{v}\times\mathbf{n}$ is another vector in the said plane.

Chapter 11 Technology Exercises

120. Write a program for a computer algebra system that finds a unit vector pointing in the direction of $a\mathbf{u}+b\mathbf{v}$ for given vectors **u**, **v** and scalars *a* and *b*. Use it to find the unit vector pointing in the direction of $2\mathbf{u}-3\mathbf{v}$ of Exercise 13.

121. Write a program for a computer algebra system that returns parametric equations for the line formed by the intersection of two given planes. (The program should accept the equations of the planes and return parametric equations for the line, displaying an appropriate message if the planes are parallel.) Use your program to check your answers for Exercises 75 and 76.

122. Write a short program for a computer algebra system that finds the distance between a point and a plane in three-dimensional space. Use it to revisit Exercises 1–6.

123. Write a program for your computer algebra system to determine the angle, in degrees, between two given three-dimensional vectors. Use it to check your answers for Exercises 27–28.

124–129. Use a graphing utility to check your answers for Exercises 88–93 by graphing the equations.