

Answer Key

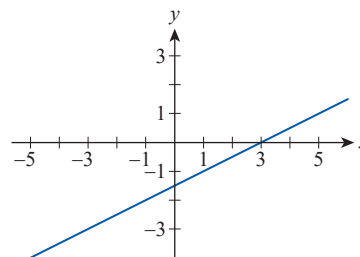
Chapter 1

Section 1.1

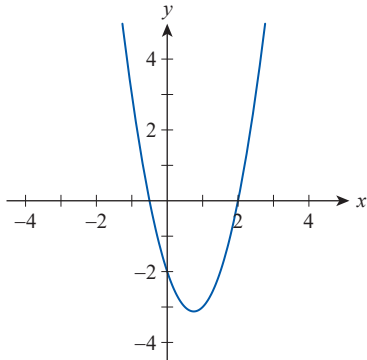
1. Dom = $\{-3, 0, 1\}$, Ran = $\{-1, 0, 1, 2, 5\}$
3. Dom = $\{-2, 3, 4\}$, Ran = $\{-8, 0, \cos 3, 5.98\}$
5. Dom = $\{\text{Tanisha, Don, Peter, David}\}$,
Ran = $\{\text{swimming, biking, skating, skateboarding}\}$
7. Dom = $\mathbb{Z}$, Ran = $\{\dots, -1, 1, 3, 5, \dots\}$
9. Dom = $\mathbb{R}$, Ran = $\{\dots, 3, 5, 7, 9, \dots\}$
11. Dom = $\mathbb{R}$, Ran = $\mathbb{R}$ 13. Dom = $\{5\}$, Ran = $\mathbb{R}$
15. Dom = $[-1, \infty)$, Ran = $\mathbb{R}$
17. Dom = $[-2, 4]$, Ran = $[-3, 3]$
19. Dom = $\mathbb{R}$, Ran = $[0, 1]$
21. Dom = $\mathbb{R}$, Ran = $\mathbb{R}$
23. Dom = $\{\text{Registered students}\}$,
Ran = $\{\text{Courses offered}\}$
25. Dom = $\{\text{People in a certain group}\}$,
Ran = $\{\text{Fathers of the people in the group}\}$
27. $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$
29. $\left\{ \begin{array}{l} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), \\ (2, 2), (2, 4), (3, 3), (4, 4), (5, 5) \end{array} \right\}$
31. Function
33. Not a function; -1 appears twice as first coordinate.
35. Not a function; it doesn't pass the vertical line test.
37. Function 39. Function 41. Function
43. Not a function; it doesn't pass the vertical line test.
45. Function 47. Function
49. Not a function; it doesn't pass the vertical line test.
(However, x is a function of y .)
51. Function (F is a function of r .)
53. $y = -\frac{1}{3}x + \frac{10}{3}$ 55. $y = x^2 + \frac{1}{3}x - \frac{5}{3}$
57. $y = \frac{3x+1}{x^2-1}$
59. a. $\frac{4}{3}$ b. $\frac{1}{3}x + \frac{7}{3}$ c. $\frac{1}{3}(x+h)+2$ d. $\frac{1}{3}$
61. a. 1 b. $x^2 + 2x - 2$ c. $(x+h)^2 - 3$ d. $2x + h$
63. a. Undefined b. $\sqrt{x+1}$
c. $\sqrt{x+h}$ d. $\frac{\sqrt{x+h}-\sqrt{x}}{h}$
65. a. -1 b. $\frac{1}{x+2}$
c. $\frac{1}{x+h+1}$ d. $-\frac{1}{(x+1)(x+h+1)}$
67. Dom = $\mathbb{N}$, Cod = $\mathbb{N}$, Ran = $\{2, 3, 4, 5, \dots\}$
69. Dom = $\mathbb{Z}$, Cod = $\mathbb{Z}$, Ran = $\{0, 1, 4, 9, 16, \dots\}$
71. Dom = $[0, \infty)$, Cod = $\mathbb{R}$, Ran = $[0, \infty)$
73. $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$
75. $(-\infty, 1) \cup (3, \infty)$ 77. $[0, 2]$
79. $(-\infty, -3/2) \cup (-3/2, \infty)$ 81. $\{\theta \in \mathbb{R} \mid \theta \neq 2k\pi, k \in \mathbb{Z}\}$
83. $C(r) = 2\pi r$ 85. $C(F) = \frac{5}{9}(F - 32)$
87. $V(b, h) = \frac{1}{3}b^2h$; a function of two variables
89. Function 91. Not a function 93. Not a function
95. Decreasing on $(-\infty, 1)$; increasing on $(1, \infty)$
97. Increasing on $(-\infty, -2), (2, \infty)$; decreasing on $(-2, 2)$
99. Decreasing on $(-\infty, 1)$; increasing on $(1, \infty)$
101. Decreasing on $(-\infty, -1)$; constant on $(-1, 2)$;
increasing on $(2, \infty)$
103. Symmetric with respect to the x -axis
105. Symmetric with respect to both axes and the origin
107. Symmetric with respect to the origin
109. Symmetric with respect to the x -axis
111. $P(A) = 4\sqrt{A}$ 113. $V(x) = x(30 - 2x)(20 - 2x)$
115. $V(r) = \frac{2\pi r^3}{3}$ 117. $C(F) = \frac{5}{9}(F - 32)$

Section 1.2

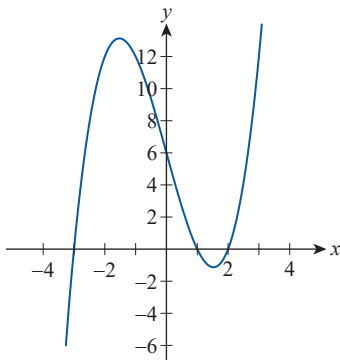
1. Deg.: 1; lead. coeff.: $1/2$;
 x -int.: 3; y -int.: $-3/2$; Ran = $\mathbb{R}$



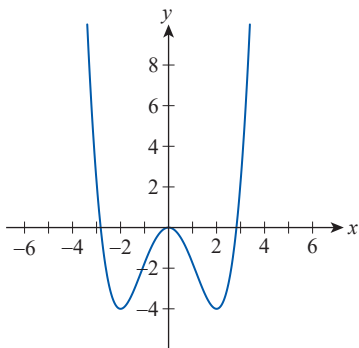
3. Deg.: 2; lead. coeff.: 2;
 x -int.: 2, $-1/2$; y -int.: -2 ; $\text{Ran} = [-25/8, \infty)$



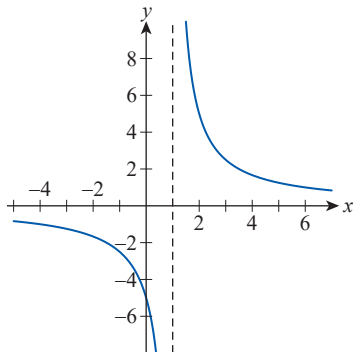
5. Deg.: 3; lead. coeff.: 1;
 x -int.: $-3, 1, 2$; y -int.: 6; $\text{Ran} = \mathbb{R}$



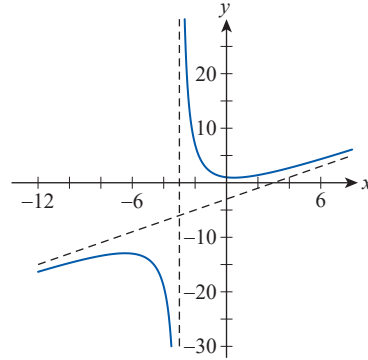
7. Deg.: 4; lead. coeff.: $1/4$;
 x -int.: $\pm 2\sqrt{2}, 0$; y -int.: 0; $\text{Ran} = [-4, \infty)$



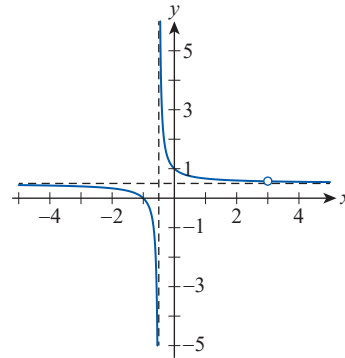
9. Vert. asym.: $x = 1$; horiz. asym.: $y = 0$;
 x -int.: none; y -int.: -5



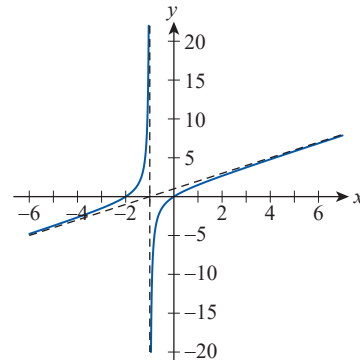
11. Vert. asym.: $x = -3$; slant asym.: $y = x - 3$;
 x -int.: none; y -int.: 1



13. Vert. asym.: $x = -1/2$; horiz. asym.: $y = 1/2$;
 x -int.: -1 ; y -int.: 1



15. Vert. asym.: $x = -1$; slant asym.: $y = x + 1$;
 x -int.: 0, -2 ; y -int.: 0



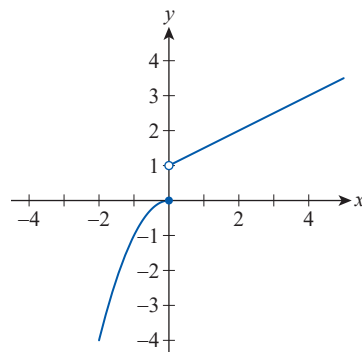
17. Answers will vary. 19. Answers will vary.

21. Answers will vary. 23. Answers will vary.

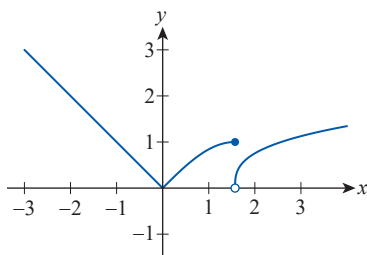
25. $\cos x$ 27. $\cos^2 \alpha$

29. $\cos^2 \theta$ 31. $\csc \beta$ 33. $\tan^2 x$

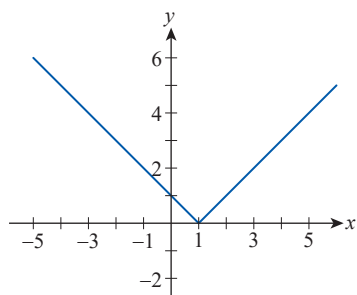
- 35.



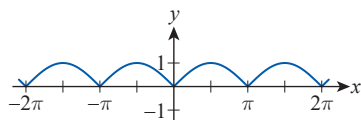
37.



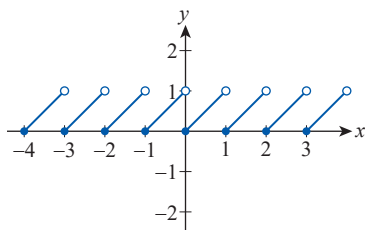
39.
$$f(x) = \begin{cases} -x+1 & \text{if } x < 1 \\ x-1 & \text{if } x \geq 1 \end{cases}$$



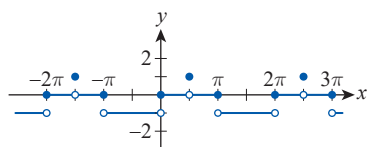
41.
$$h(x) = \begin{cases} -\sin x & \text{if } 2\pi n - \pi < x < 2\pi n \\ \sin x & \text{if } 2\pi n \leq x \leq 2\pi n + \pi \end{cases}, \quad n \in \mathbb{Z}$$



43.



45.



47. 3.5 hours

49. Width = 50 yd; length = 100 yd

51. 50 ft by 100 ft 53. (2,1) 55. 12 and 24

57. 25 sets 59. No; yes

 61. $y = -4 \sin(6\pi t)$; it takes a mere $1/3$ s to complete a period.

 63. a. 3 years b. 9 years 65. $V \approx 178$ people

67. a. 0.999567 b. 0.958 g c. 0.648 g

69. Approx. \$201,587

71. a. 0.965936 b. 0.707 kg c. 7.628 mg

73. True

 75. False; the line perpendicular to $y = Ax + B$ has a slope of $-1/A$.

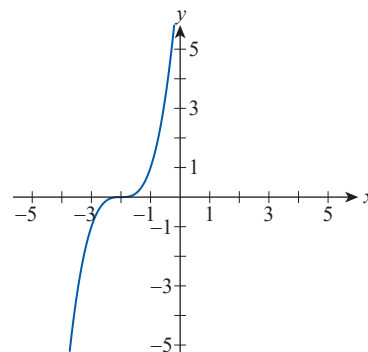
77. True

 79. False; consider $\frac{x^4}{1+x^2}$, which has no asymptotes.

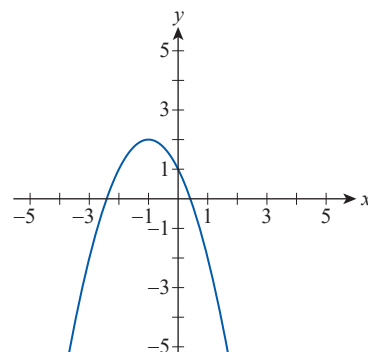
81. True

Section 1.3

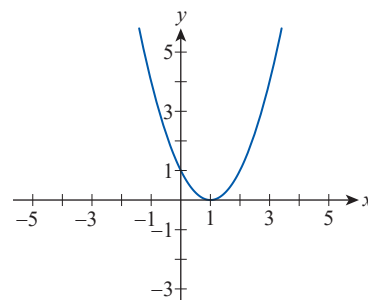
1.


 Dom = Ran = $\mathbb{R}$

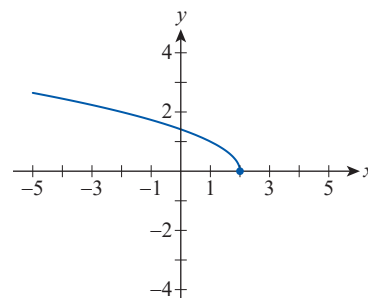
3.


 Dom = $\mathbb{R}$, Ran = $(-\infty, 2]$

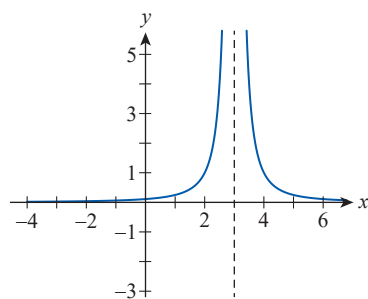
5.


 Dom = $\mathbb{R}$, Ran = $[0, \infty)$

7.

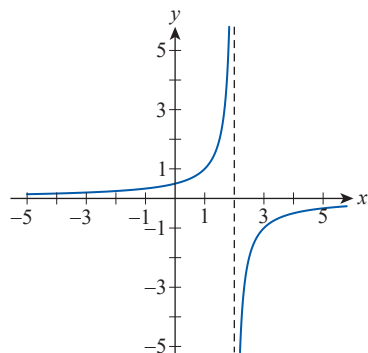

 Dom = $(-\infty, 2]$, Ran = $[0, \infty)$

9.



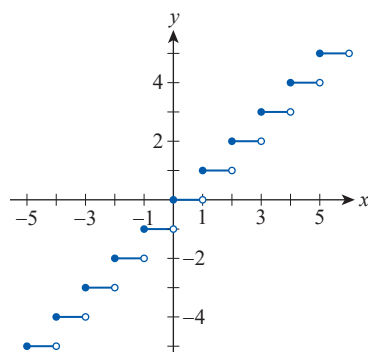
$$\text{Dom} = (-\infty, 3) \cup (3, \infty), \text{Ran} = (0, \infty)$$

11.



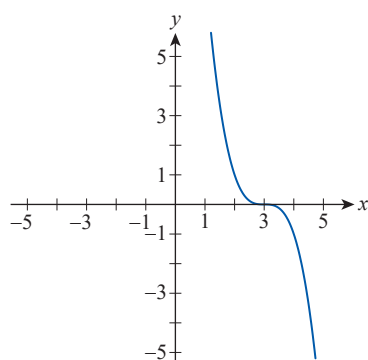
$$\text{Dom} = (-\infty, 2) \cup (2, \infty), \text{Ran} = (-\infty, 0) \cup (0, \infty)$$

13.



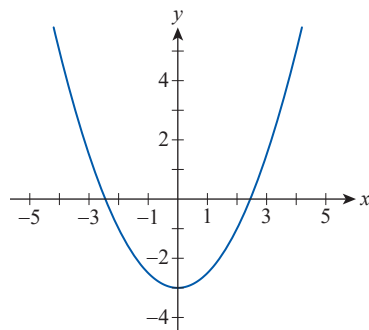
$$\text{Dom} = \mathbb{R}, \text{Ran} = \mathbb{Z}$$

15.



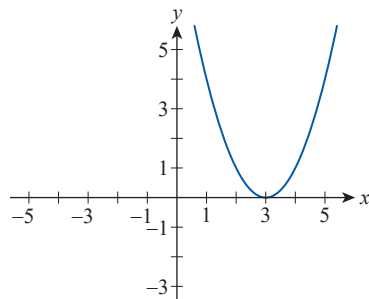
$$\text{Dom} = \text{Ran} = \mathbb{R}$$

17.



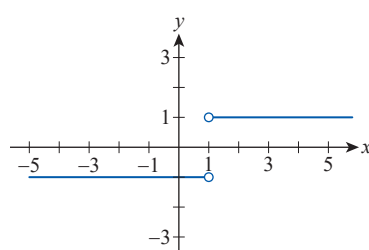
$$\text{Dom} = \mathbb{R}, \text{Ran} = [-3, \infty)$$

19.



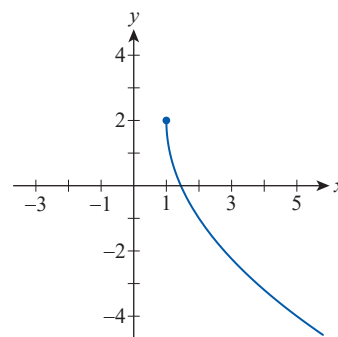
$$\text{Dom} = \mathbb{R}, \text{Ran} = [0, \infty)$$

21.



$$\text{Dom} = (-\infty, 1) \cup (5, \infty), \text{Ran} = \{-1, 1\}$$

23.



$$\text{Dom} = [1, \infty), \text{Ran} = (-\infty, 2]$$

25. $y = -x^2 + 6$ 27. $y = -\sqrt{x+5}$

29. $y = -|-x+7|$ 31. $f(x) = -\sqrt{x+4}$

33. $f(x) = 1 - (x-3)^3$

35. a. -1 b. 1 c. 0 d. 0

37. a. -3 b. 1 c. 2 d. 1/2

39. a. -4 b. 8 c. -12 d. -1/3

41. a. 4 b. 3 c. 7/4 d. 7

43. a. -1 b. 3 c. -2 d. -1/2

45. a. -1 b. -3 c. -2 d. -2

47. a. $x^2 + \sqrt[3]{x} - 1$; Dom = $\mathbb{R}$

b. $\frac{x^2 - 1}{\sqrt[3]{x}}$; Dom = $(-\infty, 0) \cup (0, \infty)$

49. a. $x^{3/2} + x - 3$; Dom = $[0, \infty)$

b. $\frac{x^{3/2}}{x - 3}$; Dom = $[0, 3) \cup (3, \infty)$

51. a. $x^3 + \sqrt{x - 2} + 4$; Dom = $[2, \infty)$

b. $\frac{x^3 + 4}{\sqrt{x - 2}}$; Dom = $(2, \infty)$

53. a. $x^{2/3} + 6x - 1$; Dom = $\mathbb{R}$

b. $\frac{6x - 1}{x^{2/3}}$; Dom = $(-\infty, 0) \cup (0, \infty)$

55. -1699/100 57. Undefined 59. -1/3

61. Undefined 63. -324

65. π^2 67. 4 69. 5 71. 3 73. 2

75. a. $\frac{1}{x - 1}$; Dom = $(-\infty, 1) \cup (1, \infty)$

b. $\frac{1}{x} - 1$; Dom = $(-\infty, 0) \cup (0, \infty)$

77. a. $1 - \sqrt{x}$; Dom = $[0, \infty)$

b. $\sqrt{1 - x}$; Dom = $(-\infty, 1]$

79. a. $9x^4 + 36x^2 + 35$; Dom = $\mathbb{R}$

b. $3x^4 + 12x^3 + 12x^2 + 5$; Dom = $\mathbb{R}$

81. a. $\sqrt{2x}$; Dom = $[0, \infty)$

b. $2\sqrt{x}$; Dom = $[0, \infty)$

83. a. x ; Dom = $(-\infty, 0) \cup (0, \infty)$

b. x ; Dom = $(-\infty, 0) \cup (0, \infty)$

85. a. x ; Dom = $\mathbb{R}$ b. x ; Dom = $\mathbb{R}$

87. a. $\frac{3}{1 - 3x^2}$;

Dom = $(-\infty, -\sqrt{3}/3) \cup (-\sqrt{3}/3, \sqrt{3}/3) \cup (\sqrt{3}/3, \infty)$

b. $\frac{27}{x^2 - 2x + 1}$; Dom = $(-\infty, 1) \cup (1, \infty)$

89. $g(x) = \frac{2}{x}$, $h(x) = 5x - 1$, $f(x) = g(h(x))$

91. $g(x) = x + \sqrt{x} - 7$, $h(x) = x + 2$, $f(x) = g(h(x))$

93. $g(x) = \frac{\sqrt{x}}{x^2}$, $h(x) = x - 3$, $f(x) = g(h(x))$

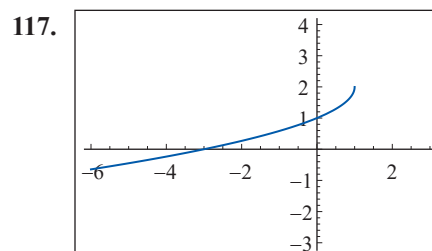
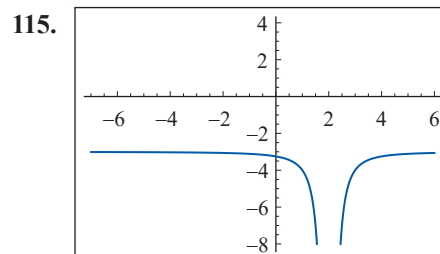
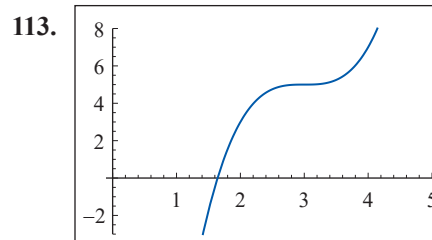
95. $S(t) = \pi r \sqrt{r^2 + \frac{t^4}{16}}$

97. $c(t) = 2040 + 17,400t - 43.5t^2$

99. $(f \circ g)(x) = \frac{-x}{\sqrt[3]{3x^2 - 9}} = -(f \circ g)(-x)$

101. Yes 103. Yes 105. No 107. No

109. True 111. True



119. $y = -(x + 3)^2 + 5$ 121. $y = -x^3 + 7$

123. $y = (x - 2)^3 - 4$

125. $(f + g)(x) = 4 + 12x + 9x^2 + \sqrt{x^2 + 5}$;

$(fg)(x) = 4\sqrt{x^2 + 5} + 12x\sqrt{x^2 + 5} + 9x^2\sqrt{x^2 + 5}$;

$(f \circ g)(x) = 49 + 9x^2 + 12\sqrt{x^2 + 5}$;

$(g \circ f)(x) = \sqrt{(3x + 2)^4 + 5}$

127. $(f + g)(x) = \frac{2x^2 - x + 1}{x(x - 1)}$;

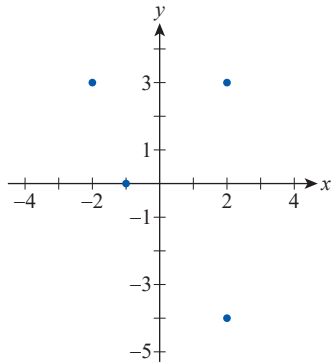
$(fg)(x) = \frac{x + 1}{x}$;

$(f \circ g)(x) = 1 - 2x$;

$(g \circ f)(x) = \frac{2}{x + 1}$

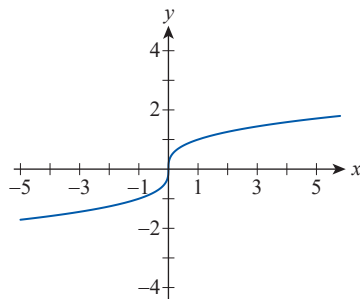
Section 1.4

1.



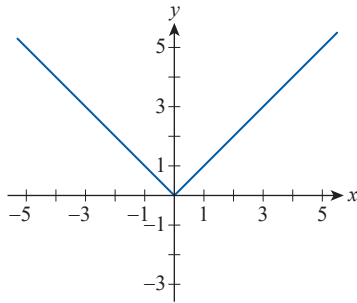
$$\text{Dom} = \{-2, -1, 2\}, \text{Ran} = \{-4, 0, 3\}$$

3.



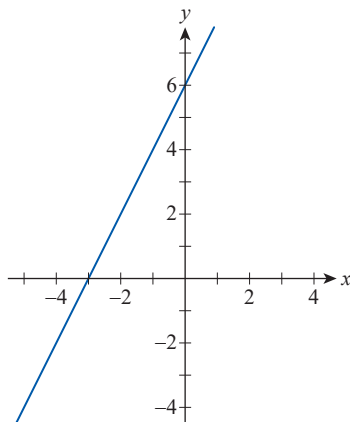
$$\text{Dom} = \text{Ran} = \mathbb{R}$$

5.



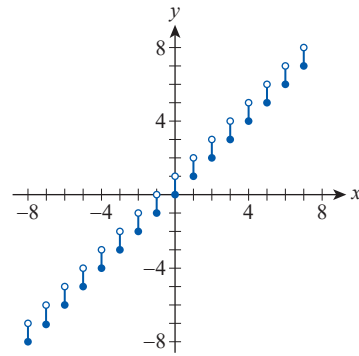
$$\text{Dom} = \mathbb{R}, \text{Ran} = [0, \infty)$$

7.



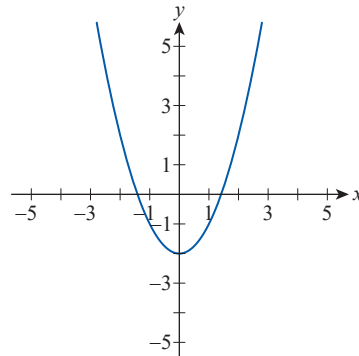
$$\text{Dom} = \text{Ran} = \mathbb{R}$$

9.



$$\text{Dom} = \mathbb{Z}, \text{Ran} = \mathbb{R}$$

11.



$$\text{Dom} = \mathbb{R}, \text{Ran} = [-2, \infty)$$

13. Restrict to $[0, \infty)$ 15. Inverse exists

17. Inverse exists 19. Inverse exists

21. Inverse exists 23. $f^{-1}(x) = (x+2)^3$

$$25. r^{-1}(x) = \frac{-2x-1}{3x-1} \quad 27. F^{-1}(x) = (x-2)^{1/3} + 5$$

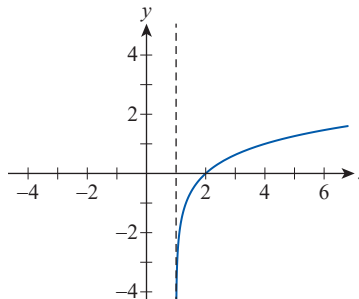
$$29. V^{-1}(x) = 2x-5 \quad 31. h^{-1}(x) = (x+2)^{5/3}$$

$$33. J^{-1}(x) = \frac{x-2}{3x} \quad 35. h^{-1}(x) = (x-6)^{1/7}$$

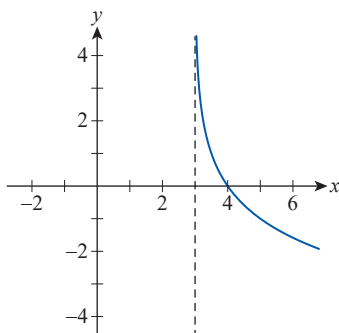
$$37. r^{-1}(x) = x^5/2 \quad 47. \text{F}$$

49. C 51. D 53. G 55. F 57. C 59. A

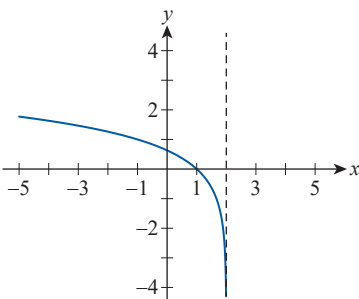
61.



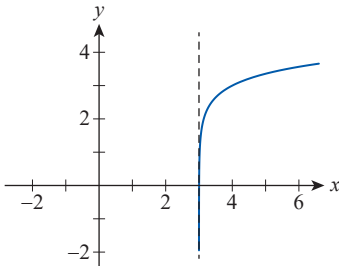
63.



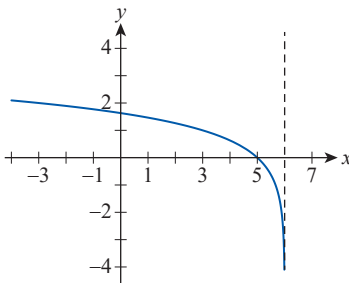
65.



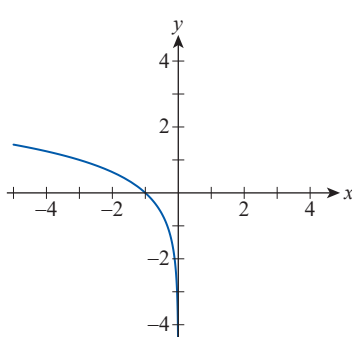
67.



69.



71.



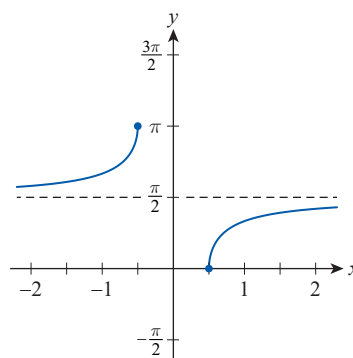
73. 2 75. 7 77. 1

79. 2.45 81. 3.30 83. 0.74 85. x^2 87. $e^2 p/x$ 89. x^3/y^4 91. $\frac{3}{2} \ln x + \ln p + 5 \ln q - 7$ 93. $\log(2 + 3 \log x)$ 95. $1 - \frac{1}{2} \log(x + y)$ 97. $\log_2(y^2 + z) - 4 - 4 \log_2 x$ 99. $2 \log_b x + \frac{1}{2} \log_b y - \log_b z$ 101. $\ln \frac{3p}{q^2}$ 103. $\log \frac{x-10}{x}$ 105. $\ln \frac{e^2}{x^3 y^3}$ 107. $-\pi/2$ 109. $-\pi/6$ 111. $\pi/4$

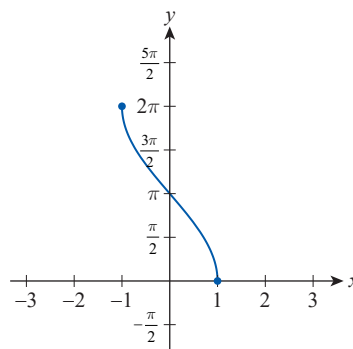
113. Does not exist 115. -1.3734 117. 1.0472

119. -1/2 121. 2 123. 2 125. $\frac{\sqrt{x^2 - 4}}{2}$ 127. $\frac{\sqrt{3}x}{3}$ 129. $\frac{4}{\sqrt{x^2 + 16}}$

131.



133.



135. 521 73 136 136 73 392 316 -7 73 188 392 217 8 -7 617

137. REMEMBER YOUR SUNBLOCK

139. 6.3 141. 13.4 decibels

143. a. 0.2898 rad b. 0.4429 rad c. 0.7956 rad

145. True 147. True

149. False; the domain of $\arcsin x$ is $[-1, 1]$.

151. True

Section 1.5

13. a. $y = 3.559x + 22.038$ b. 36,274 units15. a. $y = 10.743x - 25.829$ b. \$81,60117. a. $y = -1.345x + 79.771$ b. 69.011 cents per pound

19. $x = 0$

21. $x = \frac{(2k+1)\pi}{6} + \frac{2}{3}, k \in \mathbb{Z}$

23. $x = -1$ 25. $x = \pm\sqrt[4]{1/2}$ 27. $x \leq 0$

29. Answers will vary. 31. Answers will vary.

33. Answers will vary. 35. Answers will vary.

37. Answers will vary. 39. Answers will vary.

41. 20 43. -21.9787 45. -90.8519 47. 49.9990

49. 130,783 51. 1 53. Answers will vary.

55. Answers will vary. 57. Answers will vary.

59. -4.4213, -0.1001, 4.5213

61. 1.1974, 5.6240, 6.8003 63. 0.1586, 3.1462

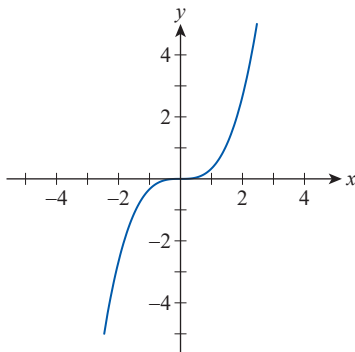
65. $f(x)$ 67. $g(x)$ 69. $g(x)$

71. Answers will vary. 73. Answers will vary.

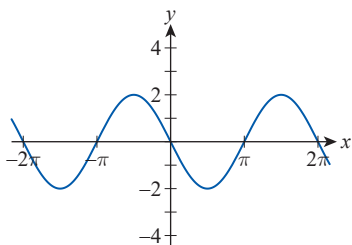
Chapter 1 Review

1. Dom = $\{-2, -3\}$, Ran = $\{-9, -3, 2, 9\}$;
not a function3. Dom = $[-6, \infty)$, Ran = $\mathbb{R}$; not a function5. Dom = $\mathbb{N}$, Cod = $\mathbb{R}$, Ran = $\left\{\frac{3x}{4} \mid x \in \mathbb{N}\right\}$ 7. Dom = $\mathbb{R}$, Cod = $\mathbb{R}$, Ran = $(0, 1]$ 9. a. $2x^2 + 6x - 8$ b. $2x^4 + 10x^2$ c. $4x + 2h + 10$ 11. a. $\frac{3}{x+1}$ b. $\frac{3}{x^2+2}$ c. $\frac{-3}{(x+h+2)(x+2)}$ 13. Decreasing on $(-\infty, 2)$; increasing on $(2, \infty)$

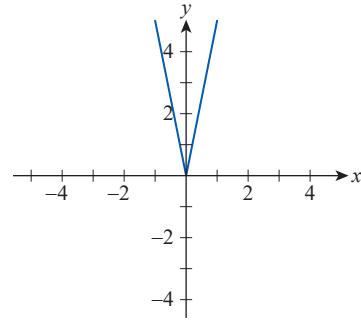
15. Odd



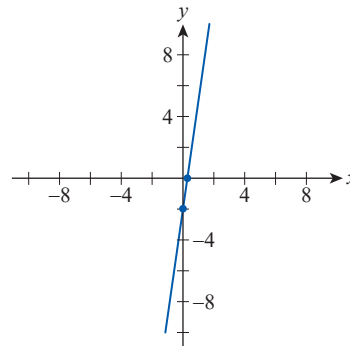
17. Odd



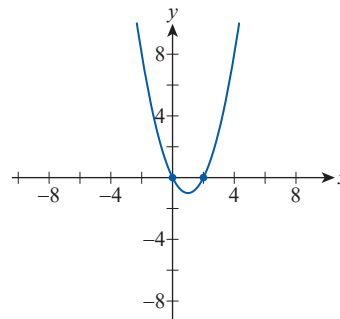
19. Symmetric with respect to the y-axis



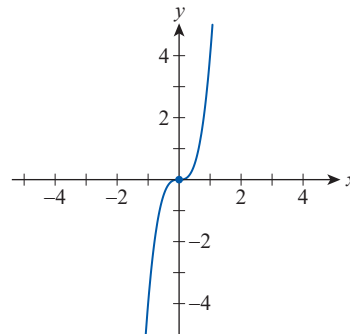
21.



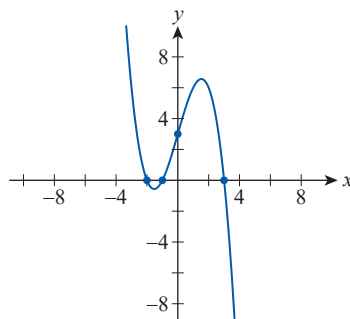
23.



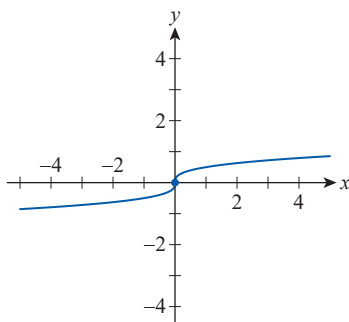
25.



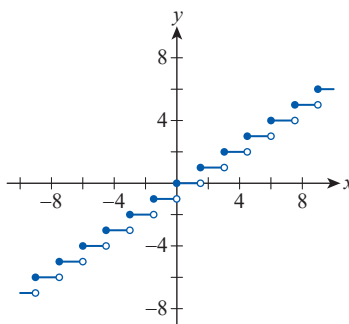
27.



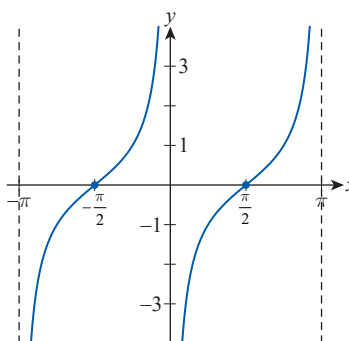
29.



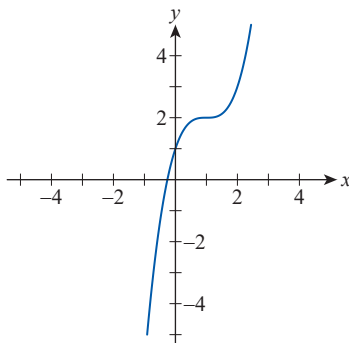
31.



33.



35.

Dom = $\mathbb{R}$, Ran = $\mathbb{R}$

37. $y = \frac{1}{x-2} + 3$

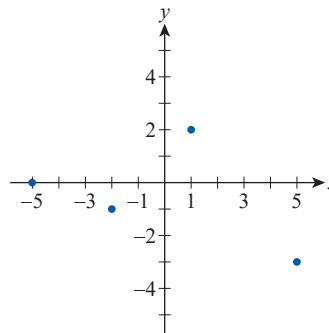
39. a. $x^2 + \sqrt{x}$; Dom = $[0, \infty)$

b. $x^{3/2}$; Dom = $(0, \infty)$

41. a. $x+2$ b. $x-2$ c. 5

43. $g(x) = \frac{\sqrt{x}+2}{x^2}$, $h(x) = x+3$; $f(x) = g(h(x))$

45.



Dom = $\{-5, -2, 1, 5\}$, Ran = $\{-3, -1, 0, 2\}$

47. $f^{-1}(x) = \frac{x+2}{7x}$ 49. $f^{-1}(x) = (x+6)^5$

53. $\frac{\sqrt{x^2-4}}{2}$ 55. $\frac{1}{3}(\log y + 2 \log z - 4 \log x)$

57. $A(t) = A_0 e^{-\frac{\ln 2}{10}t} = A_0 (1/2)^{t/10}$; approx. 17.7%

59. 15/2 and 15/2 61. Yes

63. False; a function may not be defined on (any subset of) $\mathbb{R}$ and thus may not have a graphical representation in the xy -system. Even graphs of numerical functions are not always curves.65. False; consider $f(x) = x^2$ with $a = 1$ and $b = -1$.67. False; consider $f(x) = (x-1)^2$.

69. False

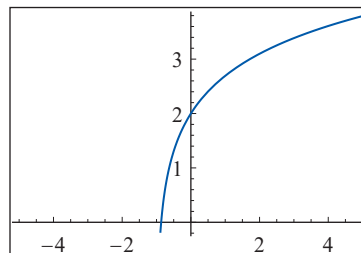
71. False; consider $c = 2$ and $f(x) = x^2$:

$$f(cx) = (2x)^2 = 4x^2, \text{ while } cf(x) = 2x^2.$$

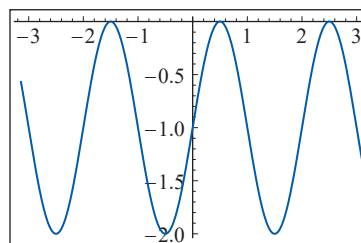
73. False; consider $y = x^2$. (The statement is only true for invertible functions.)

75. True

77.



79.



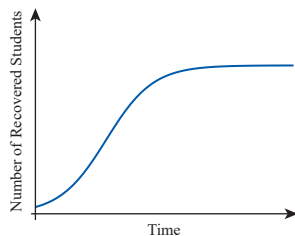
81. 1.4262 83. Answers will vary.

Chapter 1 Conceptual Project

- $r(t) = 2.6t$
- Doubling time is approximately 24 minutes and 51 seconds; the time to triple is approximately an additional 19 minutes and 4 seconds. (The exact values are $\sqrt{2} - 1$ hours and $\sqrt{3} - \sqrt{2}$ hours, respectively.)
- After 3 hours: radius 7.8 km and area approx. 191.13 km^2
After 5.5 hours: radius 14.3 km and area approx. 642.42 km^2
- Approx. $286.70 \text{ km}^2/\text{h}$
- Answers will vary, but a major flaw is that the model predicts a hole whose area will grow indefinitely, and is not a periodic function of time. A better approach is to use a periodic function.

Chapter 1 Application Project

- The independent variable is time t , and the dependent variable is the number of people infected.
 - Initially, the number of infected rises sharply, because there are few recovered and the susceptible population is large.
 - $P_i(t) + P_r(t) + P_s(t) = 25,000$
- The number is expected to increase at a growing, then at a steadily slowing rate.



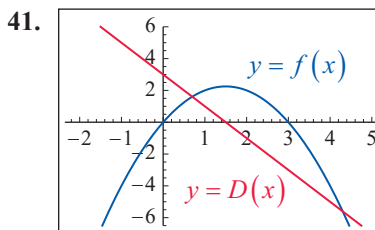
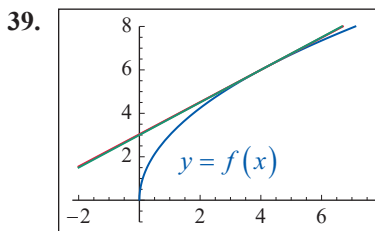
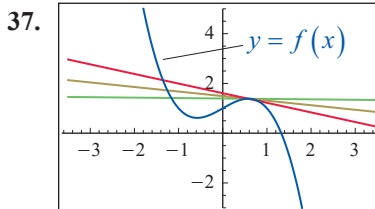
- We can partition the infected population into ten subsets according to whether they got infected one day ago, two days ago, etc. Thus, we can assume that one-tenth of them will recover daily. Limitations: The aforementioned subsets may not all be of the same size, especially at the early stages of the epidemic, or later when the rate of infections slows down.
- $P_s = 4000$ students

Chapter 2

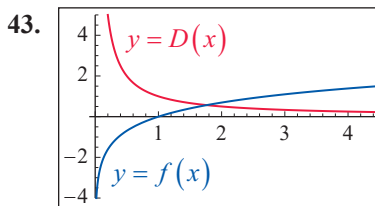
Section 2.1

- 3 3.0 5. $-1/4$
- The exact answer is -2 .

- The exact answer is 2.
- The exact answer is 2.
- The exact answer is $1/e$ or approx. 0.3679.
- The exact answer is $1/(5 \ln 10)$ or approx. 0.0869.
- The exact answer is -4 .
- a. 96 ft b. 16 ft/s c. 16 d. 2.5 s
- a. 176 ft b. 176 ft/s c. 192 ft/s
d. 64 ft/s e. 6 s
- a. 3 m/s b. 15 m/s c. $6t_0 + 3$
- a. 5 s; 45 m/s b. 10 s c. $-1.83g$
- a. 87.35 ft b. 34.94 ft/s
c. -28 ft/s d. -77.47 ft/s
- a. 60 b. 61.44
- a. 12.7234 b. 12.3129
- a. 1.0536 b. 1.0563
- a. 1.2217 b. 1.2218



The function value $D(x_0)$ at any given x_0 is approximately equal to the slope of the graph of f at x_0 .



The function value $D(x_0)$ at any given x_0 is approximately equal to the slope of the graph of f at x_0 , for any $x_0 > 0$.

45. $x = \pm 2$ 47. $x = 1$

49. a. 63.9936 b. 63.9999

51. a. 11.0508 b. 11.0050

53. a. 1.0612 b. 1.0612

55. a. 1.222 b. 1.222

Section 2.2

1. 4 3. 0

5.

x	y
1	2.414
1.4	2.814
1.41	2.824
1.414	2.828

The table points to a limit of $2\sqrt{2} \approx 2.828$.

7.

x	y
1.5	113.33
1.1	15.94
1.01	10.46
1.001	10.045

The table points to a limit of 10.

9.

x	y
3	-3.4696
3.14	-3.0048
3.141	-3.0018
3.1415	-3.0003

The table points to a limit of -3.

11.

x	y
7.5	210.5
7.1	994.1
7.01	9814.0
7.001	98014.0

The table points to an undefined limit.

13. $\lim_{x \rightarrow 1^-} f(x) = -\infty$; $\lim_{x \rightarrow 1^+} f(x) = \infty$

15. $\lim_{x \rightarrow 3^-} h(x) = \infty$; $\lim_{x \rightarrow 3^+} h(x) = \infty$

17. $\lim_{x \rightarrow -1^-} q(x) = \infty$; $\lim_{x \rightarrow -1^+} q(x) = -\infty$

19. $\lim_{x \rightarrow 1.5^-} v(x) = -\infty$; $\lim_{x \rightarrow 1.5^+} v(x) = \infty$

21. $\lim_{x \rightarrow \left(\frac{\pi}{2} + k\pi\right)^-} \tan x = \infty$; $\lim_{x \rightarrow \left(\frac{\pi}{2} + k\pi\right)^+} \tan x = -\infty$ ($k \in \mathbb{Z}$)

23. $\lim_{x \rightarrow 6^-} s(x) = -\infty$; $\lim_{x \rightarrow 6^+} s(x) = \infty$

25. $\lim_{x \rightarrow \infty} f(x) = 0$; $\lim_{x \rightarrow -\infty} f(x) = 0$

27. $\lim_{x \rightarrow \infty} h(x) = 0$; $\lim_{x \rightarrow -\infty} h(x) = 0$

29. $\lim_{x \rightarrow \infty} q(x) = -1$; $\lim_{x \rightarrow -\infty} q(x) = -1$

31. $\lim_{x \rightarrow \infty} v(x) = \infty$; $\lim_{x \rightarrow -\infty} v(x) = -\infty$

33. $\lim_{x \rightarrow \infty} \tan x$ does not exist; $\lim_{x \rightarrow -\infty} \tan x$ does not exist

35. $\lim_{x \rightarrow \infty} s(x) = 1$; $\lim_{x \rightarrow -\infty} s(x) = -1$

37. a. -2 b. 1 39. a. 1 b. 4

41. a. 1 b. $-\infty$ 43. a. 1 b. 1

45. a. Does not exist b. 0

47. $\lim_{x \rightarrow \infty} f(x) = \infty$; $\lim_{x \rightarrow -\infty} f(x) = -\infty$

49. $\lim_{x \rightarrow \infty} h(x) = -\infty$; $\lim_{x \rightarrow -\infty} h(x) = -\infty$

51. $\lim_{x \rightarrow \infty} F(x) = \infty$

53. $\lim_{x \rightarrow \infty} H(x) = \infty$; $\lim_{x \rightarrow -\infty} H(x) = \infty$

55. $\lim_{x \rightarrow \infty} u(x) = \infty$; $\lim_{x \rightarrow -\infty} u(x) = \infty$

57. $\lim_{x \rightarrow -\infty} s(x) = -\infty$

59. False; see $h(x)$ of Example 5 at $x = 2$.

61. True 63. True 65. Does not exist

67. 2 69. $3/2$ 71. 0

75. See the answer given for Exercise 65.

77. See the answer given for Exercise 67.

79. See the answer given for Exercise 69.

81. See the answer given for Exercise 71.

Section 2.3

1. $\delta \approx 0.23$ or smaller 3. $\delta \approx 0.2$ or smaller

5. L is incorrectly quantified and switched with c .

7. ε is incorrectly quantified.

9. The inequality $0 \leq |x - c|$ is incorrect.

11. $\delta = 0.02$ or smaller 13. $\delta = 0.1$ or smaller

15. $\delta = \sqrt{0.1}$ or smaller 17. $\delta = 0.09$ or smaller

19. $\delta = 1 - e^{-0.1} \approx 0.0952$ or smaller

21. $N = 10$ or larger 23. $N = -10$ or smaller

25. $N = \ln 0.1$ or smaller 27. $\delta = 0.1$ or smaller

29. $\delta = 0.1$ or smaller

31. $\delta = \frac{\pi}{2} - \arctan 100 \approx 0.0099997$ or smaller

63. The limit does not exist.

65. The limit does not exist.

67. The limit is 0. 69. 0.02256 mm

71. False; the function value and limit at c need not be equal. (See Example 2.)
73. False; consider $f(x) = -x^2$ and $g(x) = x^2$ at $c = 0$.
75. Answers will vary. 77. Answers will vary.
79. Answers will vary. 81. Answers will vary.
83. Answers will vary.
85. Vert. asym.: $x \approx -1.3340, x \approx 1.1759$
87. Vert. asym.: $x = (2n+1)\pi - 6, n \in \mathbb{Z}$
89. Vert. asym.: $x = (2n-1)\pi/2, n \in \mathbb{Z}, n \neq 0$

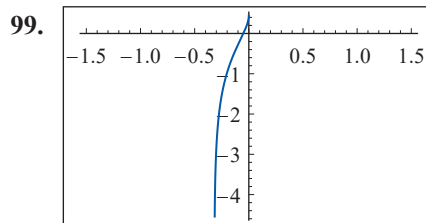
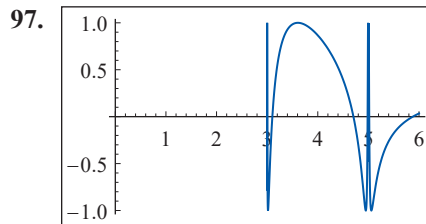
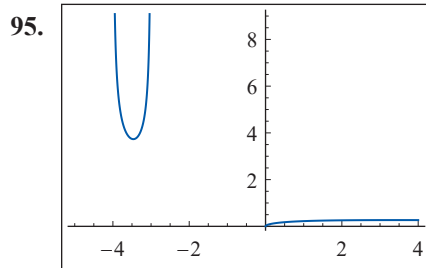
Section 2.4

1. a. -4 b. 21
3. 5 5. 7 7. 9 9. 22 11. -4 13. -2 15. $\sqrt[3]{2}$
17. $\sqrt[3]{100}$ 19. 16 21. 12 23. 11 25. $33/5$
27. $1/6$ 29. $1/(2\sqrt{5})$ 31. $1/9$ 33. 6 35. 4
37. $1/(2\sqrt{x})$ 39. $32/7$ 41. $-1/6$ 43. -1
45. 8 47. 11 49. $9-2(c-2)$ 51. 1 53. 0
55. 0 57. $2e^2$ 71. The limit is 0. 79. $r/2$

Section 2.5

1. Points of continuity: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$
Points of discontinuity: $c = 0, \lim_{x \rightarrow 0} f(x)$ does not exist;
 $c = 3, \lim_{x \rightarrow 3} f(x) \neq f(3)$
3. Answers will vary. 5. $c = 0$, nonremovable
7. $c = 3$, removable 9. $c = 2$, nonremovable
11. None
13. $c = -(2n+1)\pi/2$ for $n \in \{-1, 0, 1, 2, \dots\}$, nonremovable
15. $c = 4$, removable
17. None 19. None 21. $c = \pm 1$, nonremovable
23. None 25. All integers, nonremovable
27. $c = \pm\sqrt{n}$, n a positive integer, nonremovable
29. $c = 1/n$, n a nonzero integer, nonremovable
35. Continuous on $[-3, -1] \cup (0, 2] \cup [3, \infty)$
37. Continuous on $(-\sqrt{3}, \sqrt{3})$
39. Continuous except on
 $\{n-1/\pi | n \in \mathbb{Z}\} \cup \{\ln k - 2 | k \in \mathbb{N}\}$
45. $f(3) = 7$ will make f continuous.
47. $h(1) = 2$ will make h continuous.
49. $G(0) = 0$ will make G continuous.
51. Continuous on $[-4, 4]$

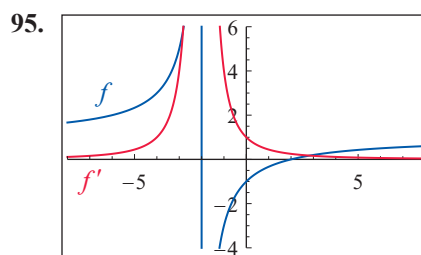
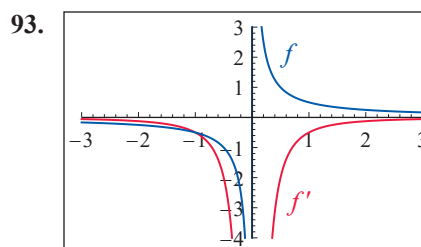
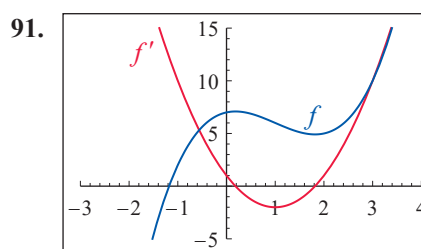
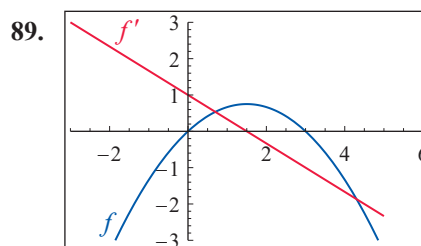
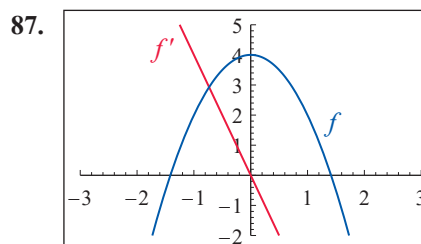
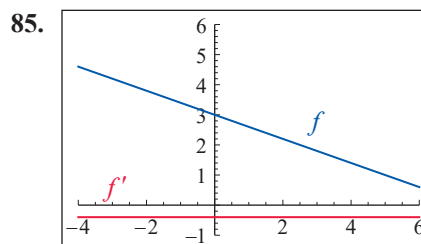
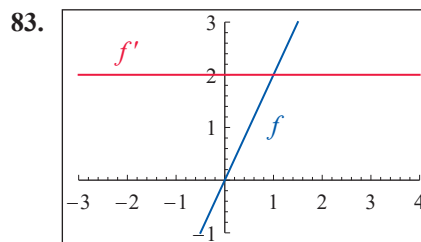
53. Not continuous at the endpoints 55. $a = 5$
57. $a = 3/2, b = -5/2$
63. Yes; $c = 2$ 65. Yes; $c = 2$
67. No; G is not continuous on $[-2, 2]$
83. a. Because speeds of everyday objects are smaller than c by orders of magnitude, the denominator of ΔT is approximately 1.
b. No moving object can reach the speed of light.
85. False; consider the Dirichlet function.
87. True
89. -0.2895 91. 0.8241 93. -1.7321

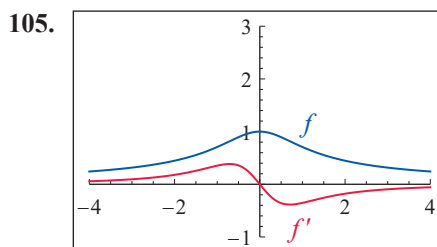
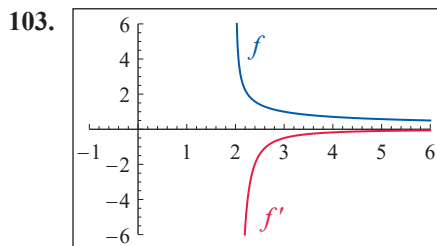
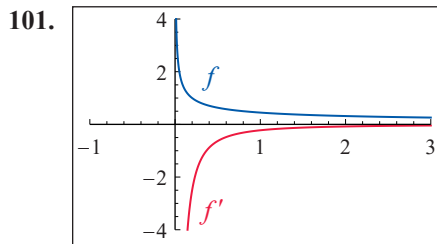
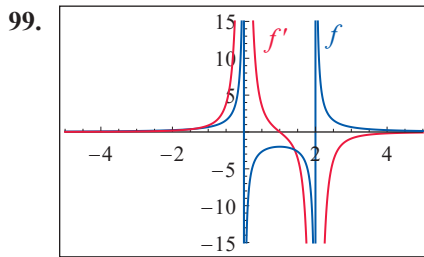
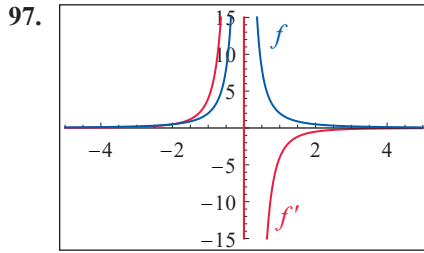


Section 2.6

1. a. $f'(-1) = 2$ b. $f'(1) = -2$
3. $y = 4x - 6$ 5. $y = \frac{1}{2}x + 4$
7. $y = 12x - 16$ 9. $y = \frac{1}{2}x + 1$
11. $y = -4x + 4$ 13. $y = -\frac{1}{16}x + \frac{3}{4}$
15. $f'(x) = 0$; tangent is horizontal for all x
17. $f'(x) = 4$; tangent is never horizontal
19. $f'(x) = 6x$; tangent is horizontal at $x = 0$
21. $f'(x) = x + 5$; tangent is horizontal at $x = -5$
23. $f'(x) = 3x^2 + 1$; tangent is never horizontal
25. $f'(x) = 4x^3$; tangent is horizontal at $x = 0$

27. $f'(x) = -\frac{10}{(2x-4)^2}$; tangent is never horizontal
29. $f'(x) = -\frac{7}{(x-3)^2}$; tangent is never horizontal
31. $f'(x) = -\frac{2x}{(x^2+1)^2}$; tangent is horizontal at $x = 0$
33. $f'(x) = \frac{5}{2\sqrt{5x}}$; tangent is never horizontal
35. $f'(x) = \frac{1}{\sqrt{2x+1}}$; tangent is never horizontal
37. $f'(x) = \frac{x}{\sqrt{x^2+1}}$; tangent is horizontal at $x = 0$
39. $y = 6x - 6$
41. $y = -\frac{1}{2}x + 1$ or $y = -\frac{1}{2}x - 1$
43. $54y + x = 27$ 45. $f'(3.6) = -1/4$
47. $h'(3) = 10$ 49. $G'(7) = 1/2$
51. $u'(-3) = -1/2$ 53. $w'(5) = -1/54$
55. $G'(-2) = -32$
57. C 59. D
61. Answers will vary. 63. Answers will vary.
65. Answers will vary.
73. $t = 1, t = 5$; the particle stops at 2 s and at 4 s.
75. a. The respective velocities are 16 ft/s and -16 ft/s; the speed is 16 ft/s in both cases.
 b. It rises 36 ft above the cliff and reaches the bottom at approx. 4.91 s.
 c. Approx. -109.12 ft/s
77. She seems to have stopped for an extended time period (probably a red light); soon after, she turned around and sped back home (perhaps forgot something at home). Then she sped back to class, this time managing to do so without a red light, and also driving faster.
79. a. $P(x) = 0.1x^2 + 40x - 190$
 b. Approx. 5 suitcases
 c. $P(25) = 872.50, P(30) = 1100, P(40) = 1570$
 d. $P'(x) = 0.2x + 40$
 e. $P'(25) = 45, P'(30) = 46, P'(40) = 48$
81. $C(x) = 20 + 3x; C'(x) = 3$





Chapter 2 Review

- The limit is 3; the function is continuous.
- The limit is 2; there is a nonremovable (jump) discontinuity at 1.
- The exact answer is 3.
- The exact answer is $1/2$.
- a. 598 ft b. 576 ft/s c. 608 ft/s
d. 544 ft/s e. At 19 seconds
- 1.5568 13. 21.7102

15.

x	y
0.5	0.7071
0.1	0.7943
0.01	0.9550
0.001	0.9931

The table points to a limit of 1.

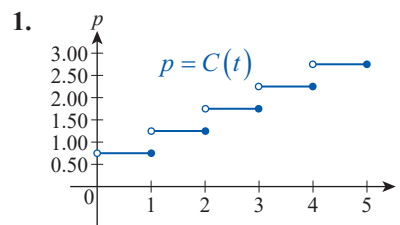
17.

x	y
0.5	0.47943
0.1	0.11969
0.01	-0.00264
0.001	-0.00194

The table points to a limit of 0.

- $\delta = 0.005$ or smaller
- 14 27. 7 29. 8 31. $-1/10$ 33. ∞
- $1/4$ 37. -2 39. 5 41. $2x$
- Answers will vary. 47. $c = 4$, nonremovable
- All integers, nonremovable
- No discontinuities 57. $y = 3x - 1$
- $f'(x) = 2 - 2x$; tangent is horizontal at $x = 1$
- Answers will vary.
- a. $P(x) = 0.1x^2 + 15x - 247.5$
b. 15 toys c. $P'(x) = 0.2x + 15$
- False; there could be, for example, a jump or oscillating discontinuity at $x = c$.
- False; the condition $B \neq 0$ is needed.
- True
- False; while $\lim_{x \rightarrow c} f(x) = L$ does follow, $f(c)$ doesn't even have to be defined.
- 5.3586 and 21.7491, respectively
- -0.6891 and 0.7729 , respectively
- 1 81. 3

Chapter 2 Conceptual Project



- No, the left- and right-hand limits are unequal.
- A call lasting for about two and a half minutes costs \$1.75.

7. A call lasting just over three minutes costs \$2.25.

9. Does not exist

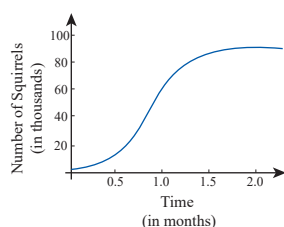
Chapter 2 Application Project

3. No population can grow toward infinity. Sooner or later, external (and/or internal) factors will slow down growth. A graph of a function whose (positive) derivative eventually decreases towards zero is a better choice.

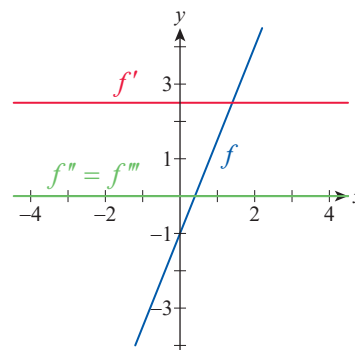
5. a. $\lim_{t \rightarrow \infty} P(t) = 90$

It means that according to the model, the population won't be able to grow beyond 90,000 squirrels (most likely due to limitations in food supply or the appearance of predators).

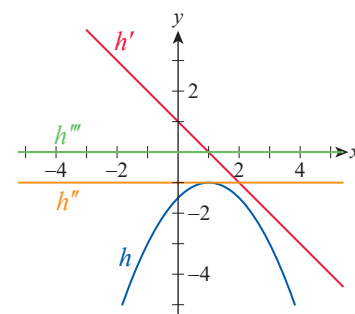
b. The derivative increases first, then approaches zero as $t \rightarrow \infty$.



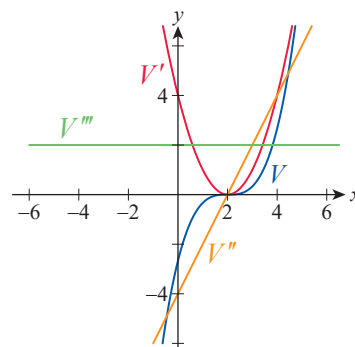
25. $f'(x) = 5/2$; $f''(x) = 0$; $f'''(x) = 0$



27. $h'(x) = -x + 1$; $h''(x) = -1$; $h'''(x) = 0$



29. $V'(x) = (x-2)^2$; $V''(x) = 2x-4$; $V'''(x) = 2$



Chapter 3

Section 3.1

1. $\frac{df}{dx} = 0$; $\frac{df}{dx}\bigg|_{x=1} = 0$ 3. $\frac{dh}{dx} = x$; $\frac{dh}{dx}\bigg|_{x=0} = 0$

5. $\frac{dG}{ds} = s^2 - 1$; $\frac{dG}{ds}\bigg|_{s=-3} = 8$

7. $\frac{dK}{dz} = \frac{-15}{(3z+1)^2}$; $\frac{dK}{dz}\bigg|_{z=0} = -15$

9. $\frac{dw}{dz} = \frac{-2z}{(z^2+2)^2}$; $\frac{dw}{dz}\bigg|_{z=\sqrt{2}} = \frac{-\sqrt{2}}{8}$

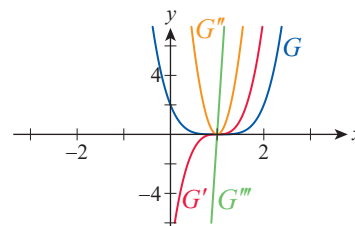
11. $\frac{dQ}{dy} = \frac{-1}{2y\sqrt{3y}}$; $\frac{dQ}{dy}\bigg|_{y=1} = \frac{-\sqrt{3}}{6}$

13. $D_x f(x) = 0$ 15. $D_t h(t) = 3t + 10$

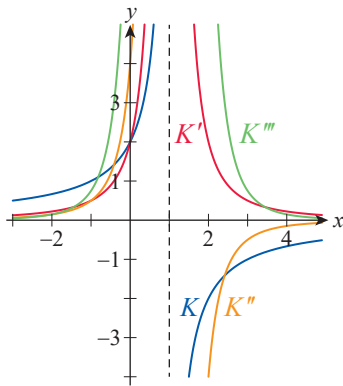
17. $D_y G(y) = -1/(3y^2)$ 19. $D_z S(z) = 6/z^3$

21. $D_v R(v) = 1/\sqrt{2v-3}$ 23. $D_y X(y) = 8y^3$

31. $G'(x) = 8(x-1)^3$; $G''(x) = 24(x-1)^2$;
 $G'''(x) = 48x - 48$



$$33. \quad K'(s) = 2/(s-1)^2; \quad K''(s) = -4/(s-1)^3; \\ K'''(s) = 12/(s-1)^4$$



35. Position: k ; velocity: g ; acceleration: h ; jerk: f

37. Position: k ; velocity: h ; acceleration: g ; jerk: f

39. Differentiable on $(-\infty, 3) \cup (3, \infty)$

41. Differentiable on $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

43. Differentiable on $(-\infty, 1) \cup (1, \infty)$

45. Not differentiable at -2 and 4

Left-hand derivative at -2 : 0

Right-hand derivative at -2 : 2

Left-hand derivative at 4 : 2

Right-hand derivative at 4 : 0

47. Not differentiable at 1.5

One-sided derivatives do not exist at 1.5

49. Undefined and therefore not differentiable on $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

One-sided derivatives do not exist at $\pm\sqrt{3}$

51. Not differentiable at 1 and 5

Left-hand derivative at 1 : -4

Right-hand derivative at 1 : 4

Left-hand derivative at 5 : -4

Right-hand derivative at 5 : 4

53. Not differentiable at $k \in \mathbb{Z}$

Left-hand derivative at each point of nondifferentiability: 1

Right-hand derivative at each point of nondifferentiability: 1

55. Undefined and therefore not differentiable for $z < 0$, not differentiable at 0

Right-hand derivative does not exist at 0

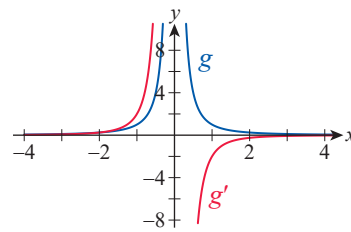
57. Differentiable on the entire real line

61. a. $a(t) = 2.2 \text{ m/s}^2$ b. $t = 15 \text{ s}$

c. The runway must be at least 247.5 m long.

63. b. $g(x) = |x|$; $c = 0$

65.



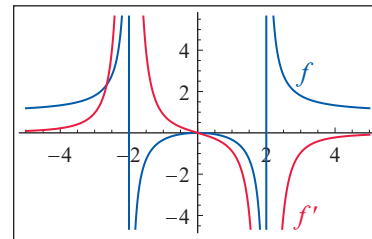
$g' > 0$ and g is increasing for $x < 0$;

$g' < 0$ and g is decreasing for $x > 0$

67. True

69. False; consider $f(x) = |x|$ at $c = 0$.

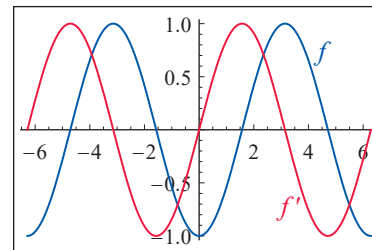
71.



$f' > 0$ and f is increasing for $x < -2$ and $-2 < x < 0$;

$f' < 0$ and f is decreasing for $0 < x < 2$ and $x > 2$

73.



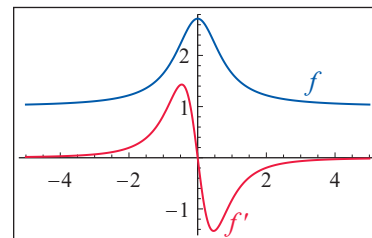
$f' > 0$ and f is increasing for

$2k\pi < x < (2k+1)\pi, k \in \mathbb{Z}$;

$f' < 0$ and f is decreasing for

$(2k-1)\pi < x < 2k\pi, k \in \mathbb{Z}$

75.

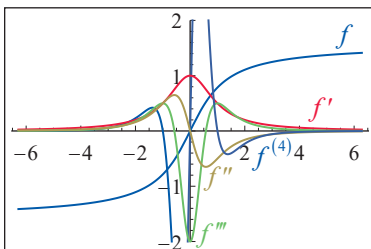


$f' > 0$ and f is increasing for $x < 0$;

$f' < 0$ and f is decreasing for $x > 0$

$$77. f'(x) = \frac{1}{1+x^2}; f''(x) = -\frac{2x}{(1+x^2)^2};$$

$$f'''(x) = \frac{-2+6x^2}{(1+x^2)^3}; f^{(4)}(x) = -\frac{24x(-1+x^2)}{(1+x^2)^4}$$

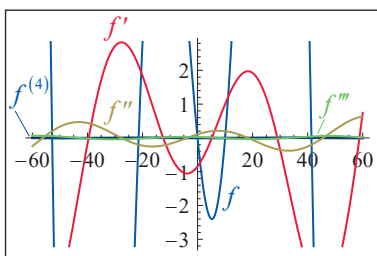


$$79. f'(x) = \frac{1}{10}x \cos\left(1 - \frac{x}{10}\right) - \sin\left(1 - \frac{x}{10}\right);$$

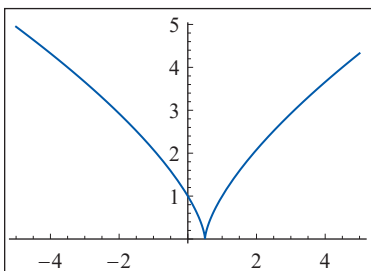
$$f''(x) = \frac{1}{5} \cos\left(1 - \frac{x}{10}\right) + \frac{1}{100}x \sin\left(1 - \frac{x}{10}\right);$$

$$f'''(x) = -\frac{1}{1000}x \cos\left(1 - \frac{x}{10}\right) + \frac{3}{100} \sin\left(1 - \frac{x}{10}\right);$$

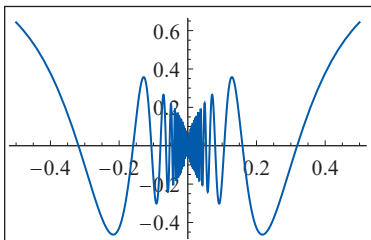
$$f^{(4)}(x) = -\frac{1}{250} \cos\left(1 - \frac{x}{10}\right) - \frac{1}{10000}x \sin\left(1 - \frac{x}{10}\right)$$



81.

Not differentiable at $t = 1/2$ (cusp)

83.

Not differentiable at $t = 0$ (oscillating discontinuity)

Section 3.2

1. $f'(x) = -2$ 3. $h'(x) = -6x + 2$

5. $G'(x) = 2x^3 + 6x^2 - 2x + 3.2$

7. $H'(x) = 8x^7 + 5\sqrt{2}x^4 - 8x^3$

9. $S'(z) = 12z^2 - \frac{3}{2\sqrt{z}}$ 11. $T'(r) = 2\pi r + 2e^r$

13. $6x + 11$ 15. $8x^3 + 15x^2 - 24x - 30$

17. $-\frac{196}{5}x^6 - \frac{20}{3}x^4 + \frac{56}{5}x^3 + \frac{4}{3}x$ 19. $\frac{1}{\sqrt{t}} + 8$

21. $f'(x) = \frac{2}{(1-2x)^2}$ 23. $h'(x) = \frac{-12x^2 + 20x - 6}{(2x^3 - 5x^2 + 3x + 1)^2}$

25. $G'(x) = \frac{-11}{(x-5)^2}$ 27. $H'(x) = \frac{11x^4 + 3x^2 - 6x}{(2x^3 + 5x^2 + 1)^2}$

29. $B'(u) = \frac{3u^{3/2} + 4u}{2(\sqrt{u} + 1)^2}$ 31. $g'(t) = \frac{5t - 8\sqrt{t} + 15}{2\sqrt{t}(4 - 5\sqrt{t})^2}$

33. $f'(x) = 6 - 10x^4$ 35. $h'(x) = -27x^2 + 14x - 5$

37. $G'(x) = 1$ 39. $f'(t) = \frac{3}{2} \left(\frac{4t+1}{\sqrt{t}} \right)$

41. $h'(s) = -\frac{1}{5s^2} + 10s + 1$

43. $G'(x) = -15x^{-6} - 6x^{-4}$ 45. $H'(t) = \frac{9}{2\sqrt{t}} - \frac{5}{2}t\sqrt{t}$

47. $w'(z) = 2 + \frac{4}{4 - \sqrt{z}} + \frac{2\sqrt{z}}{(4 - \sqrt{z})^2}$

49. $r'(x) = \frac{2a^2}{(x+a^2)^2}$

51. $F'(x) = e^x \left(2 + \sqrt{x} + \frac{1}{2\sqrt{x}} \right)$

53. $C'(x) = \frac{1}{(x+1)^2}$

55. $G'(s) = \frac{6s}{2e^s + s} - \frac{3s^2(1+2e^s)}{(2e^s + s)^2}$

57. $L'(y) = -18y^8 + 48y^7 + 6y^5 - 15y^4$

59. $H'(s) = -\frac{ace^s}{(b+ce^s)^2}$

61. $L'(t) = \frac{1}{2\sqrt{t}} - \frac{3\sqrt{t}}{2} - 2te'$

63. $g'(x) = \frac{1}{(x+1)^2}; g''(x) = \frac{-2}{(x+1)^3}; g'''(x) = \frac{6}{(x+1)^4}$

65. $F'(x) = -3\pi x^2 + 10x - 1$; $F''(x) = -6\pi x + 10$;

$$F'''(x) = -6\pi$$

67. $W'(t) = 6t + 3e^t$; $W''(t) = 6 + 3e^t$; $W'''(t) = 3e^t$

69. $f(x) = \frac{1}{3}x^3 - 4x$ (Answers will vary.)

71. $f(x) = x^3 + 2x^2 + x + 1$ (Answers will vary.)

73. $g^{(k)}(x) = \frac{(-1)^k k!}{x^{k+1}}$

75. $q^{(k)}(x) = e^x \left[\sum_{i=0}^k \binom{k}{i} D^i x^n \right]$

77. a. 0 b. 18 c. 1

79. $y = -\frac{1}{2}x + 2$ 81. $y = -2ex + 4e$

83. $y = -x$ 85. $x = 1$

87. No horizontal tangent

89. No horizontal tangent

91. $x = 0$ 93. $x = 0$ 95. No horizontal tangent

97. $a = 2$, $b = -8$

 99. The slopes are $1/2$ and -2 , respectively.

101. $y = ex$ 103. $y = -4x - 4$

105. $\frac{f(x) + xf'(x)}{g(x)} - \frac{xf'(x)g'(x)}{[g(x)]^2}$

107. $e^x \left[\frac{f(x) + f'(x)}{g(x) + 2} - \frac{f'(x)g'(x)}{(g(x) + 2)^2} \right]$

111. $2f(x)f'(x)$; $3[f'(x)]^2 f'(x)$

 115. At the point $(12, 4)$

117. a. $a(2) = 125/36 \text{ ft/s}^2$ b. $a(10) = 5/4 \text{ ft/s}^2$

119. $a = -1.62 \text{ m/s}^2$; $t \approx 1.361 \text{ s}$; $v_{\text{imp}} \approx 2.205 \text{ m/s}$

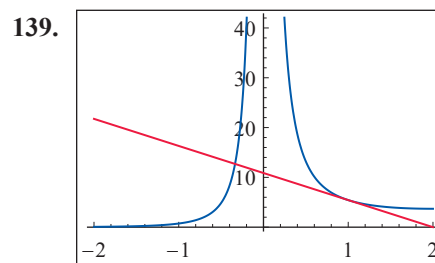
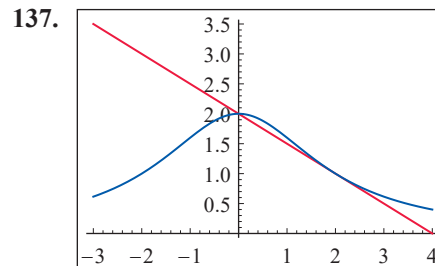
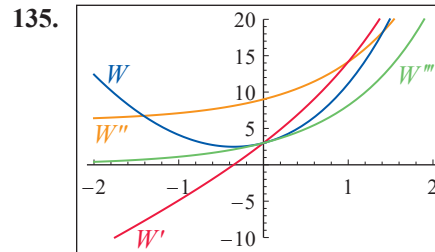
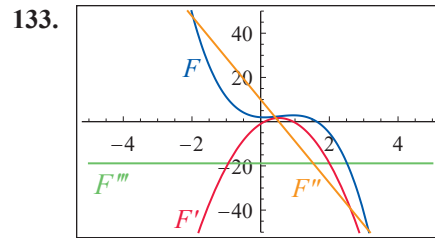
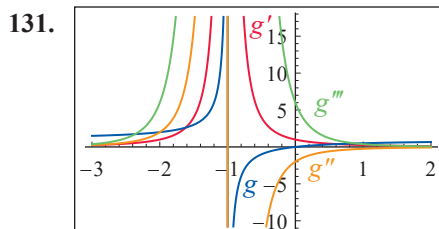
121. True

 123. False; y' is found by applying the Reciprocal Rule or the Quotient Rule.

125. True

 127. False; $\frac{d}{dx} F(x)$ is found by applying the Quotient Rule.

129. True



Section 3.3

1. $3/2$ 3. π 5. 0 7. 2 9. 1 11. 5

13. $f'(x) = 2 \cos x + 5 \sin x$

15. $h'(x) = \cos x - x \sin x$

17. $G'(x) = \sec x \left(\frac{1}{\sqrt{x}} + 2\sqrt{x} \tan x \right)$

19. $L'(x) = 3e^x (\csc x - 1)(\csc x + \cot x)$

21. $g'(x) = -2 \cot x \csc^2 x$

23. $F'(t) = \frac{t \sin t + 2 \cos t - 2}{t^3}$

25. $R'(z) = \frac{e^z (z - 1) + z \cos z - \sin z}{z^2}$

27. $B'(x) = -\csc^2 x$

29. $T'(s) = se^s (2 \cot s + s \cot s - s \csc^2 s)$

31. $x = \frac{2\pi}{3} + 2k\pi$, $x = \frac{4\pi}{3} + 2k\pi$, $k \in \mathbb{Z}$

33. $x = k\pi$, $k \in \mathbb{Z}$ 35. $u = (2k + 1)\frac{\pi}{4}$, $k \in \mathbb{Z}$

37. $x = 2k\pi, k \in \mathbb{Z}$ 39. $x = \pm \frac{\pi}{6} + k\pi, k \in \mathbb{Z}$

41. $y = 2x$ 43. $y = 1$

45. $F'(0) = 1, G'(0) = 2, H'(0) = 1$

49. $f^{(4k)}(x) = \sin x, f^{(4k+1)}(x) = \cos x,$
 $f^{(4k+2)}(x) = -\sin x, f^{(4k+3)}(x) = -\cos x$

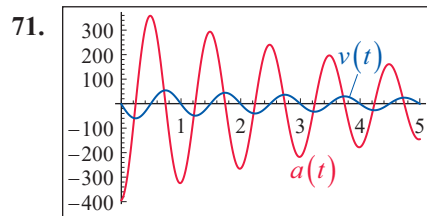
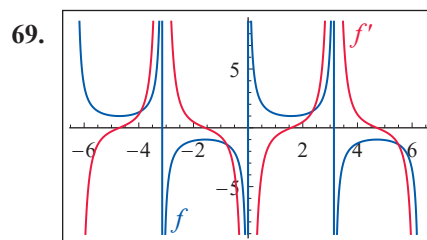
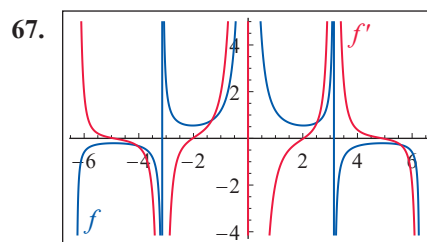
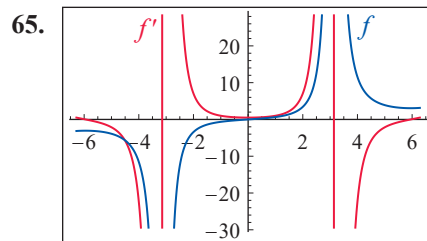
51. $f^{(4k)}(x) = (-4)^k e^x \sin x,$
 $f^{(4k+1)}(x) = (-4)^k e^x (\sin x + \cos x),$
 $f^{(4k+2)}(x) = (-4)^k 2e^x \cos x,$
 $f^{(4k+3)}(x) = (-4)^k 2e^x (\cos x - \sin x)$

61. a. $F'(x) = mg(\cos x + \mu \sin x)$

b. $x = \operatorname{arccot}(-\mu)$

63. $\left. \frac{dx}{d\beta} \right|_{\beta=\pi/6} = -15 \sin \frac{\pi}{6} = -7.5$. This rate is negative

since β is decreasing while x is increasing.



Max. velocity ≈ 54.19 cm/s;

max. acceleration ≈ 358.49 cm/s²

Section 3.4

1. $f(x) = x^6, g(x) = u = 3x - 2.5$

3. $f(x) = 2\sqrt[3]{x}, g(x) = u = x^2 - 9$

5. $f(x) = \sin x, g(x) = u = \frac{1}{x^2 + 1}$

7. $f(x) = \csc x, g(x) = u = 3e^x$

9. $f(x) = \frac{3}{\sqrt{x}}, g(x) = u = \ln x, h(x) = x^2 + 1$

11. $g'(x) = 33(x^5 - \pi x^2 + 7.5)^{10} (5x^4 - 2\pi x)$

13. $F'(x) = \frac{15}{\sqrt{x}(5 + 2\sqrt{x})^6}$

15. $k'(x) = \frac{2(5x^4 - 6x^2 + 10.5)}{(x^5 - 2x^3 + 10.5x)^{7/5}}$

17. $g'(x) = \frac{2x - 5}{2\sqrt{x^2 - 5x + 2}}$

19. $q'(x) = \frac{4}{3}(x^3 - 5x)^{-1/3} (3x^2 - 5)(x + 3)^{5/4}$
 $+ \frac{5}{2}(x^3 - 5x)^{2/3} (x + 3)^{1/4}$

21. $k'(z) = \frac{4z - 5}{(1 + 5z - 2z^2)^2}$

23. $S'(v) = \frac{(v^2 - 5)^2 (6v^2 + 6v + 30)}{(2v + 1)^4}$

25. $T'(s) = \frac{-8s}{3(s^2 + 1)^{1/3} (s^2 - 1)^{5/3}}$

27. $H'(x) = \frac{10x - 3x^3}{(x^2 + 2)^3 \sqrt{x^2 - 2}}$

29. $B'(t) = \frac{1 - 2t^2}{3t^{2/3} (1 + 2t^2)^{4/3}}$

31. $t'(x) = -\sin x \cos(\cos x)$

33. $P'(x) = \tan^2 x + 2x \tan x \sec^2 x$

35. $U'(z) = 10 \sec^2 z \tan z$

37. $C'(x) = \sec^2 x \sin(2 \tan x)$

39. $V'(x) = (-\sin x)e^{\cos x}$

41. $w'(x) = \frac{1}{\sqrt{2x+1}} \left(\cos \sqrt{2x+1} + e^{\tan \sqrt{2x+1}} \sec^2 \sqrt{2x+1} \right)$

43. $f'(x) = \pi^2 (\ln 2) \cos(\pi x) 2^{\sin \pi x}$

45. $t'(s) = (\ln 2) 2^s \sec^2(2^s)$

47. $E'(x) = (\ln 5)^2 5^x 5^{5^x}$

49. $N'(x) = 2x \sin(x^2) \sin\left(2e^{\cos(x^2)}\right) e^{\cos(x^2)}$

51. $C'(x) = -2x \sin(2x^2)$

53. $t'(s) = -\frac{(\ln 10)10^s \sin(10^s)}{2\sqrt{\cos(10^s)}}$

55. $H'(s) = 2^s \ln 2 \left(\cos(2^s) \tan(2^s) + \sin(2^s) \sec^2(2^s) \right)$

57. $T'(z) = e^z \cos(e^z) + e^{\sin z} \cos z$

59. $U'(\theta) = (1 + \sec^2(\theta + \tan(\theta + \tan \theta))) (1 + \sec^2(\theta + \tan \theta)) (1 + \sec^2 \theta)$

61. $y = \frac{4}{3}x + \frac{1}{3}$ 63. $y = 1$

65. $y = -\frac{1}{e\pi^2}x + \frac{2}{e\pi}$ 67. $y = x + 1$

69. $x = 3, x = 4, x = 5$ 71. $x = 0$

73. $x = 0, x = \pm\sqrt{k\pi - 2}, k \in \mathbb{Z}$ 75. $x = \pm 1$

77. $p''(x) = 1520x^2(x^2 + 5)^{18} + 40(x^2 + 5)^{19}$

79. $g''(x) = 10(\sin^2 x - \cos^2 x)$

81. $F''(t) = 6t \cos(t^2) - 4t^3 \sin(t^2)$

83. $G''(x) = 0$

85. $F'(1) + G'(1) = -20$

87. If $n = 4m$, $f^{(n)}(x) = k^n \cos(kx)$;

if $n = 4m + 1$, $f^{(n)}(x) = -k^n \sin(kx)$;

if $n = 4m + 2$, $f^{(n)}(x) = -k^n \cos(kx)$;

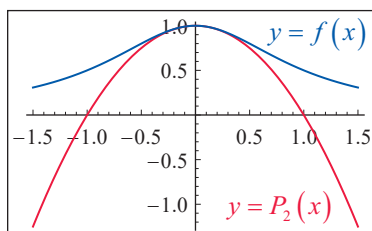
if $n = 4m + 3$, $f^{(n)}(x) = k^n \sin(kx)$.

93. $\frac{dV}{dt} = 6.4\pi \text{ in.}^3/\text{s}$

95. a. $v_{\max} = 1.5 \text{ m/s}$ b. $a_{\max} = 2250 \text{ m/s}^2$

97. $\frac{dP}{dt} \approx 243.63 \text{ Pa/s}$

99. $P_2(x) = 1 - x^2$



Section 3.5

1. $\frac{dy}{dx} = -\frac{1}{2y}$ 3. $\frac{dy}{dx} = \frac{x}{y}$

5. $\frac{dy}{dx} = \frac{1-3y^2}{6xy}$ 7. $\frac{dy}{dx} = \frac{y(2\sqrt{x+2}-1)}{2x-2x\sqrt{x+2}+4}$

9. $\frac{dy}{dx} = \frac{1+y(2\sin x+1)}{2\cos x-x}$ 11. $\frac{dy}{dx} = \frac{2y^2}{3x^2}$

13. $\frac{dx}{dy} = 2y$ 15. $\frac{dx}{dy} = -\frac{y^2}{x^2}$

17. $\frac{dx}{dy} = \frac{e^y - 3\cos y - x}{y}$ 19. $\frac{dx}{dy} = \frac{5y^3}{6x^2}$

21. Tangent line: $y = x - 1$; normal line: $y = -x + 1$

23. Tangent line: $y = -\frac{1}{2}x + 2$; normal line: $y = 2x - 3$

25. Tangent line: $y = -\frac{3}{2}x + \frac{9}{2}$; normal line: $y = \frac{2}{3}x - 2$

27. Tangent line: $y = \frac{7}{4}x - \frac{3}{2}$;
normal line: $y = -\frac{4}{7}x + \frac{22}{7}$

29. $\frac{dy}{dx} = -\frac{x^3}{y^3}$ 31. $\frac{dy}{dx} = \frac{4xy-3y^2}{4xy-3x^2}$ 33. $\frac{dy}{dx} = -1$

35. $\frac{dy}{dx} = \frac{\sin 2x - 2x \sec^2(x^2 + y^2)}{\sin 2y + 2y \sec^2(x^2 + y^2)}$

37. $\frac{dy}{dx} = \frac{2x^4 - 4x^2y - 6xy^2 - x^2 + 2y^2 - y}{3y^2 - 6x^2y - x}$

39. $\frac{dy}{dx} = \frac{5\sqrt{2xy} + y}{3\sqrt{2xy} - x}$ 41. $\frac{dy}{dx} = \frac{\sec^2 x + 2y}{\cos y - 2x}$

43. $\frac{dy}{dx} = \frac{\cos x - \sin x}{2\sqrt{\sin x + \cos x} \sec(x+y) \tan(x+y)} - 1$

45. $\frac{ds}{dt} = \frac{4s-6s^3t^2}{4s^2t^3-\sqrt{s}}$ 47. $\frac{d^2y}{dx^2} = \frac{4y^2-x^2}{16y^3}$

49. $\frac{d^2y}{dx^2} = \frac{(y^2-1)(3y^2+1)}{4x^2y^3}$ 51. $\frac{d^2y}{dx^2} = \frac{-2x(x^3+y^3)}{y^5}$

55. Horizontal tangent: $(1/2, 1)$;

vertical tangent: $(-1, -1/2)$

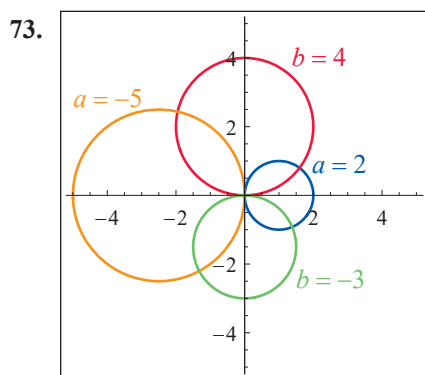
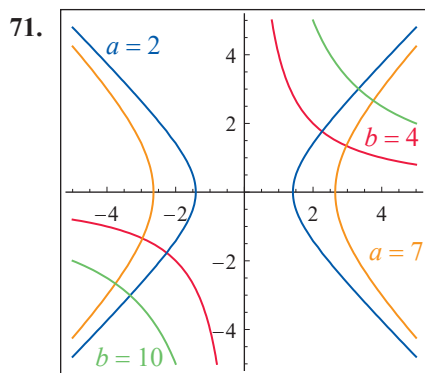
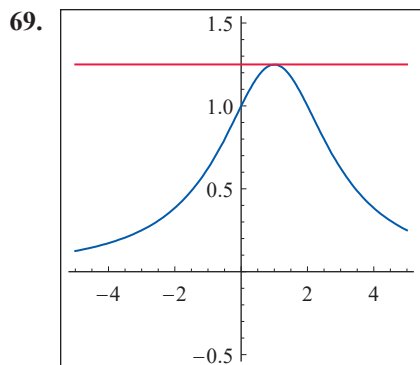
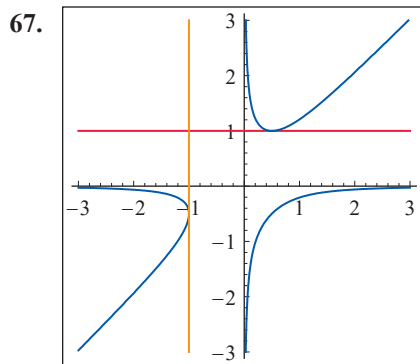
57. Horizontal tangent: $(1, 5/4)$;

vertical tangent: none

59. $y' = x/y$ and $y' = -y/x$, respectively

61. $y' = -1/b$ and $y' = b$, respectively

65. $y = \sqrt{2}x + 2$, $y = -\sqrt{2}x + 2$



Section 3.6

1. $1/12$ 3. 6 5. $1/4$ 7. $-1/2$ 9. $-3/2$
 11. $1/(10 \ln 10)$ 13. 2 15. $5/\cos 0.01$ 17. $1/39$
 19. $-\sqrt{5}/9$ 21. $-5/33$ 23. $1/37$ 25. $-16/7$
 27. 1 29. $2/\sec^2 1$ 31. $f'(x) = 3/x$

33. $h'(x) = \frac{2x}{x^2 + 3}$

35. $G'(x) = \ln \sqrt{x^2 + 4} + \frac{x^2}{x^2 + 4}$

37. $L'(x) = \frac{x^2 + 1 - 2x^2 \ln 2x}{x(x^2 + 1)^2}$

39. $g'(x) = \frac{2}{9 - x^2}$

41. $F'(t) = \frac{t + 2\sqrt{t^2 + 4}}{t^2 + 4 + 2t\sqrt{t^2 + 4}}$

43. $T'(x) = -\tan x$

45. $v'(x) = -2 \sin 2x (\ln(\cos 2x) + 1)$

47. $w'(x) = \log x + \frac{1}{\ln 10}$

49. $y' = (x+2)(x+3)(x+4) + (x+1)(x+3)(x+4) + (x+1)(x+2)(x+4) + (x+1)(x+2)(x+3)$

51. $y' = \sqrt[3]{(2x-1)(x-5)(3x+1)} \left(\frac{2}{6x-3} + \frac{1}{3x-15} + \frac{1}{3x+1} \right)$

53. $y' = \frac{\sqrt[3]{x^3 - 5x^2 + 7}(x+2)}{x^{2/3} \sqrt{3x^2 + 4}} \left(\frac{3x^2 - 10x}{3x^3 - 15x^2 + 21} + \frac{1}{x+2} - \frac{2}{3x} - \frac{3x}{3x^2 + 4} \right)$

55. $y' = \frac{x^2 \sqrt[5]{x^3 + 3}}{\sqrt[4]{x^4 + 4}} \left(\frac{2}{x} + \frac{3x^2}{5(x^3 + 3)} - \frac{x^3}{x^4 + 4} \right)$

57. $y' = (\sin x)^{1/x} \left(\frac{x \cot x - \ln(\sin x)}{x^2} \right)$

59. $y' = (\cos x)^{\sqrt{x}} \left(\frac{\ln(\cos x)}{2\sqrt{x}} - \sqrt{x} \tan x \right)$

61. $y' = (\ln x)^x \left(\ln(\ln x) + \frac{1}{\ln x} \right)$

63. $y' = x^{x^x} (x^x (\ln x + 1) \ln x + x^{x-1})$

65. $y' = (\ln x)^{\sin x} \left(\cos x \ln(\ln x) + \frac{\sin x}{x \ln x} \right)$

67. $y' = -\frac{1}{\sqrt{1-x^2}}$ 69. $y' = -\frac{1}{1+x^2}$

71. $\frac{dy}{dx} = \frac{1}{2x^2 + 2x + 1}$

73. $\frac{dy}{dx} = \frac{1}{(1+x^2) \arctan x}$ 75. $\frac{dy}{dx} = \frac{-1}{2\sqrt{x-x^2}}$

77. $\frac{dy}{dx} = -2\sqrt{1-x^2}$ 79. $\frac{dy}{dx} = \frac{e^x}{1+e^{2x}}$
81. $\frac{dy}{dx} = \frac{-2}{(1+x^2)(1+\arctan x)^2}$
83. $\frac{dy}{dx} = \frac{6x^2 \arcsin(x^3)}{\sqrt{1-x^6}}$ 85. $\frac{dy}{dx} = \frac{2}{(x^2+1)\sqrt{x^2+2}}$
87. $\frac{dy}{dx} = \frac{1}{x^2+1}$
89. $\frac{dy}{dx} = (\arctan x)^x \left[\ln(\arctan x) + \frac{x}{(1+x^2)\arctan x} \right]$
91. $\frac{dy}{dx} = \frac{2}{(x^2+1)^2 \sqrt{x^2+2}}$ 93. $\frac{dy}{dx} = \frac{1}{x^2}$
95. $y = -2.178x + 3.282$ 97. $y = 1.270x + 1.164$
99. $y = \ln(\pi/4)x + 1 - \ln(\pi/4)$
101. $f'(x) = 0$; as remarked in the text,
 $\arcsin(1/x) = \operatorname{arccsc} x$ for $|x| \geq 1$.
103. $\theta = \tan^{-1}(28/s) - \tan^{-1}(8/s)$;
 $\frac{d\theta}{ds} = \frac{8}{s^2+64} - \frac{28}{s^2+784}$
105. Restrict the domain of $\cot x$ to $(0, \pi)$.
107. False; $y = \log \pi$ is a constant function and therefore $y' = 0$.
109. True

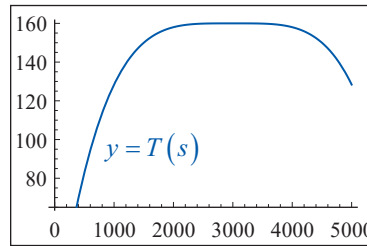
Section 3.7

3. 197,959 people 5. Approx. 367.88 g
7. a. $a = \sqrt[20]{1/2} \approx 0.9659$ b. Approx. 0.707 kg
 c. Approx. 7.629 mg
9. Approx. 8.75 more minutes (i.e., 13.75 total minutes)
11. 300 N 13. Approx. 58.86x N 15. 350 N
19. 0.073 atm/s
21. 0.75 mol/(L·s); 27 mL of water
23. a. \$152.80 b. \$152.50
25. a. \$792.29 b. \$2331 c. \$861.71; \$2311.50
27. \$1077.22 29. Approx. 36π in.³
31. The cost increases by \$465.
33. a. Cost decreases by approx. \$1250.
 b. Profit decreases by approx. \$1050.
 c. The actual decreases are \$1250.01 and \$1052.49, respectively.

35. Approx. 0.8042 cm³ 37. 1.5 percent increase
39. 0.3 percent decrease

Section 3.8

1. -3 ft/s 3. 1/6 5. 8 7. -5.2 9. 0
11. Increasing at 35 in.²/s
13. Approx. 0.8436 units/s
17. -10 cm/min 19. 2.641 cm/min
21. 15.524 mph 23. 0.0808 in./s 25. 12 ft/s
27. 1187.5 mph 29. 25 cm/s; 0.02 rad/s
31. -20 mm²/s 33. 1.52 in./min
35. 1 cm/min 37. $\sqrt{2}/4$ ft/s 39. 15 ft/s
41. 429.29 km/h 43. Approx. 0.494 in./min
45. a.



- b. Approx. 30.465 hp/s

Section 3.9

1. $L(x) = 2x - 2$ 3. $L(x) = 1$
5. $L(\theta) = 1$ 7. $L(t) = -\frac{3}{16}t + \frac{7}{8}$
9. $L(u) = -15u - 62$ 11. $L(z) = z + 1$
13. $dy = 1.4$ 15. $dy = -1.3$
17. $dy = 189/1024$ 19. $dy = -\frac{\pi+2}{8}$
21. $dy = -1/(2e^3)$ 23. $dy \approx -0.1083$
25. $dy = 0.5$; $\Delta y = 0.625$
27. $dy = (\ln 2)/2 \approx 0.3466$; $\Delta y = 2\sqrt[4]{2} - 2 \approx 0.3784$
29. $\Delta y = 1.52$ 31. $\Delta y = -1.4$
33. $\Delta y \approx 0.1854$ 35. $\Delta y = -0.4678$
37. $\Delta y \approx -0.02587$ 39. $\Delta y \approx -0.0934$
41. Approx. 3.0167; calculator value: 3.0166
43. Approx. 3.9667; calculator value: 3.9666
45. Approx. 1.9875; calculator value: 1.9873
47. Approx. 0.9933; calculator value: 0.9933
53. Approx. 0.1266 in. 55. 8.1566 mm²
57. 0.1% 59. Approx. 1 in.³

61. Approx. $\pm 1.24^\circ$

63. a. $F = 0.1875 \text{ N}$ b. $dF \approx 0.017 \text{ N}$

c. $dF \approx 0.0167 \text{ N}$

65. $dE_{\text{kin}} \approx 129.63 \text{ kJ}$; about a 24% increase

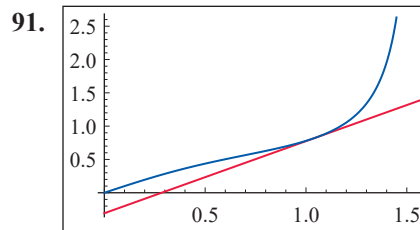
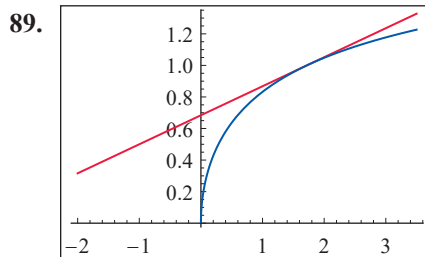
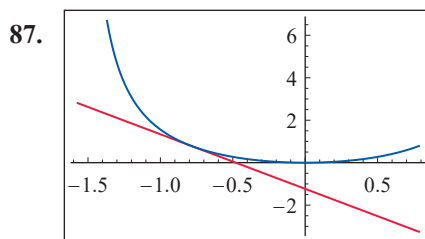
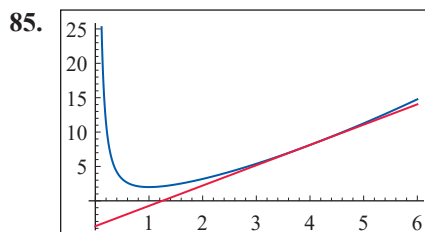
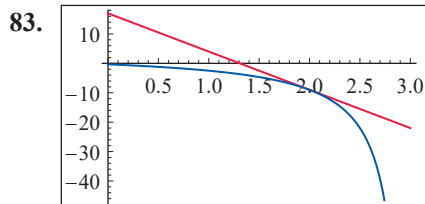
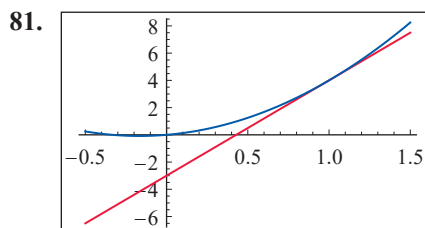
67. 3%; the generalization is $\frac{dA}{A} = \frac{2}{3} \cdot \frac{dV}{V}$

69. Up to 113.615 units

71. $x \approx 5.1$ 73. Yes, we can prove $\frac{dA}{A} = 2 \cdot \frac{dr}{r}$.

75. True 77. True

79. False; consider $f(x) = \sqrt{x}$, $x = 1$, $dx = 0.1$.



Chapter 3 Review

1. $\frac{df}{dx} = 3x^2 + 1$; $\left. \frac{df}{dx} \right|_{x=1} = 4$

3. $D_x s(x) = \frac{1}{2\sqrt{x-2}}$

5. $f'(x) = 2x$; $f''(x) = 2$; $f'''(x) = 0$

7. Not differentiable at 0
One-sided derivatives do not exist at 0

9. Not differentiable at $k \in \mathbb{Z}$
Left-hand derivative at each point of nondifferentiability: 1
Right-hand derivative at each point of nondifferentiability: 1

11. $f'(x) = x^4 - 8x^3 + 3x^2 + x$

13. $h'(x) = 6x^2 - 2x + 8$

15. $F'(t) = \frac{7}{2}t^{5/2} + 4t^3 + \frac{1}{2}t^{-1/2} - \frac{5}{2}t^{3/2} - 3t^2 + \frac{1}{2}t^{-3/2} + 1$

17. $u'(x) = \frac{7e^x}{(3e^x + 5)^2}$ 19. $v'(x) = \frac{2x}{x^2 + 2}$

21. $f'(x) = \frac{1}{(x+1)^2}$; $f''(x) = \frac{-2}{(x+1)^3}$;

$$f'''(x) = \frac{6}{(x+1)^4}$$

23. $f'(x) = \sec^2 x$; $f''(x) = 2\sec^2 x \tan x$;

$$f'''(x) = 2\sec^2 x (\sec^2 x + 2\tan^2 x)$$

25. $f(x) = x^3 - 4x^2 + 6x$ (Answers will vary.)

27. $y = -\frac{3}{8}x + \frac{7}{8}$

29. $y = x$, $y = 13x - 24$ 31. $-1/2$

33. $v(1) = 12.5 \text{ ft/s}$; $a(1) = -12.5 \text{ ft/s}^2$

35. $V'(1) = 784\pi/3 \text{ cm}^3/\text{s}$; $A'(1) = 224\pi/3 \text{ cm}^2/\text{s}$

37. $\frac{dy}{dx} = \frac{4x-5y}{5x-4y}$ 39. $\frac{dx}{dy} = \frac{y(3-y^2)}{3x}$

41. $y = x + 1$ 43. $y = -\frac{20}{9}x + \frac{56}{9}$

45. $\frac{d^2y}{dx^2} = \frac{x^{2/3} + y^{2/3}}{3x^{4/3}y^{1/3}}$ 47. 32 49. $1/2$

51. $f'(x) = \frac{1}{2\sqrt{x}(x+1)}$

53. $f'(x) = \frac{1}{x|\ln x|\sqrt{\ln^2 x - 1}}$ 55. $y' = \frac{(\sqrt{x})^{\ln x} \ln x}{x}$

57. Approx. 8 minutes (8 minutes 3 seconds)

59. Approx. 0.1 in./s 61. 2000 km/h

63. 5 ft 4 in. 65. $L(x) = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}(4-\pi)}{8}$

67. Approx. 0.7354 69. Approx. 10%

71. If $a > 1$, the slope of y increases as we scan the graph from left to right, while in the other case it decreases.

73. True

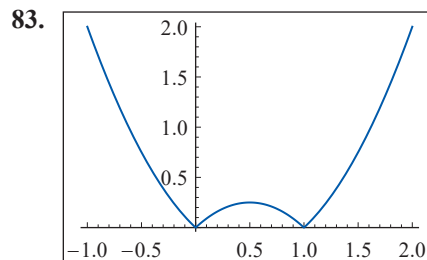
75. False; the Reciprocal Rule along with the Chain Rule (or Quotient Rule) should be used, which yields

$$y' = \frac{3-2x}{(x^2-3x+1)^2}.$$

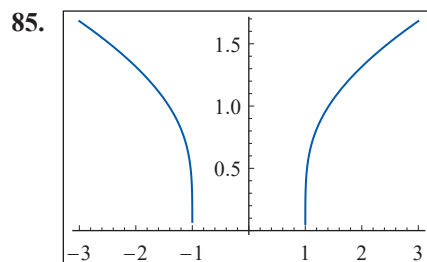
77. False; logarithmic differentiation should be used, which yields $y' = x^x(\ln x + 1)$.

79. True

81. False; $\Delta x = dx$ and even though $\Delta y \rightarrow dy$ might be true under certain circumstances, $x \rightarrow 0$ is not a necessary condition.

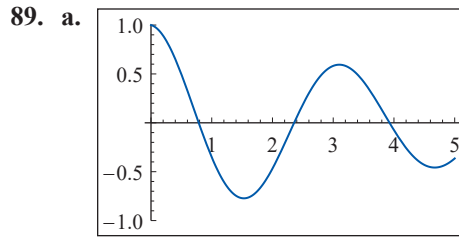


Not differentiable at $x = 0, 1$ (cusps)

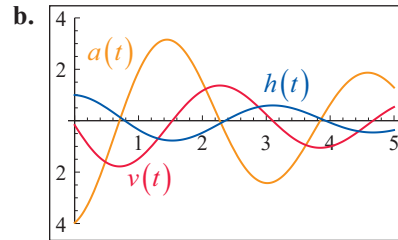


Not differentiable at $x = \pm 1$ (vertical tangents, undefined on $(-1, 1)$)

87. The algorithm employed by the graphing utility may not simplify before differentiating. Yes, we can simplify with an appropriate command. The simplified answer is $f'(x) = \cos x$.



It is an oscillating motion with the amplitude “dying down” as a result of retarding forces.



The object is near its equilibrium position when velocity is maximum and near its extreme positions when velocity is zero. Acceleration, on the other hand, is maximum near the extreme positions and zero near equilibrium.

Chapter 3 Conceptual Project

1. -0.0265 mm Hg/ft 3. -0.0225 mm Hg/ft
5. The symmetric difference quotient is the average of the difference quotients $\frac{f(c+h)-f(c)}{h}$ and $\frac{f(c-h)-f(c)}{-h}$.
7. -0.02302 mm Hg/ft

Chapter 3 Application Project

1. 3.953 s; 126.5 ft/s
3. When an object is dropped, its initial velocity is the same as in Question 2, and it is under the influence of gravity only, so the answer is the same as in Question 2.
5. 6.124 seconds; 196.0 ft/s
This is approximately 133.6 miles per hour, which is not realistic. In Gronk's case, a ball with such speed would most likely have caused significant injury. In reality, air resistance would slow down the ball to a lower speed.

7. $v_t = \sqrt{\frac{2mg}{\rho AC_d}}$

Chapter 4

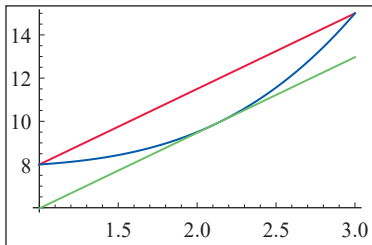
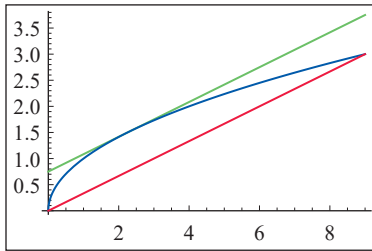
Section 4.1

1. Abs. min.: $f(-1) = f(1) = 0$; no maximum
3. Abs. min.: $f(3\pi/2) = -2$; abs. max.: $f(\pi/2) = 2$
5. Critical points at $x = 0, 1, 3$; rel. min. at $x = 0, 3$; abs. min. at $x = 3$; rel. max. at $x = -1, 1, 4.2$; abs. max. at $x = 4.2$
7. Critical points at $x = 0, 2$; rel. min. at $x = -2, 2$; abs. min. at $x = -2$; rel. max. at $x = 0$; abs. max. at $x = 0$
9. Abs. min.: $f(0) = 1$; abs. max.: $f(3) = 7$
11. No extrema
13. Abs. min.: $v(1) = -4$; abs. max.: $v(-2) = 5$ and $v(4) = 5$
15. No extrema
17. Abs. min.: $n(1) = -1$; no maximum
19. No minimum; abs. max.: $G(0) = 1$
21. Answers will vary. 23. Answers will vary.
25. Answers will vary. 27. Answers will vary.
29. Answers will vary. 31. Answers will vary.
33. Answers will vary. 35. Answers will vary.
37. Answers will vary. 39. $x = 1, -2$ 41. None
43. $x = 1.5$ 45. $x = 0, 4$
47. $t = 0$ 49. $s = 0, 0.5$
51. $\alpha = k\pi, \pm \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}$
53. $x = e^{-2}$ 55. $t = 0$
57. Abs. min.: $g(5) = -68$; abs. max.: $g(0) = 7$
59. Abs. min.: $u(2) = -49$; abs. max.: $u(3) = 0$
61. Abs. min.: $k(-3) = k(1) = -4$; abs. max.: $k(-1) = 4$
63. Abs. min.: $m(x) = 6, x \in [-3, 3]$; abs. max.: $m(-4) = m(4) = 8$
65. Abs. min.: $g(1) = 2$; abs. max.: $g(-1.5) = 14.5$
67. Abs. min.: $G(1) = 2$; abs. max.: $G(1/4) = G(4) = 5/2$
69. Abs. min.: $r(-1) = r(1) = 0$; abs. max.: $r(0) = 1$
71. Abs. min.: $w(-2) = -4$; abs. max.: $w(2) = 4$
73. Abs. min.: $r(5\pi/4) = (-\sqrt{2}/2)e^{5\pi/4}$; abs. max.: $r(\pi/4) = (\sqrt{2}/2)e^{\pi/4}$
75. Abs. min.: $t(0) = t(1) = 0$; abs. max.: $t(0.5) = 1$
77. Abs. min.: $V(-25/12) = 5 + (35/12)(-25/12)^{5/7}$; abs. max.: $V(1) = 11$
79. Abs. min.: $g(0) = -4$; no maximum
81. No minimum; abs. max.: $K(0) = 2$
83. No extrema
85. Abs. min.: $L(0) = 0$; no maximum
87. Abs. min.: $t(2n+1) = 0, n \in \mathbb{Z}$; abs. max.: $t(2n) = 4, n \in \mathbb{Z}$
89. No extrema
91. The wire needs to be cut at $21.995 \approx 22$ inches, and this first piece is to be bent into a circle.
93. Approx. 11,500 calculators
95. False; consider $f(x) = \lfloor x \rfloor$ on $[0, 1]$.
97. True
99. False; the critical points shift by k units to the left.
101. True
103. False; f can only have one largest value (however, it can be attained at multiple x -values).
105. True

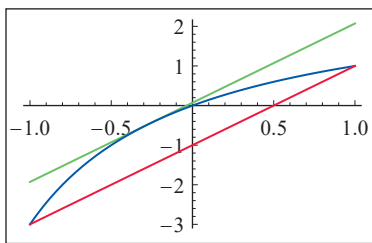
Section 4.2

1. $c = 5/2$ 3. $c = 3$ 9. $c = 2$
11. Rolle's Theorem does not apply; $h(1) \neq h(3)$.
13. $c = 0$
15. Rolle's Theorem does not apply; $H(x)$ is not differentiable at $x = 0$.
17. Rolle's Theorem does not apply; $T(z)$ is not continuous on $[0, 5\pi]$.
19. Rolle's Theorem does not apply; $w(t)$ is not continuous on $[-\pi/4, \pi/4]$.
21. The MVT does not apply; $f(x)$ is not differentiable at $x = 1$.
23. $c = 3/2$
25. The MVT does not apply; $G(x)$ is not continuous on $[-2, 3]$.
27. $c = 0$ 29. $c = -\sqrt{2}/2$
31. The MVT does not apply; $u(z)$ is not continuous on $[0, \pi]$.
33. 10
35. If $f(-5) = -1$ and $f'(x) \leq 0.1$, then $f(5) \leq 0$.
37. $f(x) = x^3 - x^2 + x - 20$
39. $p(t) = -t^2 + 5t - 12$

43. The average velocity is 460 mph, and thus, by the Mean Value Theorem, there must be a time t_0 when the velocity is $v(t_0) = 460$ mph. Use the Intermediate Value Theorem to argue that the equation $v(t) = 450$ has solutions on both intervals $[0, t_0]$ and $[t_0, 10.5]$.
51. No; consider for example $f(x) = \sin x$ on $[0, 2\pi/3]$ with $c = \pi/2$.
53. The average growth rate is 50 rabbits per month, and thus, by the Mean Value Theorem, there must exist a time when the instantaneous growth rate equals 50.
65. False; Rolle's Theorem is a special case of the Mean Value Theorem.
67. True
69. Tangent line: $y = \frac{1}{3}x + \frac{3}{4}$



71. Tangent line: $y = 2x + 7 - 4\sqrt{3}$



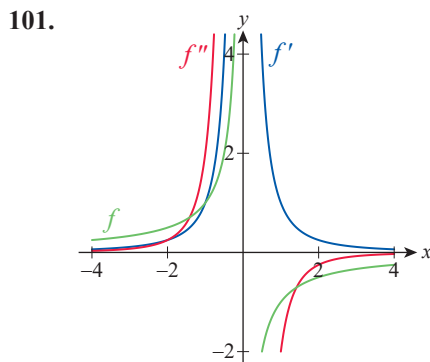
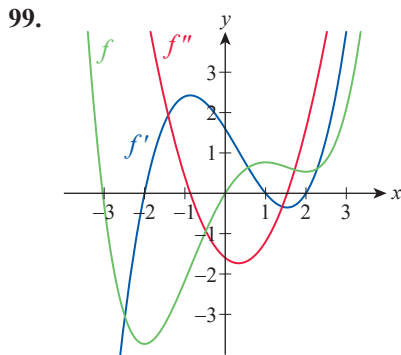
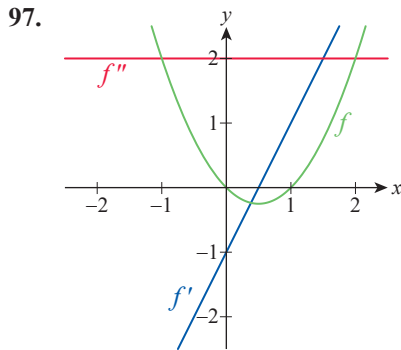
Section 4.3

- Decreasing on $(-\infty, 2)$; increasing on $(2, \infty)$
- Increasing on $(-\infty, -5)$, $(1, \infty)$; decreasing on $(-5, 1)$
- Increasing on $(-\infty, -3)$, $(1/2, 1)$;
decreasing on $(-3, 1/2)$, $(1, \infty)$
- Increasing on $(-\infty, 1)$; decreasing on $(1, \infty)$
- Increasing on $(-\infty, -3 - 2\sqrt{3})$, $(-3 + 2\sqrt{3}, \infty)$;
decreasing on $(-3 - 2\sqrt{3}, -3)$, $(-3, -3 + 2\sqrt{3})$
- Increasing on $(-\infty, 2 - \frac{\sqrt{39}}{3})$, $(2 + \frac{\sqrt{39}}{3}, \infty)$;
decreasing on $(2 - \frac{\sqrt{39}}{3}, 2)$, $(2, 2 + \frac{\sqrt{39}}{3})$
- Increasing on $(-\infty, 3.125)$; decreasing on $(3.125, \infty)$
- Decreasing on $(0, 1/3)$; increasing on $(1/3, \infty)$
- Increasing on $(k\pi, k\pi + \frac{\pi}{2})$, $k \in \mathbb{Z}$;
decreasing on $(k\pi + \frac{\pi}{2}, (k+1)\pi)$, $k \in \mathbb{Z}$
- Decreasing on $(0, 1/4)$; increasing on $(1/4, \infty)$
- Increasing on $(-\infty, -2/\sqrt{11})$, $(2/\sqrt{11}, \infty)$;
decreasing on $(-2/\sqrt{11}, 2/\sqrt{11})$
- Rel. max. at $x = -5$; rel. min. at $x = 1$
- Rel. max. at $x = -3, 1$; rel. min. at $x = 0.5$
- Rel. max. at $x = 1$
- Rel. max. at $x = -3 - 2\sqrt{3}$; rel. min. at $x = -3 + 2\sqrt{3}$
- Rel. max. at $x = 2 - \frac{\sqrt{39}}{3}$; rel. min. at $x = 2 + \frac{\sqrt{39}}{3}$
- Rel. max. at $t = 3.125$ 35. Rel. min. at $s = 1/3$
- Rel. min. at $x = k\pi$, $k \in \mathbb{Z}$;
rel. max. at $x = (2k+1)\frac{\pi}{2}$, $k \in \mathbb{Z}$
- Rel. min. at $t = 1/4$
- Rel. max. at $x = -2/\sqrt{11}$; rel. min. at $x = 2/\sqrt{11}$
- Decreasing on $(-\infty, -1)$; increasing on $(-1, \infty)$;
local min. at $x = -1$; concave up on $(-\infty, \infty)$
- Decreasing on $(-\infty, -2)$, $(2, \infty)$;
increasing on $(-2, 2)$;
local min. at $x = -2$; local max. at $x = 2$;
concave down on $(-\infty, -2\sqrt{3})$, $(0, 2\sqrt{3})$;
concave up on $(-2\sqrt{3}, 0)$, $(2\sqrt{3}, \infty)$;
inflection points at $x = \pm 2\sqrt{3}, 0$
- Increasing on $(-\infty, -1)$, $(1, \infty)$;
decreasing on $(-1, 1)$;
local max. at $x = -1$; local min. at $x = 1$;
concave down on $(-\infty, -\sqrt{2}/2)$, $(0, \sqrt{2}/2)$;
concave up on $(-\sqrt{2}/2, 0)$, $(\sqrt{2}/2, \infty)$;
inflection points at $x = \pm \sqrt{2}/2, 0$

49. Decreasing on $(-\pi/2, -\pi/4), (\pi/4, \pi/2)$;
 increasing on $(-\pi/4, \pi/4)$;
 local min. at $x = -\pi/4$; local max. at $x = \pi/4$;
 concave up on $(-\pi/2, 0)$; concave down on $(0, \pi/2)$;
 inflection point at $x = 0$
51. Concave down on $(-\infty, \infty)$
53. Concave down on $(-\infty, 1/4)$; concave up on $(1/4, \infty)$
55. Concave up on $(-\infty, -2), (0, \infty)$;
 concave down on $(-2, 0)$
57. Concave down on $(-\infty, -3), (0, 1)$;
 concave up on $(-3, 0), (1, \infty)$
59. Concave down on $(-\infty, 5)$; concave up on $(5, \infty)$
61. Concave up on $(-\infty, \infty)$
63. Increasing on $(-\infty, \infty)$; no extrema;
 no inflection points
65. Increasing on $(-\infty, 5)$; decreasing on $(5, \infty)$;
 rel. max. (abs. max.) at $x = 5$;
 concave down on $(-\infty, \infty)$; no inflection points
67. Decreasing on $(-\infty, -3/2), (1, \infty)$;
 increasing on $(-3/2, 1)$;
 rel. min. at $x = -3/2$; rel. max. at $x = 1$;
 concave up on $(-\infty, -1/4)$;
 concave down on $(-1/4, \infty)$;
 inflection point at $x = -1/4$
69. Increasing on $\left(-\infty, \frac{-1-\sqrt{21}}{2}\right), \left(1, \frac{-1+\sqrt{21}}{2}\right)$;
 decreasing on $\left(\frac{-1-\sqrt{21}}{2}, 1\right), \left(\frac{-1+\sqrt{21}}{2}, \infty\right)$;
 rel. max. (abs. max.) at $x = \frac{-1-\sqrt{21}}{2}$;
 rel. max. at $x = \frac{-1+\sqrt{21}}{2}$; rel. min. at $x = 1$;
 concave down on $(-\infty, -\sqrt{2}), (\sqrt{2}, \infty)$;
 concave up on $(-\sqrt{2}, \sqrt{2})$;
 inflection points at $x = \pm\sqrt{2}$
71. Decreasing on $(-\infty, 2), (2, \infty)$; no extrema;
 concave down on $(-\infty, 2)$; concave up on $(2, \infty)$;
 no inflection points
73. Increasing on $\left(-\infty, 4 - \frac{\sqrt{66}}{2}\right), \left(4 + \frac{\sqrt{66}}{2}, \infty\right)$;
 decreasing on $\left(4 - \frac{\sqrt{66}}{2}, 4\right), \left(4, 4 + \frac{\sqrt{66}}{2}\right)$;
 rel. max. at $x = 4 - \frac{\sqrt{66}}{2}$; rel. min. at $x = 4 + \frac{\sqrt{66}}{2}$;
 concave down on $(-\infty, 4)$; concave up on $(4, \infty)$;
 no inflection points
75. Increasing on $(-\infty, -1), (-1, 1), (1, \infty)$;
 no extrema; concave up on $(-\infty, -1)$;
 concave down on $(-1, 1), (1, \infty)$;
 no inflection points
77. Decreasing on $(-\infty, 0), (4/15, \infty)$;
 increasing on $(0, 4/15)$;
 rel. min. at $x = 0$; rel. max. at $x = 4/15$;
 concave up on $(-\infty, -2/15)$;
 concave down on $(-2/15, \infty)$;
 inflection point at $x = -2/15$
79. Increasing on $(0, 1/2)$; decreasing on $(1/2, \infty)$;
 rel. min. (abs. min.) at $x = 0$;
 rel. max. (abs. max.) at $x = 1/2$;
 concave down on $\left(0, \frac{1+\sqrt{2}}{2}\right)$;
 concave up on $\left(\frac{1+\sqrt{2}}{2}, \infty\right)$;
 inflection point at $x = \frac{1+\sqrt{2}}{2}$
81. For $k \in \mathbb{Z}$: increasing on $\left(\frac{3\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi\right)$;
 decreasing on $\left(-\frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi\right)$;
 rel. max. (abs. max.) at $x = -\frac{\pi}{4} + 2k\pi$;
 rel. min. (abs. min.) at $x = \frac{3\pi}{4} + 2k\pi$;
 concave down on $\left(-\frac{3\pi}{4} + 2k\pi, \frac{\pi}{4} + 2k\pi\right)$;
 concave up on $\left(\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi\right)$;
 inflection points at $x = \frac{\pi}{4} + k\pi$
83. Answers will vary. 85. Answers will vary.
87. Answers will vary. 89. Answers will vary.
91. Direction is negative on $(0, 3/4)$, positive on $(3/4, \infty)$; changes direction at $t = 3/4$; acceleration is constant $a = 4$, never zero

93. Direction is positive on $(0,1)$, $(4,\infty)$, negative on $(1,4)$; changes direction at $t=1$ and $t=4$; acceleration is negative on $(0,2.5)$, positive on $(2.5,\infty)$, zero at $t=2.5$

95. Direction is negative on $\left(\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi\right)$, $k \in \mathbb{Z}$, positive elsewhere; changes direction at $t = \frac{\pi}{4} + k\pi$; acceleration is positive on $\left(\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi\right)$, $k \in \mathbb{N} \cup \{0\}$, negative elsewhere, zero at $t = \frac{\pi}{2} + k\pi$



103. a. Midday (temp. is rising, rate of increase is slowing)
 b. Morning (temp. is rising, rate of increase is accelerating)
 c. Early morning (temp. has local minimum)
 d. Evening or nighttime (temp. is falling, rate of decrease is accelerating)
 e. Early morning, just before minimum temperature (temp. is falling, rate of increase is slowing)
 f. Early afternoon (temp. has reached maximum)

105. a. $P(n) = -\frac{n^2}{10} + 50n - 5000$

- b. P is increasing on $(200, 250)$, decreasing on $(250, 350)$; P is maximal when producing 250 liners

107. Local max. at $x = -4$; local min. at $x = 3$

113. $a < 0, 3ac > b^2$

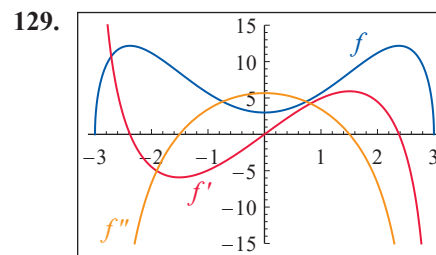
115. $a > 0, b = -6a, c = 0, d$ is arbitrary

121. True

123. False; consider $f(x) = -1/x^2$ with $c = 0$.

125. False; consider $f(x) = g(x) = -x$.

127. False; consider $f(x) = x^3$ with $c = 0$.



131. Answers will vary. 133. Answers will vary.

135. Answers will vary.

Section 4.4

1. 12 3. $1/2$

5. Limit does not exist; L'Hôpital's Rule does not apply.

7. 0

9. Limit does not exist; L'Hôpital's Rule does not apply.

11. $1/2$ 13. ∞/∞ ; $0.2x^2 + 1$ dominates

15. $0 \cdot \infty$; e^{-x} dominates 17. 0 19. $-\infty$

21. L'Hôpital's Rule does not apply; the limit is 0.

23. L'Hôpital's Rule does not apply; the limit is 0.

25. 0 27. 0 29. $-\infty$ 31. $1/2$

33. $2/\ln 3$ 35. 0 37. 1

39. L'Hôpital's Rule does not apply; the limit does not exist.

41. -2 43. ∞ 45. $\frac{\ln 3}{\ln 2}$

47. L'Hôpital's Rule does not apply; the limit is 0.

49. $0 \cdot \infty$; the limit is 0.

51. Not indeterminate form; the limit is 0.

53. ∞^0 ; the limit is 1. 55. $\infty - \infty$; the limit is $-\infty$.

57. $\infty - \infty$; the limit is $-3/2$. 59. 1^∞ ; the limit is $2/e$.

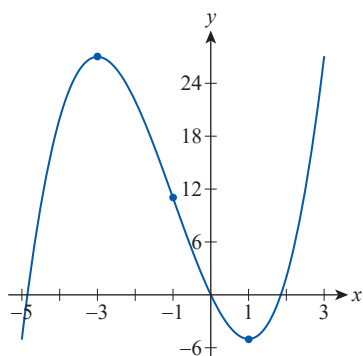
61. Not indeterminate form; the limit is ∞ .

63. ∞^0 ; the limit is 1. 65. 1^∞ ; the limit is 1.

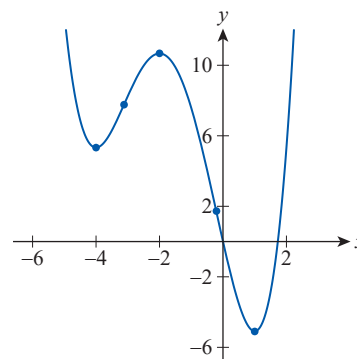
67. Not indeterminate form; the limit is 0.
 69. ∞/∞ ; the limit is $1/100$.
 71. 1^∞ ; the limit is e^2 . 73. $0/0$; the limit is $2/3$.
 75. 0 77. 1
 79. L'Hôpital's Rule does not apply; the limit is 0.
 81. 1 83. $1/2$ 85. $-1/2$
 87. $2/0$ is not indeterminate; L'Hôpital's Rule does not apply.
 89. L'Hôpital's Rule is applied incorrectly; the factors of a product are differentiated.
 91. The third use of L'Hôpital's Rule is unjustified; the limit is $-\infty$.
 93. The limit is 0. 95. The limit is 0.
 97. The limit is ∞ . 99. The limit is 1.
 101. The limit is -5 . 103. The limit is 1.
 105. The limit is 1. 107. The limit is 0.
 109. The limit is 1. 115. $c = 1/2$
 117. The theorem does not apply since $g(a) = g(b)$.
 119. $c = 1$ 121. $c = -\pi/4$ 123. $c = 1/3$
 127. The limit is $16a/9$. 129. $0 \cdot \infty$; the limit is 0.
 131. $\infty - \infty$; the limit is $1/3$.
 133. The limit of the limits is 0.

Section 4.5

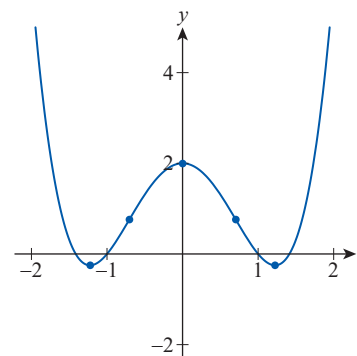
1. First derivative: a ; second derivative: b
 3. First derivative: b ; second derivative: a
 5. Dom = $\mathbb{R}$; x -int.: $0, \frac{-3 \pm 3\sqrt{5}}{2}$; y -int.: 0;
 no asymptotes;
 increasing on $(-\infty, -3), (1, \infty)$;
 decreasing on $(-3, 1)$;
 rel. max. at $x = -3$; rel. min. at $x = 1$;
 concave down on $(-\infty, -1)$;
 concave up on $(-1, \infty)$;
 inflection point at $x = -1$



7. Dom = $\mathbb{R}$; y -int.: 0; no asymptotes;
 decreasing on $(-\infty, -4), (-2, 1)$;
 increasing on $(-4, -2), (1, \infty)$;
 rel. min. at $x = -4$; rel. max. at $x = -2$;
 abs. min. at $x = 1$;
 concave up on $(-\infty, \frac{-5 - \sqrt{19}}{3}), (\frac{-5 + \sqrt{19}}{3}, \infty)$;
 concave down on $(\frac{-5 - \sqrt{19}}{3}, \frac{-5 + \sqrt{19}}{3})$;
 inflection points at $x = \frac{-5 \pm \sqrt{19}}{3}$



9. Dom = $\mathbb{R}$; x -int.: $\pm 1, \pm \sqrt{2}$; y -int.: 2;
 no asymptotes; even function;
 decreasing on $(-\infty, -\sqrt{3/2}), (0, \sqrt{3/2})$;
 increasing on $(-\sqrt{3/2}, 0), (\sqrt{3/2}, \infty)$;
 rel. max. at $x = 0$;
 abs. min. at $x = \pm \sqrt{3/2}$;
 concave up on $(-\infty, -\sqrt{2}/2), (\sqrt{2}/2, \infty)$;
 concave down on $(-\sqrt{2}/2, \sqrt{2}/2)$;
 inflection points at $x = \pm \sqrt{2}/2$



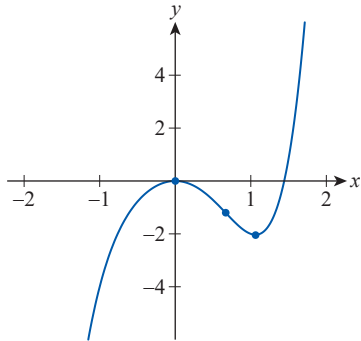
11. Dom = $\mathbb{R}$; x -int.: $0, \sqrt[3]{3}$; y -int.: 0;
 no asymptotes;
 increasing on $(-\infty, 0), (\sqrt[3]{6/5}, \infty)$;
 decreasing on $(0, \sqrt[3]{6/5})$;

rel. max. at $x = 0$; rel. min. at $x = \sqrt[3]{6/5}$;

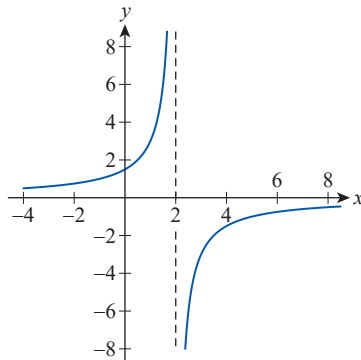
concave down on $(-\infty, \sqrt[3]{3/10})$;

concave up on $(\sqrt[3]{3/10}, \infty)$;

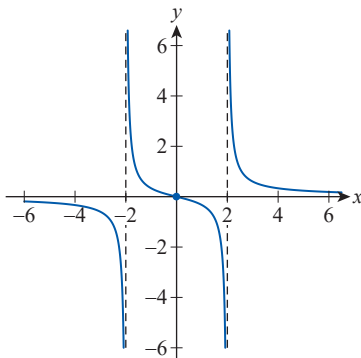
inflection point at $x = \sqrt[3]{3/10}$



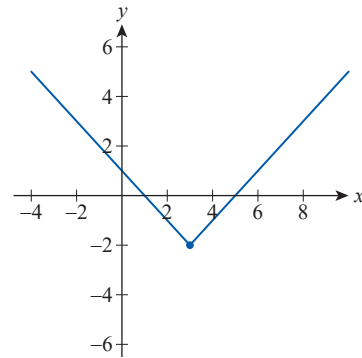
13. Dom = $(-\infty, 2) \cup (2, \infty)$; y-int.: $3/2$;
 vert. asym.: $x = 2$; horiz. asym.: $y = 0$;
 increasing on $(-\infty, 2)$, $(2, \infty)$; no extrema;
 concave up on $(-\infty, 2)$; concave down on $(2, \infty)$;
 no inflection points



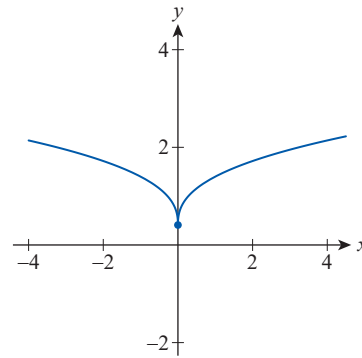
15. Dom = $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$;
 odd function; x-int.: 0; y-int.: 0;
 vert. asym.: $x = \pm 2$; horiz. asym.: $y = 0$;
 decreasing on $(-\infty, -2)$, $(-2, 2)$, $(2, \infty)$;
 no extrema; concave down on $(-\infty, -2)$, $(0, 2)$;
 concave up on $(-2, 0)$, $(2, \infty)$; inflection point at $x = 0$



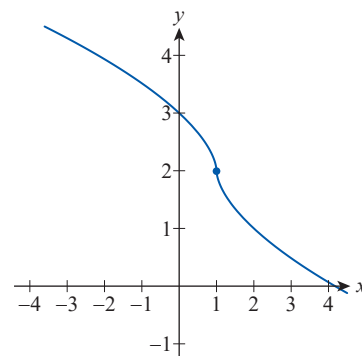
17. Dom = $\mathbb{R}$; x-int.: 1, 5; y-int.: 1; no asymptotes;
 decreasing on $(-\infty, 3)$; increasing on $(3, \infty)$;
 abs. min. at $x = 3$



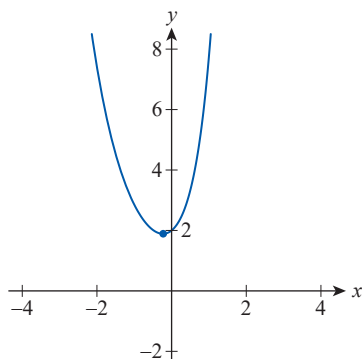
19. Dom = $\mathbb{R}$; even function; y-int.: $2/5$;
 no asymptotes; decreasing on $(-\infty, 0)$;
 increasing on $(0, \infty)$; abs. min. at $x = 0$;
 concave down everywhere on $\mathbb{R}$;
 no inflection points



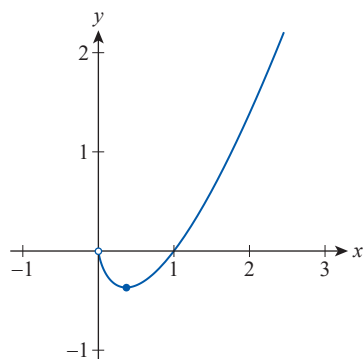
21. Dom = $\mathbb{R}$; x-int.: $1 + 2\sqrt[3]{4}$; y-int.: 3;
 no asymptotes; decreasing everywhere on $\mathbb{R}$;
 no extrema; concave down on $(-\infty, 1)$;
 concave up on $(1, \infty)$; inflection point at $x = 1$



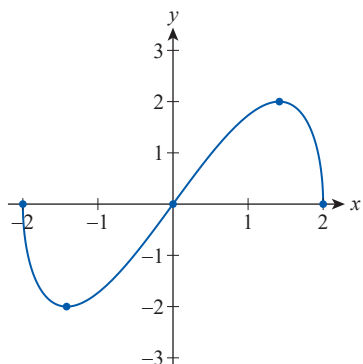
23. Dom = $\mathbb{R}$; y-int.: 2; no asymptotes;
 decreasing on $\left(-\infty, -\frac{\ln 2}{3}\right)$;
 increasing on $\left(-\frac{\ln 2}{3}, \infty\right)$; abs. min. at $x = -\frac{\ln 2}{3}$;
 concave up everywhere on $\mathbb{R}$; no inflection points



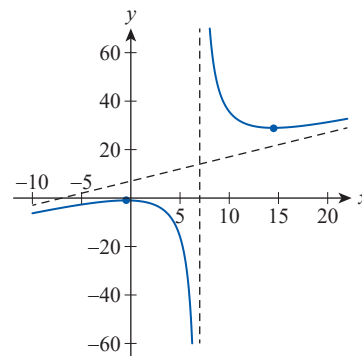
25. Dom = $(0, \infty)$; x-int.: 1; no asymptotes;
 decreasing on $(0, 1/e)$; increasing on $(1/e, \infty)$;
 abs. min. at $x = 1/e$; concave up on $(0, \infty)$;
 no inflection points



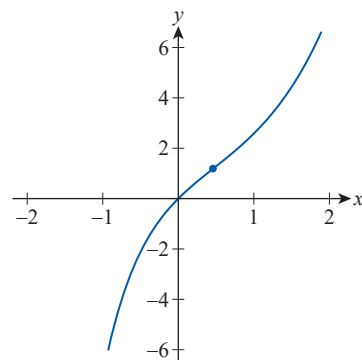
27. Dom = $[-2, 2]$; odd function;
 x-int.: 0, ± 2 ; y-int.: 0; no asymptotes;
 decreasing on $(-2, -\sqrt{2})$, $(\sqrt{2}, 2)$;
 increasing on $(-\sqrt{2}, \sqrt{2})$;
 abs. min. at $x = -\sqrt{2}$; abs. max. at $x = \sqrt{2}$;
 concave up on $(-2, 0)$; concave down on $(0, 2)$;
 inflection point at $x = 0$



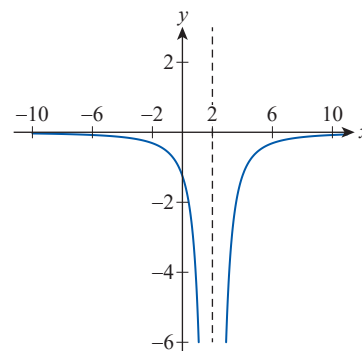
29. Dom = $(-\infty, 7) \cup (7, \infty)$; y-int.: -1;
 vert. asym.: $x = 7$; slant asym.: $y = x + 7$;
 increasing on $(-\infty, 7 - 2\sqrt{14})$, $(7 + 2\sqrt{14}, \infty)$;
 decreasing on $(7 - 2\sqrt{14}, 7)$, $(7, 7 + 2\sqrt{14})$;
 rel. max. at $x = 7 - 2\sqrt{14}$; rel. min. at $x = 7 + 2\sqrt{14}$;
 concave down on $(-\infty, 7)$; concave up on $(7, \infty)$;
 no inflection points



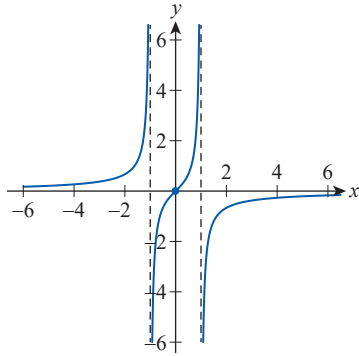
31. Dom = $\mathbb{R}$; x-int.: 0; y-int.: 0; no asymptotes;
 increasing everywhere on $\mathbb{R}$; no extrema;
 concave down on $\left(-\infty, \frac{\ln 4}{3}\right)$;
 concave up on $\left(\frac{\ln 4}{3}, \infty\right)$; inflection point at $x = \frac{\ln 4}{3}$



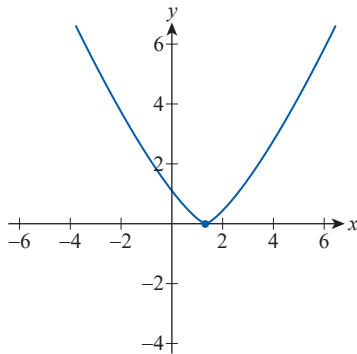
33. Dom = $(-\infty, 2) \cup (2, \infty)$; y-int.: $-5/4$;
 vert. asym.: $x = 2$; horiz. asym.: $y = 0$;
 decreasing on $(-\infty, 2)$; increasing on $(2, \infty)$;
 no extrema;
 concave down on $(-\infty, 2)$, $(2, \infty)$; no inflection points



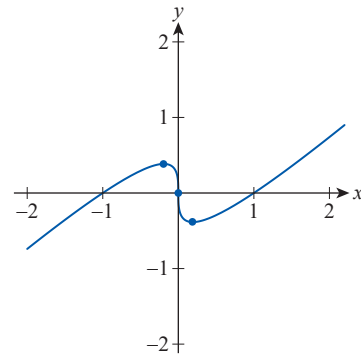
35. Dom = $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$;
 odd function; x-int.: 0, y-int.: 0;
 vert. asym.: $x = \pm 1$; horiz. asym.: $y = 0$;
 increasing on $(-\infty, -1), (-1, 1), (1, \infty)$; no extrema;
 concave up on $(-\infty, -1), (0, 1)$;
 concave down on $(-1, 0), (1, \infty)$;
 inflection point at $x = 0$



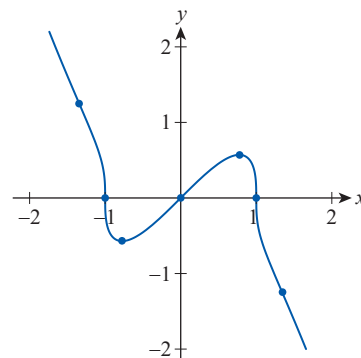
37. Dom = $\mathbb{R}$; x-int.: $4/3$; y-int.: $(4/3)^{1/3}$;
 no asymptotes;
 decreasing on $(-\infty, 4/3)$; increasing on $(4/3, \infty)$;
 abs. min. at $x = 4/3$;
 concave up everywhere on $\mathbb{R}$; no inflection points



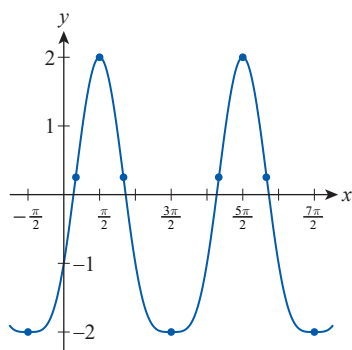
39. Dom = $\mathbb{R}$; odd function; x-int.: 0, ± 1 ; y-int.: 0;
 no asymptotes;
 increasing on $(-\infty, -\frac{1}{3\sqrt{3}}), (\frac{1}{3\sqrt{3}}, \infty)$;
 decreasing on $(-\frac{1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}})$;
 rel. max. at $x = -\frac{1}{3\sqrt{3}}$; rel. min. at $x = \frac{1}{3\sqrt{3}}$;
 concave down on $(-\infty, 0)$;
 concave up on $(0, \infty)$; inflection point at $x = 0$



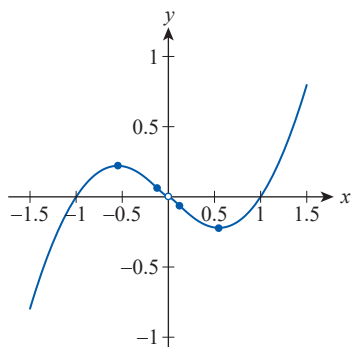
41. Dom = $\mathbb{R}$; odd function;
 x-int.: 0, ± 1 ; y-int.: 0; no asymptotes;
 decreasing on $(-\infty, -\sqrt{3/5}), (\sqrt{3/5}, \infty)$;
 increasing on $(-\sqrt{3/5}, \sqrt{3/5})$;
 rel. min. at $x = -\sqrt{3/5}$; rel. max. at $x = \sqrt{3/5}$;
 concave up on $(-\infty, -3/\sqrt{5}), (-1, 0), (1, 3/\sqrt{5})$;
 concave down on $(-3/\sqrt{5}, -1), (0, 1), (3/\sqrt{5}, \infty)$;
 inflection points at $x = 0, \pm \frac{3}{\sqrt{5}}, \pm 1$



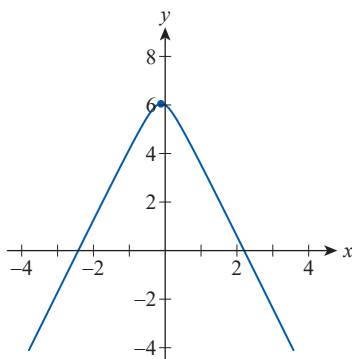
43. Dom = $\mathbb{R}$; 2π -periodic; for the period $-\pi/2 \leq x \leq 3\pi/2$ we have the following: y-int.: -1;
 no asymptotes;
 increasing on $(-\pi/2, \pi/2)$;
 decreasing on $(\pi/2, 3\pi/2)$;
 abs. min. at $x = -\pi/2, 3\pi/2$;
 abs. max. at $x = \pi/2$;
 concave up on $(-\pi/2, \pi/6), (5\pi/6, 3\pi/2)$;
 concave down on $(\pi/6, 5\pi/6)$;
 inflection points at $x = \pi/6, 5\pi/6$



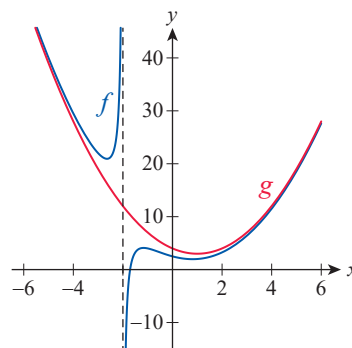
45. Dom = $(-\infty, 0) \cup (0, \infty)$; odd function;
 x-int.: ± 1 ; no asymptotes;
 decreasing on $(-e^{-3/5}, 0), (0, e^{-3/5})$;
 increasing on $(-\infty, -e^{-3/5}), (e^{-3/5}, \infty)$;
 rel. max. at $x = -e^{-3/5}$; rel. min. at $x = e^{-3/5}$;
 concave down on $(-\infty, -e^{-21/10}), (0, e^{-21/10})$;
 concave up on $(-e^{-21/10}, 0), (e^{-21/10}, \infty)$;
 inflection points at $x = \pm e^{-21/10}$



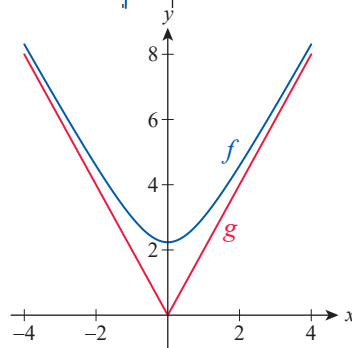
47. Dom = $\mathbb{R}$; x-int.: $\frac{-1 \pm \sqrt{433}}{9}$; y-int.: 6;
 no asymptotes;
 increasing on $(-\infty, -1/9)$;
 decreasing on $(-1/9, \infty)$;
 abs. max. at $x = -1/9$;
 concave down everywhere on $\mathbb{R}$;
 no inflection points



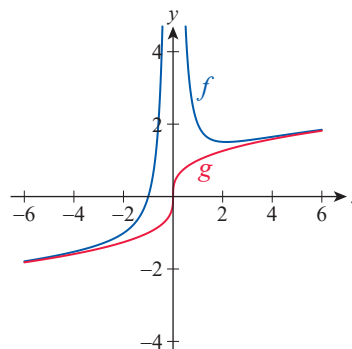
49.



51.



53.



55. No; depending on the starting point, the method may converge to a different root, or not converge at all.

57. 2.65914 59. 1.60943 61. -1.52138

63. 0.48046, 5.61953 65. -0.75487 67. 0.44171

69. 0.69005 71. 0.876726 73. 0.930407

75. 2.475353 77. 0.5671 79. 1.0768, 3.6435

 81. ± 1.15932 83. $f'(x_1) = 0$

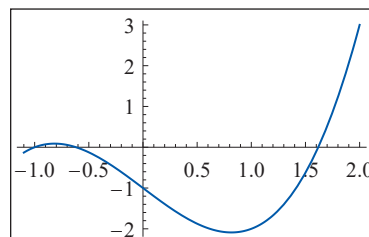
 85. $x_{n+1} = -x_n$ for all n .

 87. x_n alternates between the values of 3 and 1.

 89. $x_{n+1} = \frac{k-1}{k}x_n + \frac{a}{k(x_n)^{k-1}}$

91. 1.41421 93. 2.15443 95. 0.14285

99.



Section 4.6

1. 60 and 60 3. -18 and 18 5. n and n
7. 1 and 1 9. 120 and 80 11. 4 and 4
13. 1 and 0 15. $w = 2\sqrt{k/3}$; $h = 2k/3$
17. $w = 2$, $h = 24$ 19. $y = -\frac{1}{\sqrt[3]{3}}x + \sqrt[3]{9} + 1$
21. $(1/2, \sqrt{2}/2)$
23. Width: $2\sqrt{3}$, height: 2 25. 60 ft by 30 ft
27. Each pen is to be $30\sqrt{2} \times 20\sqrt{2}$ ft, joining along the $20\sqrt{2}$ ft side.
29. Width of cross-section: $w = 2r/\sqrt{3}$ in.,
depth of cross-section: $h = 2\sqrt{2}r/\sqrt{3}$ in.
31. Cut at $\frac{30\pi}{4+\pi}$ in., and use the first piece to form the circle.
33. In the first case, form a circle without cutting. In the second case, form a square.
37. A square of side length $\sqrt{A}$ units
39. $r = h = \sqrt{\frac{S}{3\pi}}$ in.
41. A $2 \times 2 \times 8$ in. box is cheapest, costing \$2.88.
43. $\alpha = 2\pi - 2\pi\sqrt{2/3}$ 45. 14×21 in.
47. $\alpha = 2$ rad, $r = 13$ ft 49. $s = 4\sqrt{14}$ ft
51. The wire needs to be staked 18 ft from the taller antenna.
53. The wire needs to be staked approx. 15.865 ft from the taller antenna.
55. We need to fold $(1/4)(8.5) = 2.125$ in. from the right lower corner.
57. Radius of base: $r = 2\sqrt{2}R/3$, height: $h = 4R/3$
59. Radius of base: $r = \frac{HR}{2(H-R)}$,
height: $h = \frac{H(H-2R)}{2(H-R)}$
61. $\frac{65}{3} \times \frac{65}{3} \times \frac{130}{3}$ in. 63. $b = P/4$
65. He should land approx. 9 miles from the store.
67. Approx. 10 minutes and 21.2 seconds later
69. $t = \pi/4$ s; $v(\pi/4) = -3\sqrt{2}$ ft/s
71. $\theta = \pi/4$
73. \$420; maximum revenue: \$588,000
75. \$1900; apartments rented: 95

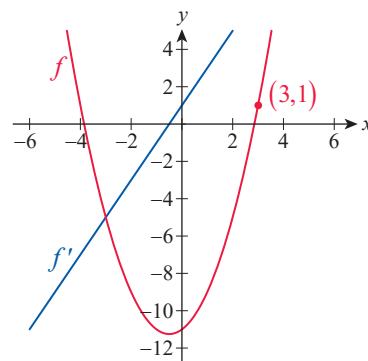
77. 2600 cars; \$29.75 each

81. $x = 7.5$ units

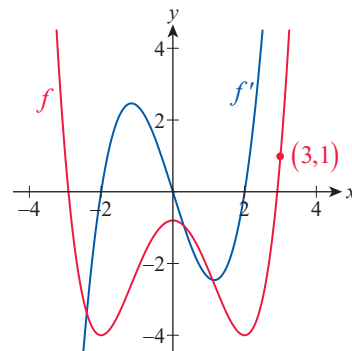
83. The total volume is maximum when we paint a sphere of radius $\frac{1}{2\sqrt{\pi}}$ ft (approx. 3.39 in.) and no tetrahedron. The total volume is a minimum when we paint a sphere of radius approx. 1.63 in., and a tetrahedron of side length approx. 7.99 in.

Section 4.7

9. $F(x) = x$ 11. $H(x) = x^4 - \frac{1}{2}x^2$
13. $V(x) = \tan x + \frac{3}{2}x^2$ 15. $F(x) = 5e^x$
17. $U(t) = 2/t^2$ 19. $W(z) = \arcsin z$
21. $F(x) = x^2 - 3x + C$, $C = 3$
23. $F(x) = 2\sqrt{x} + C$, $C = -1$
25. $F(x) = C$, $C = 1$
27. $F(x) = \frac{x^4}{4} + \frac{1}{x} + C$, $C = -1/4$
29. $F(x) = 10^x + C$, $C = -9$
31. $F(x) = \tan\left(\frac{\pi}{4}x\right) + x + C$, $C = -1$
33. Answers will vary. Sample graph:



35. Answers will vary. Sample graph:

37. $F(x) = 2x^3 - 2x^2 + 1.5x + C$ 39. $H(x) = \frac{x^6}{2} - 2x^5 + \frac{x^3}{3} + 7x + C$

41. $V(x) = 2x^3 + \frac{39}{2}x^2 + 18x + C$

43. $H(x) = \frac{2x^{9/2}}{9} + C$

45. $N(x) = \frac{x^2}{2} + 7\ln|x| + C$

47. $A(y) = \frac{3}{11}y^{11/3} - \frac{6}{7}y^{7/3} + y + C$

49. $G(t) = \frac{e^{3t}}{3} - 3\sec t + C$

51. $T(\theta) = \frac{\theta^2}{2} + \sin \theta + C$

53. $V(x) = \csc x - \cot x + C = \tan(x/2) + C$

55. $W(x) = \csc x + C$

57. $A(x) = \frac{5}{3}\arctan 3x + C$

59. $C(x) = \frac{4}{5}\operatorname{arcsec} 5x + C$

61. $f(x) = \frac{\pi}{2}x^2 - \pi x + \frac{\pi}{2}$

63. $f(x) = x^2 - 2x + 2$

65. $f(x) = \frac{9}{28}x^{7/3} - \frac{3}{4}x + \frac{4}{7}$

67. $f(x) = \frac{8}{105}x^{7/2} + \frac{x^3}{6} + \frac{x^2}{2} - x + 7$

69. $f(x) = 2\arctan 2x + \frac{\pi}{2}$

71. $f(x) = \frac{\sin 2x}{8} + \frac{x^2}{2} + \frac{3x}{4} - 1$

73. $f(x) = \frac{2^{5x}}{(\ln 32)^2} + Cx + D$

75. $f(x) = -\cos x - \frac{e^{2x}}{4} - \frac{3}{2}x + \frac{9}{4}$

77. 39 ft

79. It falls for approx. 7.49 s, and the speed of impact is approx. 83.47 m/s.

81. $16\sqrt{15} \approx 61.97$ ft/s 83. Approx. 22.12 m

87. Approx. 54.77 m 89. 5275

91. The position function (assuming the initial position is 0) is $p(t) = 5.5t^2$. It covers 12.5 m in the first 1.5 s. Its acceleration time from 0 to 60 mph is 2.4 s.

93. 8.25 ft/s^2

95. $p(t) = -\frac{4}{105}t^{5/2}(3t-14)+3$; in 5 seconds it will be at $x = 3 - \frac{20\sqrt{5}}{21} \approx 0.87$. Its instantaneous velocity will be zero at $t = 0$ and $t = 10/3$.

97. True

99. False; if $F(x)$ is an antiderivative, then $F(x) + C$ is also an antiderivative for any $C \in \mathbb{R}$.

101. True

103. False; consider $f(x) = g(x) = 1$ with $F(x) = G(x) = x$. Note that $x \cdot x = x^2$ is not an antiderivative of $1 \cdot 1 = 1$.

Chapter 4 Review

1. Answers will vary. 3. Answers will vary.

5. Abs. min.: $f(1) = -1/3$; abs. max.: $f(-1) = 7/3$

7. Abs. min.: $f(-2) = f(4) = 0$; abs. max.: $f(1) = 9$

9. Abs. min.: $f(2) = -\sqrt{2}$; abs. max.: $f(1/3) = 2/(3\sqrt{3})$

11. No extrema

13. Abs. min.: $f(\pi) = 1$; no maximum

17. $c = 4/3$ 19. $c = \sqrt[3]{65/4}$

23. $f(x) = \sin x + e^x + 2$

27. Increasing on $(-\infty, 3)$; decreasing on $(3, \infty)$

29. Decreasing on $(-\infty, -3)$, $(2, 4)$;
increasing on $(-3, 2)$, $(4, \infty)$

31. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$

33. Concave up on $(-\infty, -1)$, $(1, \infty)$;
concave down on $(-1, 1)$

35. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$;
rel. min. (abs. min.) at $x = 0$;
concave down on $(-\infty, -1/\sqrt[4]{3})$, $(1/\sqrt[4]{3}, \infty)$;
concave up on $(-1/\sqrt[4]{3}, 1/\sqrt[4]{3})$;
inflection points at $x = \pm 1/\sqrt[4]{3}$

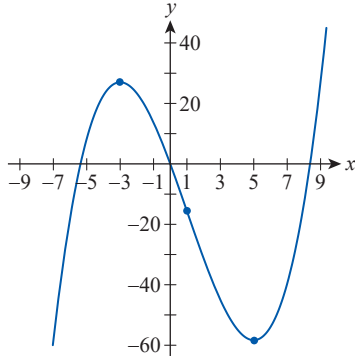
37. Direction is positive on $(0, \sqrt{3})$,
negative on $(\sqrt{3}, 3)$;
changes direction at $t = \sqrt{3}$;
acceleration is positive on $(0, 1)$,
negative on $(1, 3)$, zero at $t = 1$

39. 0

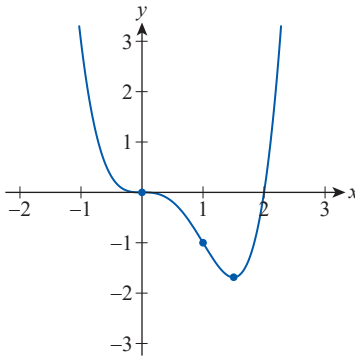
41. L'Hôpital's Rule does not apply; the limit is ∞ .

43. $-\infty$ 45. ∞ 47. $1/2$ 49. 0

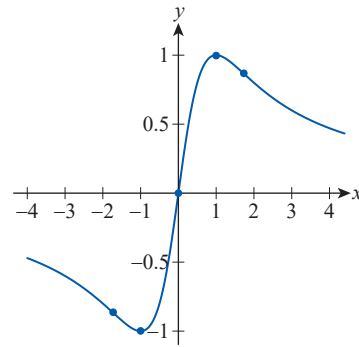
51. Dom = $\mathbb{R}$; x-int.: $0, \frac{3 \pm 3\sqrt{21}}{2}$; y-int.: 0;
 no asymptotes; increasing on $(-\infty, -3), (5, \infty)$;
 decreasing on $(-3, 5)$;
 rel. max. at $x = -3$; rel. min. at $x = 5$;
 concave down on $(-\infty, 1)$; concave up on $(1, \infty)$;
 inflection point at $x = 1$



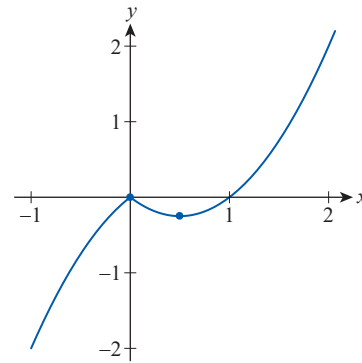
53. Dom = $\mathbb{R}$; x-int.: 0, 2; y-int.: 0;
 no asymptotes;
 decreasing on $(-\infty, 3/2)$;
 increasing on $(3/2, \infty)$;
 abs. min. at $x = 3/2$;
 concave up on $(-\infty, 0), (1, \infty)$;
 concave down on $(0, 1)$;
 inflection points at $x = 0, 1$



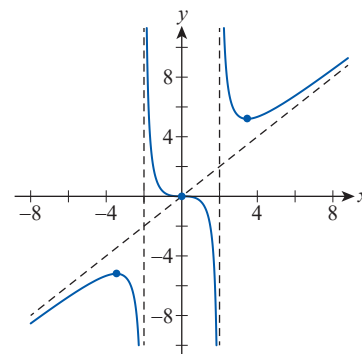
55. Dom = $\mathbb{R}$; odd function; x-int.: 0; y-int.: 0;
 horiz. asym.: $y = 0$;
 decreasing on $(-\infty, -1), (1, \infty)$;
 increasing on $(-1, 1)$;
 abs. min. at $x = -1$; abs. max. at $x = 1$;
 concave down on $(-\infty, -\sqrt{3}), (0, \sqrt{3})$;
 concave up on $(-\sqrt{3}, 0), (\sqrt{3}, \infty)$;
 inflection points at $x = 0, \pm\sqrt{3}$



57. Dom = $\mathbb{R}$; x-int.: 0, 1; y-int.: 0; no asymptotes;
 increasing on $(-\infty, 0), (1/2, \infty)$;
 decreasing on $(0, 1/2)$;
 rel. max. at $x = 0$; rel. min. at $x = 1/2$;
 concave down on $(-\infty, 0)$;
 concave up on $(0, \infty)$;
 inflection point at $x = 0$



59. Dom = $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$;
 odd function; x-int.: 0; y-int.: 0;
 vert. asym.: $x = \pm 2$; slant asym.: $y = x$;
 increasing on $(-\infty, -2\sqrt{3}), (2\sqrt{3}, \infty)$;
 decreasing on $(-2\sqrt{3}, -2), (-2, 2), (2, 2\sqrt{3})$;
 rel. max. at $x = -2\sqrt{3}$; rel. min. at $x = 2\sqrt{3}$;
 concave down on $(-\infty, -2), (0, 2)$;
 concave up on $(-2, 0), (2, \infty)$;
 inflection point at $x = 0$



61. 1.97435 63. 0.741912 65. 0.4797

67. $\frac{1}{n-1\sqrt{n}}$ 69. $a = (a_1 + \cdots + a_n)/n$

71. $l = 2\sqrt{2/3}$; $w = 2/\sqrt{3}$

73. $5\sqrt{10} \times 10\sqrt{10}$ 75. $4 - 2\sqrt{3}$

77. The equilateral triangle

79. The task is impossible, since $A(r)$ has no maximum. (The surface area approaches infinity as $r \rightarrow 0$ or $r \rightarrow \infty$.)

81. Nate needs to reach the other side approximately 500 ft downstream from the restaurant.

83. $F(x) = x^5 - 1.2x^4 + e^2x + C$

85. $F(x) = 0.16x^{5/2} - 4x^{1/2} + C$

87. $F(x) = x^2 + \tan 2x + C$

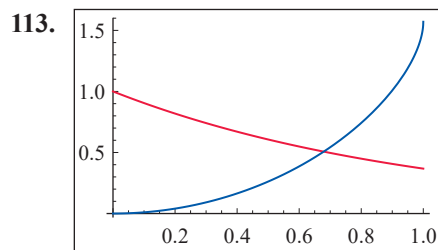
89. $F(x) = \frac{3}{2} \arctan 2x + C$

91. $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 8x - \frac{17}{3}$

93. 80 ft/s

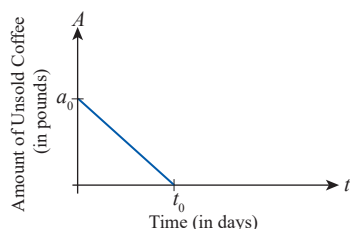
95. False; consider $y = 1/x$ on $(0, 1)$.

97. True 99. True

101. False; consider $y = x^5$ at $x = 0$.

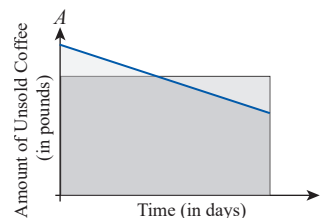
Chapter 4 Application Project

1. It is a linear function with a slope of
- $-c$
- .



3. Since the holding cost is proportional to time and the amount of unsold coffee at hand, it will also be proportional to their product. As a consequence, the holding cost is well represented by the area enclosed by the graph of $A(t)$ and the t -axis. (This is easiest to see when the amount stored is constant.) Since in Question 1 we saw that $A(t)$ was a decreasing linear

function, the area under the curve over a particular time period is the same as the area under the constant function equal to the average value over that time interval. A graphical interpretation of this is below.



5. a. $H_{t_0} = \frac{a_0}{2} h t_0$ b. $C_{t_0} = \frac{a_0}{2} h t_0 + f$

7. $x = \sqrt{\frac{2f}{ch}}$

Chapter 5

Section 5.1

1. $(O_4 + U_4)/2 = 17$ 3. $(O_8 + U_8)/2 = 16.25$

5. Approx. 126 ft; an overestimate

7. a. $O_{12} = 112$ m; $U_{12} = 97.45$ m b. $M_6 = 104.8$ m

c. $(O_{12} + U_{12})/2 = 104.725$ m;
 M_6 is greater, because the curve is likely concave down, as indicated by the data.

9. Left-endpoint est. ≈ 4.146 (underestimate);
 right-endpoint est. ≈ 6.146 (overestimate); midpoint
 est. ≈ 5.384 (overestimate)

11. Left-endpoint est. $= 25/12$ (overestimate);
 right-endpoint est. $= 77/60$ (underestimate); midpoint
 est. $= 496/315$ (underestimate)

13. Left-endpoint est. ≈ 2.367 (overestimate);
 right-endpoint est. ≈ 1.582 (underestimate); midpoint
 est. ≈ 2.013 (overestimate)

15. $\sum_{i=1}^{33} 3i$ 17. $\sum_{i=1}^{100} \frac{1}{i^2}$ 19. $\sum_{i=-2}^{38} a_{2i+1}$

21. $\sum_{i=1}^n f\left(\frac{3i}{n}\right)$ 23. $\sum_{i=0}^n f(x_i^*) \Delta x$ 25. 64 27. $501 + n$

29. $25 + 2n$ 31. $11/6$ 33. 255

35. $\frac{7n^2 + 2n}{2}$ 37. 17,080 39. $\frac{6n^3 + 15n^2 + 11n}{2}$

41. $\frac{n(n+1)(3n^2 - 5n - 4)}{6}$

43. $\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \cdots + \frac{1}{10} - \frac{1}{11} = \frac{10}{11}$

45. $\sqrt{1} - \sqrt{2} + \sqrt{2} - \cdots + \sqrt{n} - \sqrt{n+1} = 1 - \sqrt{n+1}$

47. $e^2 - e^3 + e^3 - \cdots + e^{n+3} - e^{n+4} = e^2 - e^{n+4}$

49. 88,573

51. $\frac{3(1-(2/3)^{100})}{5}$

55. $\sum_{j=1}^{n-2} \frac{1}{j}$ 57. $\sum_{l=5}^{21} \cos(2l\pi)$

59. 6 61. 1/4 63. 11/12

65. The region between the graph of $x^3 + 2x$ and the x -axis from 0 to 367. The region between the graph of $\sin x$ and the x -axis from 0 to $\pi/2$

71. Approx. 44.44 m 73. 22.5 m 85. 110

87. $\frac{mn(m+1)(n+1)}{4}$

89. $\lim_{n \rightarrow \infty} \frac{-\pi \csc \frac{\pi}{2n} \sin \frac{\pi - n\pi}{2n} + \pi \csc \frac{\pi}{2n} \sin \frac{\pi + n\pi}{2n}}{2n} = 2$

91. $\lim_{n \rightarrow \infty} \frac{3n + \csc \frac{\pi}{2n} \sin \frac{(2n+1)\pi}{2n}}{8n} = \frac{3}{8}$

Section 5.2

1. a. 10 b. $(60+9\pi)/8 \approx 11.0343$ c. 9 d. $(9\pi/8) - 18 \approx -14.4657$

3. 64 ft; -96 ft; -800 ft 5. 16

7. $7564/11,025 \approx 0.686$ 9. $1899/16,000 \approx 0.119$

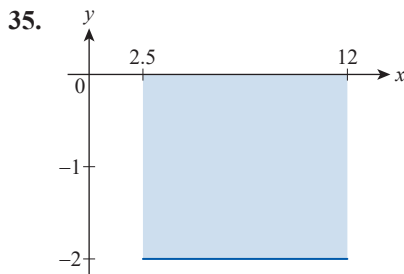
11. $1 + \frac{\ln 3}{2} \approx 1.549$

13. 19/25 15. 17/2 17. 26/3 19. 13

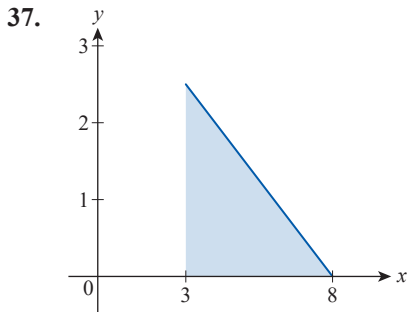
21. 4 23. 1/4 25. 201/4 27. 44/3 29. $3/\sqrt[3]{4}$

31. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{4}{n} \cdot \left[\left(\frac{4i}{n} \right)^2 - \log_2 \frac{4i}{n} \right]$

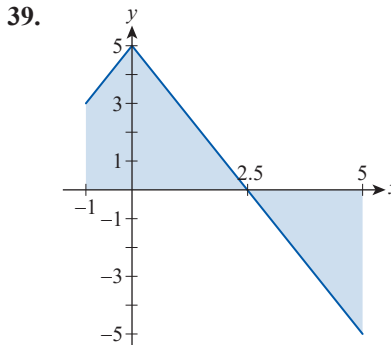
33. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{b-2}{n} \sqrt[4]{2 + \frac{(b-2)i}{n}}$



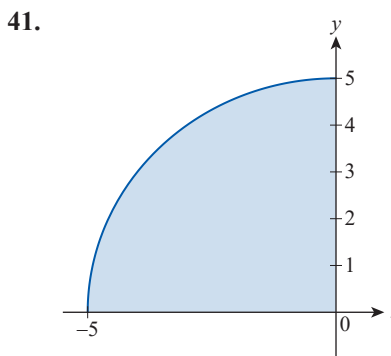
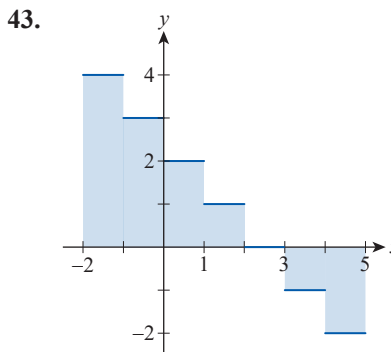
Value: -19



Value: -25/4

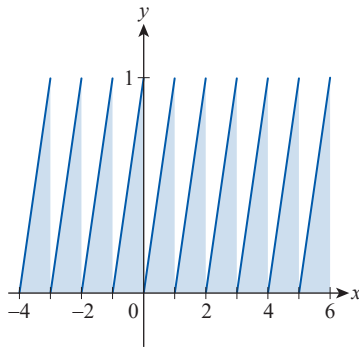


Value: 4

Value: $25\pi/4$ 

Value: 7

45.



Value: 5

53. No; see Exercise 51.

55. Yes 57. Yes 59. No; see Exercise 49. 61. E

 63. A 65. C 67. $8 + (c - a)$ 69. 0

 71. $5\sqrt{2}/2$ 73. Not possible 75. $1 + 5\pi/4$

 77. $-1 - \frac{3\sqrt{2}}{4}$ 79. $69/2$ 81. $11/6$ 83. 0 85. 2

87. 0 89. Not possible

 93. $0 \leq \int_2^3 \sqrt{3-x} \, dx \leq 1$ 95. $6 \leq \int_0^6 \left(\frac{x^2}{32} - \frac{x}{4} + \frac{3}{2} \right) dx \leq 9$

 105. -1 107. $17/2$ 109. $-5/3$ 111. $13/5$

 113. $6/5$ 115. $-9/4$ 117. $\int_0^3 \left(\frac{2}{9}x + 4 \right) dx = 13$

 119. $\int_0^2 \sqrt{4-x^2} \, dx = \pi$ 123. $\frac{\pi^3}{3} - \frac{3\pi^2}{2} + \pi$

 125. False; consider $f(x) = g(x) = x$ on $[0, 1]$.

 Note: $\int_0^1 x^2 \, dx \neq \int_0^1 x \, dx \cdot \int_0^1 x \, dx$.

127. True 129. True

Section 5.3

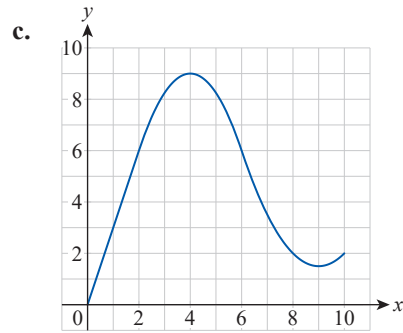
1. $c = \sqrt[3]{2}$ 3. $c = \pm 2/\sqrt[4]{5}$

5. $c = \sin^{-1}(2/\pi), \pi - \sin^{-1}(2/\pi)$

7. $c = \sin^{-1}(\sqrt{\pi}/2), \pi - \sin^{-1}(\sqrt{\pi}/2)$

9. a. $F(2) = 6, F(4) = 9, F(6) = 6, F(8) = 2,$
 $F(10) = 2$

b.
$$F(x) = \begin{cases} 3x & \text{if } 0 \leq x \leq 2 \\ -\frac{3}{4}x^2 + 6x - 3 & \text{if } 2 < x \leq 6 \\ \frac{x^2}{2} - 9x + 42 & \text{if } 6 < x \leq 10 \end{cases}$$


 11. $14/3$ 13. $\sqrt{2}$ 15. $160/7$

17. $F'(x) = \frac{1}{3}(x^2 + \sqrt{x})$

19. $G'(x) = \frac{x^4}{x^4 + 4}$ 21. $y' = -\sin \sqrt{x+1}$

23. $y' = (27x^2 + 9)e^{3x-2}$ 25. $y' = (\sin^2 x + e^{\sin x}) \cos x$

27. $y' = \pi \sin(\pi x) - \sin x$

29. $F'(x) = \frac{-\sqrt{1 + \sqrt{1 + (1/x^2)}}}{\sqrt{1-x^2}}$

31. $H'(x) = \ln x - \frac{\ln(\ln x)}{x}$ 33. $F(x) = 2x - 2$

35. $F(x) = -\frac{x^3}{3} - \frac{x^2}{2} + \frac{5}{6}$ 37. $F(x) = 1 - \frac{1}{\sqrt{x}}$

39. -30 41. 175 43. 48 45. 990 47. $1 - \ln 4$

49. $\frac{14}{3} + 4 \ln 2$ 51. $\frac{8-2\sqrt{2}}{5}$ 53. $\frac{45-36\sqrt{2}}{32}$

55. $\frac{1}{2} - \frac{\sqrt{2}}{6}\pi^{3/2}$ 57. $3\pi/4$ 59. $e^{\pi/3}$

61. $44/3$ 63. $\pi/2$ 65. $e - \frac{1}{3}$ 67. $\int_0^2 x^4 \, dx = 32/5$

69. $\int_1^e (1/x) \, dx = 1$ 71. 8 73. 3 75. $63/2$

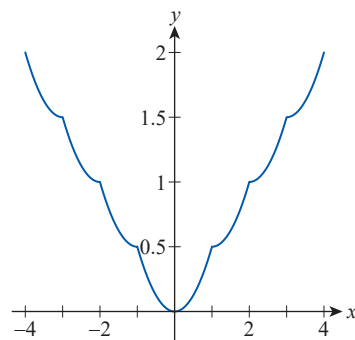
77. $2 \ln 2 - \ln 3$ 79. $1 + 2 \ln 2$ 81. $2e - 9$

83. $3 + \frac{\pi}{4}$ 85. $6 + \ln(33/13)$

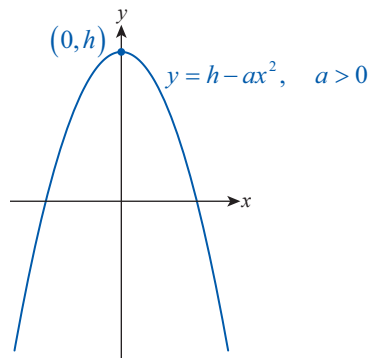
87. $2 + \ln 2$ 89. a. $\frac{3}{2} - \ln 4$ b. $1/2$

91. $f(x) = 2 \cos 2x + 1$

93.



101.



103. True 105. True

107. False; see part a. of Example 3.

109. a. \$3375; \$3250 b. 5.377 years

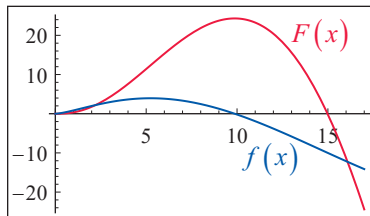
111. The following answers are based on a sine regression.

a. $N(t) = 1.637 \sin(0.023t - 0.936) + 2.583$

b. Approx. 1972 cars

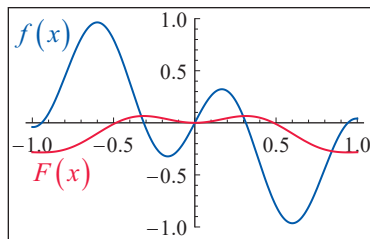
c. Approx. 3 cars per minute

113. $F(x) = (12 - 2x)\sqrt{x} \cos \sqrt{x} + 6(x - 2)\sin \sqrt{x}$



F is increasing where f is positive and decreasing where f is negative. F is concave up where f is increasing and concave down where f is decreasing. The zeros of f are critical points of F , and the critical points of f are potential inflection points for F .

115. $F(x) = \frac{\cos^2 x}{2} - \frac{\cos 8x + 7}{16}$



F is increasing where f is positive and decreasing where f is negative. F is concave up where f is increasing and concave down where f is decreasing. The zeros of f are critical points of F , and the critical points of f are potential inflection points for F .

Section 5.4

1. $2x^6 + 1.5x^5 - 0.25x^4 + 2x + C$

3. $\frac{10}{3}x^3 + 3x^2 - 4x + C$

5. $3/7$ 7. $\frac{2x^{5/2}}{5} - \frac{x^2}{\sqrt{2}} + C$

9. $\pi \tan x - \sec x + C$

11. $\frac{-\csc 2x}{2} + C$ 13. $\pi/2$ 15. $-\frac{7}{5}\sin^{-1}(5x) + C$

17. $\frac{(x^4 + 2)^{3/2}}{6} + C$ 19. $\tan^{-1}(2x) + C$

21. $2e^{2t+3} + C$ 23. $\frac{5^{\arctan x}}{\ln 5} + C$

25. $2 \tan(1 + \sqrt{s}) + C$ 27. $-\tan(1/x) + C$

29. $\frac{-\csc^3 x}{3} + C$ 31. $\frac{2z - 1 + \ln|2z - 1|}{4} + C$

33. $\frac{(6x + 20)(x - 5)^{3/2}}{15} + C$

35. $\frac{x\sqrt{1-x^2} + \sin^{-1} x}{2} + C$ 37. $\frac{(3x - 2)^8}{8} + C$

39. $\frac{(2x^2 + 3x)^{21}}{21} + C$ 41. $\ln|\sin z| + C$

43. $\frac{(x^3 - 5)^{20}}{60} + C$ 45. $e^{\sin x} + C$

47. $\frac{-(5-s)^{38}}{38} + C$ 49. $\frac{3(x^4 + 11)^{4/3}}{16} + C$

51. $\frac{-1}{x^2 + x - 7} + C$ 53. $\frac{-\sqrt{2+x^4}}{2} + C$

55. $-\frac{1}{4}\left(1 + \frac{1}{t}\right)^4 + C$ 57. $2 \ln(|\ln 2t|) + C$

59. $\frac{e^{x^2-3}}{2} + C$ 61. $\frac{\sin \pi x}{\pi} + C$

63. $\frac{\sin^4 2t}{8} + C$ 65. $e^{1-\cot v} + C$

67. $\frac{2}{3(30 - x^{3/2})} + C$ 69. $\ln(\sin^2 x + 2) + C$

71. $\sin(\sin x) + C$ 73. $\frac{\ln|1 - \cos 2x|}{2} + C$

75. $\frac{\sec^3 2x - 3 \sec 2x}{6} + C$ 77. $\frac{4(1 + \sqrt{w})^{3/2}}{3} + C$

79. $\frac{(x+1)^{13}}{13} - \frac{(x+1)^{12}}{2} + C$ 81. $\frac{1 + 4x + 7 \ln|1 + 4x|}{16} + C$

83. $\frac{(1-\ln x)^5}{5} - \frac{7(1-\ln x)^4}{8} + C$

85. $f(x) = \frac{8}{3}(1+x^2)^{3/2} - \frac{5}{3}$ 87. $y(t) = \ln(\sin^2 t + e) - 1$

89. $y(x) = \frac{16x+1-\cos 4x}{16}$ 91. $x = 5$

93. False; the indefinite integral is the set of all antiderivatives.

95. True

97. False; the integral doesn't fit the Substitution Rule with $F(g(x)) = \frac{(\cos x + 1)^3}{3}$. (You can check the falsity of the answer by differentiation.)

Section 5.5

1. $364/3$ 3. $512/27$ 5. $\frac{12^{100}-5^{100}}{300}$ 7. $2/7$

9. $\frac{3(2^{4/3}-1)}{16}$ 11. $2/7$ 13. 1 15. $129/128$

17. $\frac{\ln 2}{3}$ 19. $1/2$ 21. $2/\pi$ 23. $1/6$

25. 0 27. $1 + \ln 2$ 29. 0 31. 2

33. $\frac{4(\sqrt{2}-1)}{3}$ 35. $e^{\pi/3} - 1$ 37. $\ln 2$

39. $\frac{20\sqrt{5}-32}{3}$ 41. $6 - 7\ln 3$

43. $53/72$ 45. 0 47. $33\ln 5 - 32$ 49. $\frac{608-56\sqrt{2}}{15}$

51. $\frac{2\sqrt{2}-1}{3}$ 53. $64/3$ 55. 10.5845

57. $4/3$ 59. $9/2$ 61. $35/4$ 63. $37/12$

65. $5/12$ 67. $\ln 2$ 69. $\frac{11+5\sqrt{5}}{6}$ 71. $1/e$

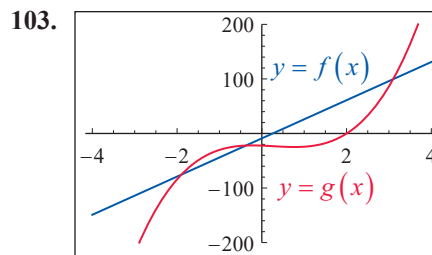
73. $324/5$ 75. $9/2$ 77. $37/32$ 79. 81

81. $7/2$ 83. $937/12$ 85. $144\sqrt{2} - \frac{2656}{15}$ 87. 1

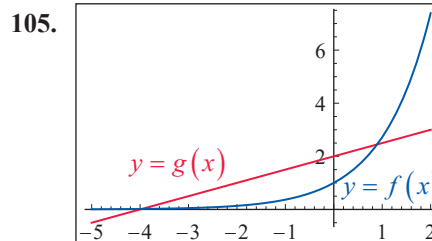
89. $\frac{4-\pi}{\sqrt{2}}$ 91. $9\pi/4$ 93. $\frac{8}{5} + \frac{4}{\pi}$ 95. 2

101. a. Bike #1

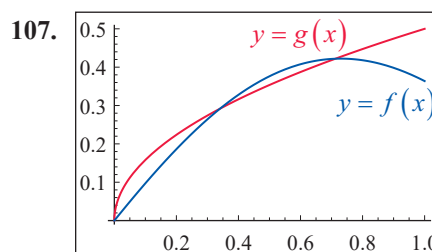
b. Bike #2 starts gaining ground on #1.

c. Yes, it happens when $\int_0^T v_1(t) dt = \int_0^T v_2(t) dt$, at about $T = 2.5$ seconds.

The area is approx. 151.932.



The area is approx. 3.562.



The area is approx. 0.019.

Chapter 5 Review

1. $(O_4 + U_4)/2 = 1.375$

3. $\sum_{i=1}^{100} \frac{(-1)^{i+1}}{i^3 + 1}$ 5. $\sum_{i=1}^{50} f\left(\frac{2i}{n^2}\right)$

7. $212 + 2n$ 9. 9065 11. $\frac{n^2 + 2n}{(n+1)^2}$

13. 0.769399 15. $\frac{mn(m+1)(n+1)(2n+1)}{12}$

17. $1/4$

19. The region between the graph of $2x - x^2$ and the x -axis from 0 to 2; the limit is $4/3$.

23. 0 25. 1.5 27. $1/4$; $x = 3/4, 13/4$

29. $F'(x) = \sqrt{1+x^2}$ 31. $87/5$ 33. $\frac{3}{2} + \ln 4$

35. π 37. $1 + 3\ln 2$ 39. $9/20$ 41. $f(x) = \cos x$

43. $-2\sin^{-1} x + C$ 45. $\sec x + \tan x + C$

47. $\frac{(2x^3 - 7)^{10}}{10} + C$ 49. $\frac{-2}{x^2 + 1} + C$

51. $2\tan\sqrt{x} + C$ 53. $e^x - \ln(e^x + 1) + C$

$$55. y(x) = \frac{2}{1-\sqrt{x}} + 2 \quad 57. x = 1 + \frac{\ln 17}{2}$$

$$59. 16\sqrt{6} \quad 61. \frac{\pi}{4\ln 2} \quad 63. 1/3 \quad 65. 4/21$$

$$67. \frac{3-e}{2} \quad 69. 2$$

73. If f is concave up, the integral is expected to be less than the average of L_n and R_n , while if f is concave down, the integral is expected to be greater than the average of L_n and R_n .

75. True

$$77. \text{False; } \int \frac{1}{e^x} dx = \int e^{-x} dx = -e^{-x} + C.$$

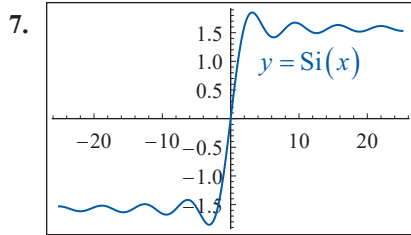
79. False; consider $f(x) = x - 1$ on $[0, 3]$.

81. False; $1/x^3$ is not continuous on $[-1, 1]$.

Chapter 5 Conceptual Project

$$3. \frac{d}{dx} \text{Si}(x) = f(x)$$

5. Rel. max. at $x = (2k-1)\pi$; rel. min. at $x = 2k\pi$, $k \in \mathbb{N}$



Chapter 5 Application Project

$$1. \text{ a. } a = 16.18 \text{ ft/s}^2 \quad \text{ b. } v(t) = at = 16.18t \text{ ft/s}$$

$$\text{ c. } v(t) = \int_0^t 16.18 du \quad \text{ d. } 1496 \text{ ft}$$

$$3. \text{ a. } 45.47 \text{ ft} \quad \text{ b. } 1975.6 \text{ ft; a Riemann sum.}$$

$$5. \text{ a. } d = \int_{t_0}^{t_1} v(t) dt$$

$$\text{ b. } d(t) = \int_{t_0}^t v(u) du, \quad t_0 \leq t \leq t_1$$

$$\text{ c. } v(t) = \int_{t_0}^t a(u) du, \quad t_0 \leq t \leq t_1$$

Chapter 6

Section 6.1

$$1. 3.2 \text{ m}^3 \quad 3. 8/(15\sqrt{3}) \quad 5. 8 \quad 7. 1/\sqrt{3}$$

$$9. 32/3 \quad 11. 4/315 \quad 13. (2\pi + 6)/3$$

$$15. 138,240\sqrt{5} \text{ cm}^3 \approx 0.309 \text{ m}^3$$

$$17. 4\sqrt{3}\pi \text{ cm}^3 \quad 19. 384\pi/7$$

$$21. 19\pi \quad 23. 287\pi/30 \quad 25. 15\pi/2 \quad 27. 18\pi$$

$$29. 4\pi/3 \quad 31. 117\pi/2 \quad 33. 128\pi/7 \quad 35. 16\pi/3$$

$$37. \text{ a. } 2\pi/3 \quad \text{ b. } 8\pi/3 \quad 39. \text{ a. } \pi/7 \quad \text{ b. } 9\pi/14$$

$$41. \text{ a. } 128\pi/45 \quad \text{ b. } 224\pi/45$$

$$43. \text{ a. } 3\pi/13 \quad \text{ b. } 51\pi/52$$

$$45. \text{ a. } 93\pi/5 \quad \text{ b. } 411\pi/10$$

$$47. \text{ a. } 4\pi/3 \quad \text{ b. } \frac{3\pi^2 + 4\pi}{3}$$

$$49. \text{ a. } 2\pi/5 \quad \text{ b. } 7\pi/5$$

$$51. \text{ a. } 8\pi/3 \quad \text{ b. } 104\pi/15$$

$$53. \text{ a. } 6\pi/11 \quad \text{ b. } 45\pi/22$$

$$55. \text{ a. } 762\pi/7 \quad \text{ b. } 1077\pi/7$$

$$57. \text{ a. } \pi^2 \quad \text{ b. } 3\pi^2$$

$$59. 261\pi \quad 61. 1250\pi/3 \quad 63. 11\pi/30$$

$$65. 56\pi/15 \quad 67. 488\pi/15$$

$$69. \pi/2 \quad 71. \pi^2/4$$

73. The solid (paraboloid) generated by rotating the region bounded by $y = 0$ and $y = \sqrt{x}$, $0 \leq x \leq 4$ about the x -axis

75. The solid generated by rotating the region bounded by $y = e$, $y = e^x$, and $x = 0$ about the x -axis

77. The solid generated by rotating the region bounded by $y = \sqrt[4]{x}$ and $y = x^4$, $0 \leq x \leq 1$ about the x -axis

79. The solid generated by rotating the region bounded by $y = \log_2 x$ and $y = x - 1$, $1 \leq x \leq 2$ about the x -axis

81. Approx. 81.5 mL

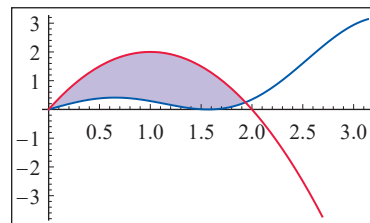
The rate from part b. should be used as a “water clock,” because the water level is falling at a constant rate.

85. For min. volume: $c = \sqrt{2}$; for max. volume: $c = 0, 2$

$$87. \text{Approx. } 45.6 \text{ mL} \quad 89. \frac{\pi(\pi^2 - 8)}{4}$$

91. Approx. 109.745

$$93. V \approx 13.021$$



Section 6.2

$$1. 4\pi \quad 3. 24\pi/5 \quad 5. \pi/3 \quad 7. 9\pi/4$$

$$9. 18\pi \quad 11. 931\pi/75 \quad 13. 81\pi/2 \quad 15. 33\pi/2$$

$$17. 10\pi/13 \quad 19. 336\pi/5 \quad 21. \pi/2 \quad 23. \pi(2\ln 2 - 1)$$

25. $2e(e-1)$ 27. $112\pi/3$ 29. $16\pi/3$ 31. $19\pi/15$

33. $8\pi/3$ 35. $\pi \ln 17$ 37. π/e

39. a. $425\pi/12$ b. $100\pi/3$ c. $125\pi/6$ d. $200\pi/3$

41. a. $544\pi/15$ b. $40\pi/3$ c. $192\pi/5$ d. $40\pi/3$

43. a. 64π b. $512\pi/15$ c. 8π d. $832\pi/15$

45. $V = 2\pi \int_0^{\ln 2} (2xe^{-x} - x) dx$

47. $V = 2\pi \int_0^1 x \arcsin x dx$

49. $V = 2\pi \int_0^{\pi/3} x(\sqrt{3} - \tan x) dx$

51. The solid (circular cone) generated by rotating the region bounded by the y -axis, $y = 2x$, and $y = 4$ about the y -axis53. The solid (sphere) generated by rotating the region bounded by the x -axis and the semicircle $y = \sqrt{2x - x^2}$ about the x -axis55. The solid generated by rotating the region bounded by the x -axis and $y = 1 - x^4$, $0 \leq x \leq 1$ about $x = -1$ 57. The solid generated by rotating the region bounded by $y = \sin x$, $\pi/2 \leq x \leq \pi$, and the x -axis about $x = \pi/2$ 59. The solid generated by rotating the region bounded by $y = \csc x$, $x = \pi/4$, $x = 3\pi/4$, and the x -axis about $x = -1$

61. $\pi/2$ 63. $1024\pi/105$ 65. $144\pi/5$

69. $4\sqrt{3}\pi \text{ cm}^3$ 71. $V = 2\pi^2 Rr^2$

73. False; the volume is k^2V .

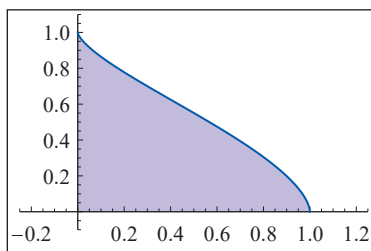
75. True

77. False; compare the formulas

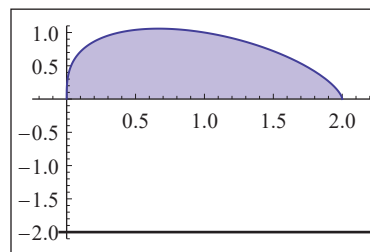
$$\int_a^b \pi \left([f(x)]^2 - [g(x)]^2 \right) dx \text{ and}$$

$$\int_a^b \pi \left([f(x)+1]^2 - [g(x)+1]^2 \right) dx.$$

79. $125\pi/312$



81. $\frac{416\pi^2}{81\sqrt{3}}$



Section 6.3

1. $6\sqrt{10}$ 3. $3\sqrt{5}$ 5. $18 - \frac{4\sqrt{2}}{3}$ 7. $\frac{e^2 + 7}{8}$

9. $521/480$ 11. $\pi/2$ 13. $\frac{e}{4} - \frac{1}{e} + \frac{3}{4}$ 15. 1

17. $\int_0^2 \sqrt{1+4x^2} dx$; $s(x) = \int_0^x \sqrt{1+4t^2} dt$

19. $\int_0^1 \sqrt{4y^2 - 4y + 2} dy$; $s(y) = \int_0^y \sqrt{4t^2 - 4t + 2} dt$

21. $\int_1^2 \sqrt{\frac{4}{x^6} + 1} dx$; $s(x) = \int_1^x \sqrt{\frac{4}{t^6} + 1} dt$

23. $\int_0^2 \sqrt{e^{4-2x} + 1} dx$; $s(x) = \int_0^x \sqrt{e^{4-2t} + 1} dt$

25. $\int_{-1/2}^{1/2} \sqrt{\frac{2-x^2}{1-x^2}} dx$; $s(x) = \int_{-1/2}^x \sqrt{\frac{2-t^2}{1-t^2}} dt$

27. $12\sqrt{2}\pi$ 29. $(40\sqrt{5} - 8)\pi/3$

31. $241\pi/192$ 33. $(16e - e^{-1} - 7)\pi/16$ 35. 2π

37. $(8\sqrt{3} - \frac{8}{3})\pi$ 39. $(16 - 4\sqrt{2})\pi/3$ 41. $515\pi/64$

43. $12\sqrt{5}\pi$ 45. $\pi \cdot \frac{[e(e+2)]^2 - 4e - 1}{4e^2}$

47. $2\pi \int_0^{\sqrt{3}/2} \sin^{-1} x \sqrt{\frac{2-x^2}{1-x^2}} dx$

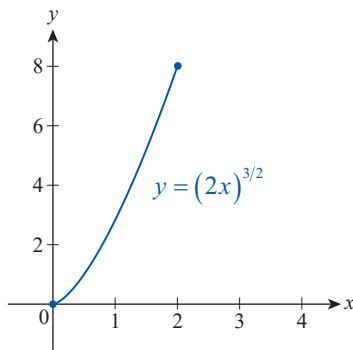
49. $2\pi \int_2^4 \sqrt{\frac{1}{x^4} + \frac{4}{x^{10}}} dx$

51. $2\pi \int_1^8 (3 + \sqrt[3]{x}) \sqrt{\frac{1}{9x^{4/3}} + 1} dx$

53. $2\pi \int_0^1 (2 + \sqrt[4]{y+1}) \sqrt{1 + \frac{1}{16(y+1)^{3/2}}} dy$

55. The surface area is 1200π square units. Yes, because of symmetry, we do obtain the same answer when rotating about the y -axis.

57. a.



b. $\frac{37\sqrt{37}-1}{27}$

61. $\frac{(61\sqrt{61}+185\sqrt{37}-130)\pi}{30}$ 63. $A = 4\pi^2 Rr$

65. $2\pi \left(b^2 + \frac{a^2 b}{\sqrt{a^2 - b^2}} \sin^{-1} \frac{\sqrt{a^2 - b^2}}{a} \right)$

67. Approx. 6.7573 69. Approx. 1.1321

71. Approx. 2.6625 73. Approx. 3.8202

75. Approx. 14.4236 77. Approx. 22.9430

79. Approx. 57.7233 81. Approx. 7.6031

83. Approx. 3.0444 yd

Section 6.4

1. $M_0 = -5; \bar{x} = -5/11$ 3. $M_0 = -6.5; \bar{x} = -0.5$

5. $M_x = 24, M_y = 30; (\bar{x}, \bar{y}) = (10/7, 8/7)$

7. She should sit on Christina's side, 1 ft ahead of her.

9. The nosepiece will land approx. 400 yd northeast from the target.

11. $\bar{x} = 63.7681$ 13. $(23/18, 4/9)$

15. $(0.7157, 2.7917)$

17. $(-2, -8/5)$ 19. $(-9/10, 18/5)$

21. $(8/5, 3/2)$ 23. $(1, 1)$ 25. $(2/\ln 3, 1/\ln 27)$

27. $(0, -18/5)$ 29. $(16/15, 64/21)$

31. $(\pi/2, 3/8)$ 33. $(\ln 4, 0)$ 35. $(0, 6/5)$

37. $\left(\frac{5-6\ln 2}{3+6\ln 2}, \frac{5}{6+12\ln 2} \right)$

39. $\left(\frac{14}{3\pi}, \frac{28}{9\pi} \right)$ 41. $(3, 1/2)$

43. $(1/2, 8/15)$ 45. $(2/3, 46/21)$

47. $(2/9, 11/21)$ 49. $(137/76, 25/76)$

51. $(0, 3r/4)$ 53. $\left(\frac{4r}{3\pi}, \frac{4r}{3\pi} \right)$ 55. $(0, r/2)$

59. $\left(\frac{n+1}{2n+1}, \frac{n+1}{2n+4} \right);$

If $n \rightarrow \infty$ the centroid converges to $(1/2, 1/2)$.

61. $(h/3, R/3)$ 63. $\frac{\pi r^3 (3\pi - 4)}{3}$

65. $828\pi/5$ 67. $240\sqrt{2}\pi$ 69. $2\pi r^2 (\pi - 2)$

73. False; consider Example 4.

75. False; he or she has to sit at two-thirds of the distance (when compared to the other child) from the pivot point (i.e., approximately 33% closer).

Section 6.5

1. 3433.5 J 3. 27 ft·lb

5. a. 200 ft·lb b. 800 ft·lb

7. a. 4 ft·lb b. 3.0625 ft·lb

9. 7600 J 11. 0.84 J

15. $5/4$ ft·lb 17. 10,950 ft·lb 19. 3977.2 kJ

21. 42.2321 kJ

23. $27,500\pi$ ft·lb $\approx 86,393.80$ ft·lb. The work is 3 times that in Example 4a.

25. a. 100,000 ft·lb b. 75,000 ft·lb

c. No. The lower half of the water needs to be pumped higher than the top half.

d. Approx. 6 minutes and 4 seconds;
approx. 4 minutes 33 seconds

27. 2995.2 ft·lb 29. 11,980.8 ft·lb

31. $\delta\pi R^4/4$ 33. 31.2 ft·lb

35. a. 2620.8 lb b. 2433.6 lb

37. Approx. 98.17 N 39. Yes

41. 499.2 lb 43. Approx. 6.27 lb

45. $450 \ln 3$ J ≈ 494.38 J 47. Approx. 620.83 J

51. 4.02286×10^9 ft·lb 53. 4.09308×10^6 kJ

55. Rw 57. 6.75 J 71. Approx. 2.89 ft

Section 6.6

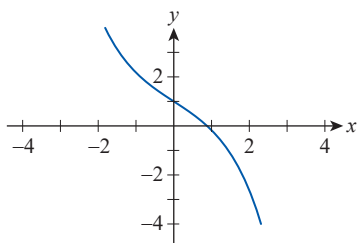
1. a. 1 b. $\frac{e^{-2} - e^2}{2} \approx -3.6269$

c. $\frac{e^{\pi/2} - e^{-\pi/2}}{2} \approx 2.3013$

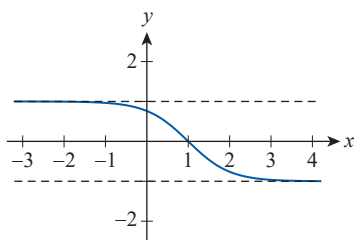
3. a. 0 b. 0 c. $\ln(2 + \sqrt{5}) \approx 1.4436$

5. Odd 7. Odd 9. Even

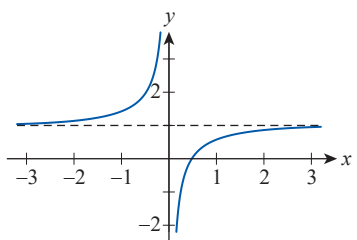
11.



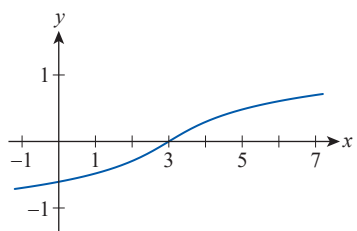
13.



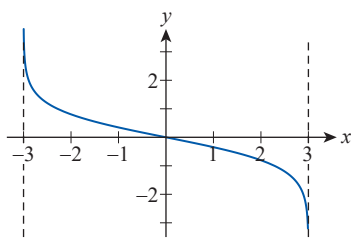
15.



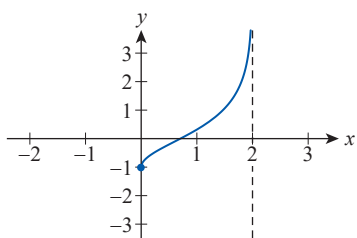
17.



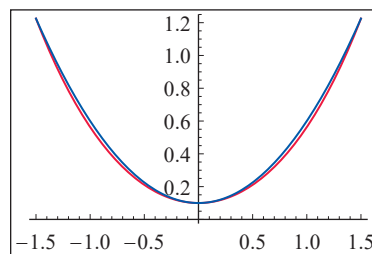
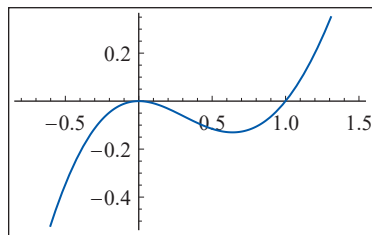
19.



21.

51. $\sinh 2x$ 53. $2 \operatorname{sech}^2 x \tanh x$ 55. $\frac{2 \cosh 2x - \sinh 2x}{e^x}$ 57. $\frac{\operatorname{sech}^2 2x}{\sqrt{1 + \tanh 2x}}$ 59. $3t^2 [\cosh(1+t^3) + (1+t^3) \sinh(1+t^3)]$ 61. $\frac{4y(1+y)}{(1+2y)^2}$ 63. $\sinh x + C$ 65. $\operatorname{sech}(1/z) + C$ 67. $-\ln(\cosh(2-w)) + C$ 69. $\frac{2e^2}{e^4 - 1}$ 71. $\csc^{-1}\left(\frac{4e^2}{e^4 + 1}\right) - \frac{\pi}{6}$ 83. $\frac{7}{\sqrt{49x^2 - 4}}$ 85. $-2 \csc 2x$ 87. $\frac{2}{1-4x^2}$ 89. $\frac{1}{4} \sinh^{-1}\left(\frac{4x}{3}\right) + C$ 91. $\tanh^{-1}(x-3) + C$ 93. $\operatorname{csch}^{-1}(\sin x) + C$ 95. $2 \sinh^{-1} 25$ 97. $\ln(2 - \sqrt{3})$ 99. $-\frac{2}{3} \tanh^{-1} \sqrt{1+x^3} + C$ 101. $\ln \frac{4 + \sqrt{15}}{2 + \sqrt{3}}$ 103. ∞ 105. 1 107. $1/2$ 109. 0113. a. At the left endpoint: approx. -0.30452 ; at the right endpoint: approx. 0.30452 ; The parabola predicts ± 0.3 .b. Approx. 3.0452 ; Yes, the answer is very close to that given by the parabolic model.115. True 117. False; $\operatorname{csch} 0$ is undefined.

119. a. Approx. 625.093 ft b. Approx. 598.45 ft

c. Approx. ± 0.641 121. $f'(x) = (x^2 - x) \operatorname{sech}^2 x + (2x - 1) \tanh x$;rel. max. at $x = 0$;rel. min. at $x \approx 0.63923$ 

Chapter 6 Review

1. $2/3$ 3. $\pi/2$ 5. $\frac{\pi(\pi-2)}{2}$ 7. $8\pi/3$ 9. $512\pi/21$

11. The solid (circular cone) generated by rotating the region bounded by the x -axis, $y = x$, $0 \leq x \leq 3$ about the x -axis
13. The solid (paraboloid) generated by rotating the first-quadrant region bounded by the graph of $y = 4 - x^2$ and the coordinate axes about the y -axis

15. 2π 17. $8\pi/15$

21. $\int_1^5 \sqrt{16y^2 + 8y + 2} \, dy$ 23. $\int_0^{\pi/3} \sec y \, dy$

25. $14/3$ 27. $\frac{3+e^4}{4}$ 29. $e - \frac{1}{e}$

31. $\pi \int_1^e \sqrt{\ln(x^4) + \frac{1}{x^2}} \, dx$

33. $2\pi \int_0^4 (\sqrt{y} + 1) \sqrt{1 + \frac{1}{4y}} \, dy$

35. $61\pi/432$ 37. $35\pi/16$ 39. $111\pi/16$

41. $(2/5, 1)$ 43. $\left(\frac{24\ln 2 - 14}{9}, \frac{13}{9}\right)$

45. $(8/7, 6/7)$ 47. $(11/2, 0)$

49. $\left(0, \frac{4b}{3\pi}\right)$ 53. $4200 \text{ ft} \cdot \text{lb}$ 55. 29.332 kJ

57. Approx. $123,686 \text{ ft} \cdot \text{lb}$

59. $0.524 \text{ ft} \cdot \text{lb}$ 63. $39,240 \text{ kN}$

65. Approx. $3.30313 \times 10^9 \text{ ft} \cdot \text{lb}$

67. a. Undefined b. 1

73. $e^{2x} (\operatorname{sech}^2 x + 2 \tanh x)$

75. $5 \coth(5x - 2)$ 77. $-\operatorname{sech} x + C$

79. $\sinh^{-1}(x - 2) + C$ 81. $\cosh^{-1}(e^x) + C$

85. $8c/9$

87. False; consider Example 3 and Exercise 60 of Section 6.2.

89. False; consider the region bounded by $y = 1$ and the x -axis over the interval $[0, 1]$.

91. True 93. Approx. 2.104

95. Approx. 26 97. Approx. 2.259

99. Approx. 14.424

Chapter 6 Conceptual Project

5. $h_{\max} = \frac{Rv_0^2}{2gR - v_0^2}$ 7. Approx. 2.36 km/s

Chapter 6 Application Project

1. $F_t = (2500 \text{ m/s})(150 \text{ kg/s}) = 375 \text{ kN} = 3.75 \times 10^5 \text{ N}$

3. From Equation 1, $dv = \frac{v_f}{m} dm - gdt$. Thus,

$$v(t) = \int_0^t dv = v_f \int_0^t \frac{dm}{m} - g \int_0^t du = v_f \ln \frac{m(t)}{m(0)} - gt.$$

Chapter 7

Section 7.1

1. $e^x(x-1) + C$ 3. $x^4 \left(\ln x - \frac{1}{4} \right) + C$

5. $\frac{e^{5t}(4+5t)}{25} + C$ 7. $2(\sin x - x \cos x) + C$

9. $\frac{x^2 [\ln(x^2) - 1]}{2} + C$ 11. $\frac{x^3}{3} \left(\ln x - \frac{1}{3} \right) + C$

13. $(s-4)e^{s-3} + C$ 15. $\frac{19+6x}{9}e^{3x-1} + C$

17. $\frac{6x+8}{15}(x-2)^{3/2} + C$ 19. $\frac{(3x)^{4/3}}{16}(4 \ln x - 3) + C$

21. $2\sqrt{x}(\ln \sqrt{x} - 1) + C$

23. $x \sec x - \ln |\sec x + \tan x| + C$

25. $\ln |\sin x| - x \cot x + C$ 27. $-(3x+2)e^{-x} + C$

29. $(t+2)\sinh t - \cosh t + C$

31. $\frac{2(3x-2)(x+1)^{3/2}}{15} + C$

33. $-3 \ln(\cosh x) + \tanh x(3x+2) + C$

35. $\frac{(3x+5)(2x-5)^{3/2}}{15} + C$

37. $2\theta \cos \theta + (\theta^2 - 2)\sin \theta + C$

39. $\frac{1}{2}e^x(\cos x + \sin x) + C$

41. $2t \sin 3t - \frac{1}{3} \cos 3t(9t^2 - 2) + C$

43. $\frac{e^{3x}(3 \sin 2x - 2 \cos 2x)}{13} + C$

45. $(2+x^2)\cosh x - 2x \sinh x + C$

47. $\frac{x[\cos(\ln x) + \sin(\ln x)]}{2} + C$

49. $2(\sqrt{x}-1)e^{\sqrt{x}} + C$

51. $\sqrt{x} \arctan(1/\sqrt{x}) + \frac{1}{2} \ln(x+1) + C$
53. $2\sqrt{x} \cos^{-1} \sqrt{x} - 2\sqrt{1-x} + C$
55. $3x^3 (\ln x)^2 - 2x^3 \ln x + \frac{2}{3} x^3 + C$
57. $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$ 59. $\frac{e^{2\pi} + 1}{5}$
61. $\frac{\sqrt{3}\pi + 9\ln 3}{18}$ 63. $\frac{\pi}{2} + \ln 2 - 2$
65. $x \sin^{-1} x + \sqrt{1-x^2} + C$
67. $x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C$
69. $x \sinh^{-1} x - \sqrt{1+x^2} + C$
71. $\sqrt{3}\pi - 3\ln 2$
73. a. $\left(\frac{e-2}{e-1}, \frac{e+1}{4e}\right)$ b. $2\pi(e-2)/e$
75. a. $(\pi/8, 1)$ b. 2π 77. $\left(\frac{e^2+1}{4}, \frac{e-2}{2}\right)$
89. $\frac{x^3 \sin 2x}{2} + \frac{3x^2 \cos 2x}{4} - \frac{3x \sin 2x}{4} - \frac{3 \cos 2x}{8} + C$
91. $e^{\sin x} (\sin^3 x - 3 \sin^2 x + 6 \sin x - 6) + C$
93. $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$
95. $9e - 24$
99. $e^{2x} (4x^3 - 6x^2 + 6x - 3) + C$
101. $e^{3x} (3 \sin 2x - 2 \cos 2x) + C$
17. $\frac{3}{z+1} + 2 \ln|z| - 2 \ln|z+1| + C$
19. $\ln|x+1| - \ln|x+2| + C$
21. $2 \ln|x| - \ln|x+1| - \ln|x-1| + C$
23. $\frac{\ln(t^2+1)}{2} - \ln|t+1| + C$
25. $\frac{3}{2x^2} - \frac{1}{x} + C$
27. $\ln|x-2| + 2 \ln|x| - 3 \ln|x+3| + C$
29. $\ln|x+1| - \ln|x-3| - \frac{2}{x+1} - \frac{2}{x-3} + C$
31. $\ln|v-2| + \ln|v+2| - \ln(v^2+4) + C$
33. $\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \ln|x-1| - \ln\sqrt{x^2+x+1} + C$
35. $\frac{a}{b} \ln|x| - \frac{a}{2b} \ln|x^2+b| + C$
37. $\ln|x+1| - 4 \ln|x-1| + 3 \ln|x-7| + C$
39. $\ln(243/4096)$ 41. $\ln(3/4)$
47. $\frac{2}{3} \ln|x+1| + \frac{1}{6} \ln(x^2-x+1) + \frac{\sqrt{3}}{3} \arctan \frac{2x-1}{\sqrt{3}} + C$
49. $\frac{\ln 3}{8}$ 51. $\ln \sqrt{3/2}$
53. $3x - \sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \ln|x-1| - \frac{\ln(x^2+x+1)}{2} + C$
55. $-2 \arctan(1-x) + \ln(x^2-2x+2) + \ln|x-3| + C$
57. $\ln(8/5) - 3/8$
59. $\frac{a}{(b-c)} \ln|x-b| - \frac{a}{(b-c)} \ln|x-c| + C$
61. $\frac{4x^2 + 2 \ln|x| - 17 \ln(x^2+4)}{8} + C$
63. $-\frac{2}{3} \ln|\cos x + 2| - \frac{1}{3} \ln|\cos x - 1| + C$
65. $2 \ln \left| \frac{\ln x - 3}{\ln x} \right| + C$
67. $2\sqrt{x} - \frac{2}{1+\sqrt{x}} - 4 \ln(1+\sqrt{x}) + C$
69. $2 \ln \left| \frac{1-\sqrt{1+\sqrt{x}}}{1+\sqrt{1+\sqrt{x}}} \right| + C$
71. $\ln|1 + \sec(\ln x)| + C$
73. $\left[(\ln w)^2 + 1\right] \arctan(\ln w) - \ln w + C$

Section 7.2

1. $\frac{A_1}{x} + \frac{A_2}{x-1}$
3. $\frac{A_1}{x+1} + \frac{A_2}{x-3} + \frac{A_3}{(x-3)^2} + \frac{A_4}{(x-3)^3}$
5. $\frac{B_1x+C_1}{x^2+x+2} + \frac{B_2x+C_2}{(x^2+x+2)^2} + \frac{B_3x+C_3}{(x^2+x+2)^3}$
7. $\frac{A_1}{2x+1} + \frac{A_2}{x-3}$
9. $\frac{A_1}{x+1} + \frac{B_1x+C_1}{x^2+1} + \frac{B_2x+C_2}{(x^2+1)^2}$
11. $4 \ln|3x-1| - 4 \ln|x| + C$
13. $2 \ln|2x+5| - 2 \ln|x+3| + C$
15. $\frac{2}{9} (\ln|x-2| - \ln|x+1|) - \frac{4}{3(x-2)} + C$
17. $\frac{3}{z+1} + 2 \ln|z| - 2 \ln|z+1| + C$
19. $\ln|x+1| - \ln|x+2| + C$
21. $2 \ln|x| - \ln|x+1| - \ln|x-1| + C$
23. $\frac{\ln(t^2+1)}{2} - \ln|t+1| + C$
25. $\frac{3}{2x^2} - \frac{1}{x} + C$
27. $\ln|x-2| + 2 \ln|x| - 3 \ln|x+3| + C$
29. $\ln|x+1| - \ln|x-3| - \frac{2}{x+1} - \frac{2}{x-3} + C$
31. $\ln|v-2| + \ln|v+2| - \ln(v^2+4) + C$
33. $\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \ln|x-1| - \ln\sqrt{x^2+x+1} + C$
35. $\frac{a}{b} \ln|x| - \frac{a}{2b} \ln|x^2+b| + C$
37. $\ln|x+1| - 4 \ln|x-1| + 3 \ln|x-7| + C$
39. $\ln(243/4096)$ 41. $\ln(3/4)$
47. $\frac{2}{3} \ln|x+1| + \frac{1}{6} \ln(x^2-x+1) + \frac{\sqrt{3}}{3} \arctan \frac{2x-1}{\sqrt{3}} + C$
49. $\frac{\ln 3}{8}$ 51. $\ln \sqrt{3/2}$
53. $3x - \sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \ln|x-1| - \frac{\ln(x^2+x+1)}{2} + C$
55. $-2 \arctan(1-x) + \ln(x^2-2x+2) + \ln|x-3| + C$
57. $\ln(8/5) - 3/8$
59. $\frac{a}{(b-c)} \ln|x-b| - \frac{a}{(b-c)} \ln|x-c| + C$
61. $\frac{4x^2 + 2 \ln|x| - 17 \ln(x^2+4)}{8} + C$
63. $-\frac{2}{3} \ln|\cos x + 2| - \frac{1}{3} \ln|\cos x - 1| + C$
65. $2 \ln \left| \frac{\ln x - 3}{\ln x} \right| + C$
67. $2\sqrt{x} - \frac{2}{1+\sqrt{x}} - 4 \ln(1+\sqrt{x}) + C$
69. $2 \ln \left| \frac{1-\sqrt{1+\sqrt{x}}}{1+\sqrt{1+\sqrt{x}}} \right| + C$
71. $\ln|1 + \sec(\ln x)| + C$
73. $\left[(\ln w)^2 + 1\right] \arctan(\ln w) - \ln w + C$

75. $\frac{2\pi}{3} \ln 4$

77. $\frac{1}{4c} \ln \left| \frac{c+y}{c-y} \right| - \frac{1}{2c} \tan^{-1} \left(\frac{y}{c} \right) = mt + C$

79. $I(t) = \frac{Ne^{Nkt}}{N-1+e^{Nkt}}$

81. $3x^2 - x + 5 + \frac{x}{x^2+2x+3} + \frac{3x-1}{x^2+1} + \frac{1}{x+2} - \frac{4}{(x+2)^2}$

Section 7.3

1. $\frac{\sin^3 x}{3} + C$

3. $\sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$

5. $\frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$

7. 1 9. $5\pi/8$

11. $\frac{\cos^{15} x}{15} - \frac{3\cos^{13} x}{13} + \frac{3\cos^{11} x}{11} - \frac{\cos^9 x}{9} + C$

13. $\frac{3\theta}{128} - \frac{\sin \theta}{32} + \frac{\sin 2\theta}{256} + C$ 15. $\pi/2$

17. $\frac{2\cos^{7/2} x}{7} - \frac{2\cos^{3/2} x}{3} + C$

19. $-\frac{\cos^6 x}{6} + \frac{\cos^8 x}{8} + C$ 21. π

23. $x - \frac{\sin 4x}{4} + \frac{\sin^3 2x}{3} + C$

25. $\frac{\sin^8 x}{8} - \frac{\sin^{10} x}{10} + C$

27. $3/5$ 29. $\frac{-\sin x}{2} + \frac{\sin 15x}{30} + C$

31. $2(2\sqrt{2}-1)/5$ 33. $-2\sqrt{1-\sin x} + C$

35. $2\sqrt{2} \left[\frac{2}{3} \cos^3 \left(\frac{\pi-2x}{4} \right) - \cos \left(\frac{\pi-2x}{4} \right) \right] + C$

37. $\frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} - \ln |\cos x| + C$

39. $\frac{\tan^3 x}{3} + \tan x + C$

41. $\frac{-2}{\sqrt{\sec \theta}} + C$ 43. $\frac{\sec^6 x}{6} + C$

45. $\frac{-2\cot^{9/2} t}{9} - \frac{2\cot^{5/2} t}{5} + C$

47. $\frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + C$

49. $848/105$ 51. $2\sqrt{3}$

53. $\frac{\tan^6 2\alpha}{12} + \frac{\tan^4 2\alpha}{8} + C$

55. $-\frac{1}{16} \csc^4 4x + C$ 57. $1/2$

59. $x \tan x - \ln |\sec x| + C$

61. $\frac{1}{4} [2z \tan 2z - 2z^2 - \ln |\sec 2z|] + C$

63. $\frac{-\cos 2x}{4} - \frac{\cos 4x}{8} + C$

65. a. $\frac{-\sin x \cos^6 x}{7} + \frac{\sin x}{7} - \frac{2\sin^3 x}{21} + \frac{\sin^5 x}{35} + C$

b. $\frac{-\sin x \cos^7 x}{8} +$

$\frac{1}{128} \left(5x + 4 \sin 2x + 3 \sin 4x - \frac{\sin^3 2x}{3} \right) + C$

67. $4\pi/\sqrt{3}$ 69. $\left(\frac{6\pi+2\pi^3}{3\pi^2+12}, \frac{15\pi+2\pi^3}{6\pi^2+24} \right)$

75. True 77. True

Section 7.4

1. d 3. c and d 5. b 7. c 9. b 11. c

13. $3 \ln |\sqrt{x^2+9}+x| + C$ 15. $-\sqrt{4-x^2} + C$

17. $\frac{1}{2} \left(25 \arcsin \frac{t}{5} - t \sqrt{25-t^2} \right) + C$

19. $\frac{\sqrt{x^2-36}}{36x} + C$ 21. $\sqrt{1-x^2} + C$

23. $2 \ln |\sqrt{x^2+4}+x| - \sqrt{x^2+4} + C$

25. $\frac{5\sqrt{5}-9}{120}$ 27. $25\pi/16$ 29. $-\frac{\sqrt{t^2+9}}{9t} + C$

31. $-2 \ln \left| \frac{2+\sqrt{4-9x^2}}{x} \right| + \sqrt{4-9x^2} + C$

33. $\frac{-(4-w^2)^{5/2}}{20w^5} + C$ 35. $\frac{3\sqrt{7}-\sqrt{3}}{16}$

37. $\frac{1}{2} x \sqrt{9x^2+16} + \frac{8}{3} \ln |\sqrt{9x^2+16}+3x| + C$

39. $-\frac{\sqrt{4-x^2}}{x} - \arcsin \frac{x}{2} + C$ 41. $\frac{\sqrt{3x^2+1}}{3} + C$

43. $-\frac{1}{\sqrt{1-x^2}} + C$ 45. $-\frac{\sqrt{25-9x^2}}{25x} + C$

47. $4/15$ 49. $\ln |2x+1+\sqrt{4x^2+4x+12}| + C$

51. $\frac{27}{2} \arcsin \frac{x-3}{3} - \frac{(x+9)\sqrt{6x-x^2}}{2} + C$

53. $\frac{\pi}{4\sqrt{2}}$ 55. $\frac{1}{\sqrt{3}} - \frac{2}{\sqrt{15}}$

57. $\frac{x-3}{2} \sqrt{7+6x-x^2} - 8 \arcsin \frac{3-x}{4} + C$

59. $\frac{x-5}{4(x^2-10x+29)} + \frac{1}{8} \arctan \frac{x-5}{2} + C$

61. $\frac{1}{2} \left[e^x \sqrt{1-e^{2x}} + \arcsin(e^x) \right] + C$

63. $-\frac{\sqrt{\sin^2 x + 1}}{\sin x} + C$

65. $\frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \ln |x + \sqrt{x^2-a^2}| + C$

67. $\frac{1}{a} \sec^{-1} \left(\frac{x}{a} \right) + C$ 69. $\ln |x + \sqrt{x^2+a^2}| + C$

71. $\frac{1}{a} \ln \left| \frac{\sqrt{x^2+a^2}+a}{x} \right| + C$ 73. $\frac{1}{a} \operatorname{sech}^{-1} \left(\frac{x}{a} \right) + C$

75. $\sinh^{-1} \left(\frac{x}{4} \right) - \frac{x}{\sqrt{x^2+16}} + C;$

$\ln |\sqrt{x^2+a^2}+x| - \frac{x}{\sqrt{x^2+a^2}} + C;$

$\sinh^{-1} \left(\frac{x}{a} \right) - \frac{x}{\sqrt{x^2+a^2}} + C$

77. $\frac{1}{4} \left[-t\sqrt{1-t^2} + (2t^2-1) \operatorname{arccost} t \right] + C$

79. $\frac{\sqrt{6}}{9} - \frac{1}{6}$ 81. $2\pi \left(\ln 3 - \frac{4}{5} \right)$

83. $\pi\sqrt{3}/2$ 85. $1 + \frac{1}{2} \ln \frac{3}{2} \approx 1.203$

89. False; $x = \sec \theta$ is a recommended substitution.

91. False; the definite integral is not multiplicative.

93. $\frac{x}{2} \sqrt{x^2+25} - \frac{25}{2} \sinh^{-1} \left(\frac{x}{5} \right) + C$

95. $-\sinh^{-1}(3-x) + C$

13. $\frac{x+1}{2} \sqrt{x^2+2x+5} + 2 \ln |x+1+\sqrt{x^2+2x+5}| + C$

15. $\frac{\sqrt{3}\pi+9\ln 3}{18}$ 17. $2 \cot \frac{\pi}{8} - \frac{\pi}{4} - 2$

19. $\frac{2}{3} \left[(z+2)^{3/2} - z^{3/2} \right] + C$

21. $\sqrt{x^2-4} + 2 \ln |x + \sqrt{x^2-4}| + C$

23. $\frac{1}{3} \left(\ln |x| - \ln |3 + \sqrt{2x^2+9}| \right) + C$

25. $-\frac{1}{x^3} + 2 \ln \left| \frac{2x^3+1}{x^3} \right| + C$

27. $\frac{1}{3} (\tan u - 5) \sqrt{5+2 \tan u} + C$

29. $\frac{x \ln x - x}{2} + C$ 31. $\frac{\sin x - 2}{\sqrt{2 \sin x - \sin^2 x}} + C$

33. $2\sqrt{3x-2} - 2\sqrt{2} \arctan \sqrt{\frac{3x-2}{2}} + C$

35. $\frac{1-x}{6} \sqrt{1+6x-3x^2} + \frac{2}{3\sqrt{3}} \arcsin \frac{\sqrt{3}(x-1)}{2} + C$

37. $\frac{17}{18} + \frac{3 \ln 3}{8}$ 39. $\pi/4$

43. $\sqrt{2} \arctan \frac{\tan(x/2)-1}{\sqrt{2}} + C$

45. $\frac{2}{\sqrt{5}} \arctan \frac{\tan(u/2)}{\sqrt{5}} + C$

57. $\frac{1}{2} (1+x) \sqrt{5+2x+x^2} + 2 \sinh^{-1} \left(\frac{1+x}{2} \right) + C$

59. $2\sqrt{1+2x} + \ln(1-\sqrt{1+2x}) - \ln(1+\sqrt{1+2x}) + C$

61. $2\sqrt{e^2+e^x} - 2e \tanh^{-1} \left(\frac{\sqrt{e^2+e^x}}{e} \right) + C$

63. $\frac{1}{3} \arctan \left(\frac{1}{3} \sqrt{-9+e^{2x}} \right) + C$

65. $\frac{1}{6} \left[\sqrt{5+2x+x^2} (7+x+2x^2) - 12 \sinh^{-1} \left(\frac{1+x}{2} \right) \right] + C$

67. $F(x) = \sqrt{x^2-4x+5} (x^2-x-1) - 3 \sinh^{-1}(2-x) + \sqrt{2} + 3 \sinh^{-1}(1)$

69. $x \approx 5.1292$ 71. $x \approx 0.095142$

Section 7.5

1. $\frac{1}{25} \ln \left| \frac{x}{x+5} \right| + C$ 3. $-\ln \left| \frac{2+\sqrt{x^2+4}}{x} \right| + C$

5. $\frac{1}{2} \tan(2x) - x + C$ 7. $\frac{1}{2} \arctan \frac{x+2}{2} + C$

9. $\frac{x^2}{2} + \frac{4x^{3/2}}{3} + C$ 11. $\frac{1}{2} \arcsin \frac{x^2}{2} + C$

Section 7.6

1. a. 40 b. 42

3. $T_8 = 6724$; $S_8 \approx 6554.6667$; exact value: 6553.6

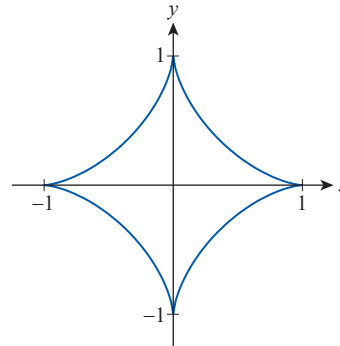
5. $T_8 \approx 0.83954$; $S_8 \approx 0.8048$; exact value: 0.8

7. $T_8 = 65$; $S_8 = 64$; exact value: 64
 9. $T_8 = 11.7296$; $S_8 = 11.8632$; exact value: 12
 11. $T_8 = 4.03068$; $S_8 = 4.0467$;
 exact value: $5 \ln 5 - 4 = 4.047189\dots$
 13. $T_8 = 84$; $S_8 \approx 85.33333$; exact value: $256/3 = 85.\bar{3}$
 15. $T_8 \approx 24.6507$; $S_8 = 24.5122$;
 exact value: $52\sqrt{2}/3 = 24.513035\dots$
 17. $T_8 \approx 6.9734$; $S_8 \approx 7.0665$;
 exact value: $\sqrt{65} - 1 = 7.062257\dots$
 19. a. 1.9541 b. 2.0009
 21. 6468.5 23. 0.7809 25. $\pi^3/300 \approx 0.1034$
 27. $1/100 = 0.01$ 29. $5/3 = 1.\bar{6}$
 31. $\pi^5/112,500 \approx 0.0027$ 33. $1/75,000 \approx 0.00001$
 35. $4/3 = 1.\bar{3}$ 37. $n > 71$ 39. $n > 36$
 41. $n > 25$ 43. $n > 10$ 45. $n > 5$ 47. $n > 3$
 49. 3.1415918; 0.00003% 51. 19.2407
 53. Approx. 1430.49 ft 55. 3750 cubic feet
 57. 3.36047; exact value: 3.361267...
 59. 1.20794; exact value: 1.207949...
 61. 3.36094 63. 1.208
 65. $S_{50} \approx 1.46265179$; error bound: 1.83635×10^{-7}
 67. $S_{50} \approx 0.56568724$; error bound: 4.35225×10^{-4}
 69. $S_{50} \approx 0.11714154$; error bound: 1.6636×10^{-7}
 71. $x \approx 1.3391165$ 73. $x \approx 2.080915$

Section 7.7

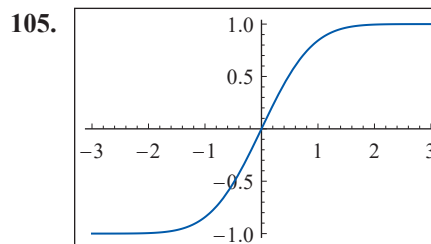
1. Yes, type I 3. Yes, type II 5. Not improper
 7. Yes, type I (doubly improper)
 9. $\lim_{b \rightarrow \infty} \int_2^b \frac{1}{x^2 - 1} dx$ 11. $\lim_{a \rightarrow -\infty} \int_a^{-1} \frac{-3}{x} dx$
 13. $\lim_{a \rightarrow -\infty} \int_a^0 e^x dx$ 15. $\lim_{c \rightarrow 0^+} \int_c^1 \frac{2}{x} dx + \lim_{b \rightarrow \infty} \int_1^b \frac{2}{x} dx$
 17. Divergent 19. Convergent; $6\sqrt{2}$
 21. Type I; convergent; 2 23. Type II; convergent; 2
 25. Type I; convergent; 1 27. Type II; divergent
 29. Type I; convergent; π 31. Type II; divergent
 33. Type I; convergent; π
 35. Both type I and type II; convergent; π
 37. Type II; convergent; π
 39. Both type I and type II; divergent
 41. Type II; divergent 43. Type I; convergent; $2/e^2$

45. Type I; convergent; 1 47. Type II; divergent
 49. Both type I and type II; convergent; $\pi/2$
 51. Type I; convergent; 0 53. Type II; convergent; $4\sqrt{2}$
 55. Type II; convergent; $\pi/3$
 57. Type II; divergent 59. Type II; divergent
 61. Type II; convergent; -4 63. Type I; convergent; $\ln 3$
 65. Diverges for $p \leq 1$; converges for $p > 1$
 67. Converges 69. Converges 71. Diverges
 73. Diverges 75. $4\pi/3$ 79. 1
 81.



The perimeter is 6.

85. $\pi/2$ (Substitute $u = \sqrt{x}$.)
 87. $\pi/4$ 89. $1/s^2$ 91. $\frac{1}{s-a}$ 93. $\frac{s}{s^2 + k^2}$
 95. \$200,000 97. \$603,448
 99. $v_0 = \sqrt{2Rg} \approx 11,180 \text{ m/s}$
 101. False; consider $\int_1^\infty \frac{dx}{x}$.
 103. True



Chapter 7 Review

1. $\frac{2x^{3/2}(3 \ln x - 2)}{9} + C$ 3. $\frac{2}{3}(x+2)\sqrt{x-1} + C$
 5. $2t \cos t + (t^2 - 2)\sin t + C$ 7. $\pi(\pi^2 - 6)$
 9. $\frac{[2 \ln(\sin t) - 1]\sin^2 t}{4} + C$ 11. $\frac{e^{\pi/2} + e^{-\pi/2}}{2}$
 13. a. $\left(\frac{27e^4 + 9}{32e^3 + 16}, \frac{459e^5 - 54}{500e^3 + 250} \right)$ b. $\frac{3\pi}{8}(3e^4 + 1)$

17. $2\ln|x+1| - \ln|x-3| + C$
19. $\ln|x| - \ln|x-1| - \frac{1}{x} - \frac{2}{x^2} + C$
21. $\frac{1}{2}\ln(x^2+2) - \frac{4x^2+7}{4(x^2+2)^2} + C$
23. $2\ln|x-1| + 2\ln|x+5| - \ln|x+3| + C$
25. $-\frac{5\ln 2}{2}$ 27. $\frac{2}{5}(\ln|\sqrt{x}-2| - \ln|\sqrt{x}+3|) + C$
29. $\pi\ln(256/9)$ 31. $16/3$
33. $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$ 35. $3/4$ 37. $-2\cot 2x + C$
39. $\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$
41. $5/12$ 43. $-\frac{\csc^2 x}{2} - \ln|\sin x| + C$
45. $\frac{\tan^5 x}{5} + C$ 47. $\frac{\sin^3 v}{3} + C$
49. 2 51. $\frac{5\pi(\pi+2)}{4}$
55. $4\sin^{-1}(x/2) - x\sqrt{4-x^2} + C$
57. $\frac{t}{2\sqrt{t^2+4}} + C$ 59. $9(\sqrt{2} - \sinh^{-1} 1)$
61. $(x-2)\sqrt{5+4x-x^2} - 9\sin^{-1}\left(\frac{2-x}{3}\right) + C$
63. $\sin^{-1}\left(\frac{e^x-1}{2}\right) + C$
65. $2\sqrt{3} - \sinh^{-1} \sqrt{3}$ 67. $\pi\left(\pi - \frac{7\sqrt{3}}{4}\right)$
69. $2\pi(\sqrt{2} + \sinh^{-1} 1) \approx 14.4236$
71. 6.3589
73. $\sqrt{2\cos x - \cos^2 x} - \sin^{-1}(\cos x - 1) + C$
75. $T_6 \approx 0.50403; |E_T| \approx 0.00403$
77. $T_6 \approx 0.70067; |E_T| \approx 0.00752$
79. $|E_T| \leq 1/72 \approx 0.01389$
81. $|E_T| \leq \frac{\pi^3}{486\sqrt{3}} \approx 0.03683$
83. $S_6 \approx 0.50009; |E_S| \approx 0.00009$
85. $S_6 \approx 0.69347; |E_S| \approx 0.00033$
87. $|E_S| \leq 1/1944 \approx 0.00051$

$$89. |E_S| \leq \frac{11\pi^5}{590,490\sqrt{3}} \approx 0.00329$$

91. Type II; divergent 93. Type II; convergent; 4

95. Type I; convergent; $\pi/2$ 97. Converges

$$99. \int_0^{\pi^2/4} \frac{\cos \sqrt{x}}{\sqrt{x}} dx = \int_0^{\pi/2} 2 \cos u du = 2$$

101. Yes, the volume is $\pi/2$; the surface area is $\pi(\sqrt{2} + \sinh^{-1} 1) \approx 7.2118$.

$$103. \frac{1}{(s-a)^2} \quad 105. \frac{2ks}{(s^2+k^2)^2}$$

107. True 109. False; consider Gabriel's horn.

111. False; consider $f(x) = 1/\sqrt{|x|}$.

113. False; see Exercises 72–74 in Section 7.4.

115. $n = 35$ 117. $n = 4; n = 12$

119. $T_{50} \approx 1.08948$; $S_{50} \approx 1.08943$; Simpson's Rule is more accurate. (In fact, S_{50} agrees with the numerical integration command's value to 11 decimal digits!)

Chapter 7 Conceptual Project

$$1. I_0 = \pi/2; I_1 = 1; I_2 = \pi/4; I_3 = 2/3$$

$$3. I_4 = 3\pi/16; I_5 = 8/15; I_6 = 5\pi/32; I_7 = 16/35$$

Chapter 7 Application Project

3. a. No; the formula obtained in Question 2c implies that torque and power are numerically equal at 5252 rpm. (Torque must be measured in lb-ft and power in hp.)
- b. From about 40 hp to 83 hp
- c. From about 60 hp to 107 hp
- d. The 2021 car
- e. A "flat" torque curve means more torque at lower rpm's, hence more performance off the line. Since city driving rarely involves high rpm's, a flat torque curve means more power and better acceleration in stop-and-go traffic.
5. a. Approx. 175 kJ b. Approx. 250 kJ
- c. As illustrated by the problems in this project, more area under the torque curve generally translates into more horsepower early on and greater values of definite integrals of the power curve over various intervals (especially in lower rpm ranges). Therefore, the engine is capable of doing more work, and earlier on the rpm range, than engines that have low torque over lower rpm ranges, and thus have less area under the curve.

Chapter 8

Section 8.1

1. $\frac{dy}{dx} = \sqrt{\frac{y}{x}}$ 3. $\frac{dy}{dx} = x\sqrt{1-y^2}$
5. $\frac{dy}{dx} = \frac{y}{x^2} - \frac{y}{x}$ 13. $y = \frac{x^3}{3} + \sin x + 5$
15. $y = \ln|x| + x - 1$ 17. $y = -\frac{1}{x} + \frac{7}{2}$
19. $y = \arcsin x + 2$
21. Yes 23. Yes 25. Yes 27. No
29. $y = \tan(x^2 + C)$ 31. $y = Ce^{3x^2/2} - 2$
33. $y = Ce^{x^3}$ 35. $y = Ce^{\arcsin(x/2)}$
37. $x = \frac{1}{\ln(t^2 + C)}$ 39. $y = Cx - 6$
41. $x = C(y+1)$ 43. $x = \pm\sqrt{\frac{y^2}{2} + 2y + C}$
45. $y = \sqrt[3]{x^3 - 3x + 8}$
47. $y = \frac{1}{5}\ln\left(\frac{5}{2}e^{2x} - \frac{3}{2}\right)$ for $e^{2x} > \frac{3}{5}$
49. $y = \frac{1}{4 - \sin x}$ 51. $y = \frac{2e^{x^2/2}}{1 - 2e^{x^2/2}}$
53. $y = \ln((1 + \sin t)^2)$ 55. Approx. 2 min 21 s
57. Approx. 31.85 g 61. Approx. 73.84 cm
63. a. $\frac{dv}{dt} + \frac{k}{m}v = g$ b. $v = \frac{mg}{k}$
65. $x^2 + 2y^2 = C$ 67. $x^3 + 3x^2 + 3y^2 = C$
69. $x^3 + y^3 = C$ 71. $y^2 + x^2 - \ln(x^2) = C$
73. $2y^2 - x^2 - \ln(x^2) = C$ 75. True
77. False; for example, when solving motion problems, finding the velocity function from the acceleration and initial velocity is an initial value problem.

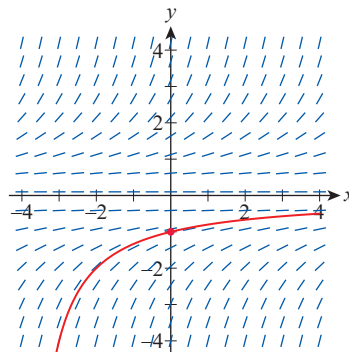
Section 8.2

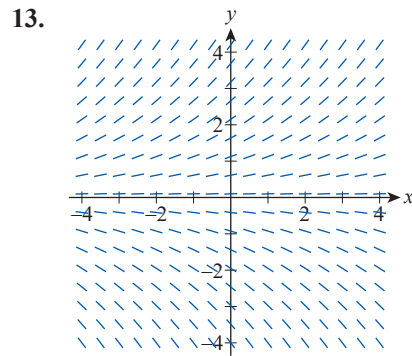
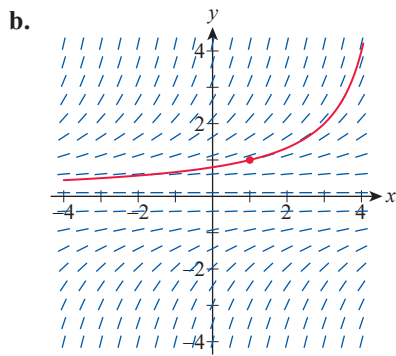
1. Linear 3. Linear 5. Linear 7. Linear in y
9. Linear in x
11. $y = \frac{C}{x^3}$ 13. $y = \frac{C}{x^4} + \frac{1}{4}$
15. $y = (2x - 3\ln x + C)x - 5$
17. $x = \frac{y^2}{10} + \frac{C}{y^8}$ 19. $y = \sin x + Ce^{-x}$
21. $y = \frac{(x^5/5) + x + C}{x^4 + 1}$ 23. $y = \frac{\ln x}{2} + \frac{C}{\ln x}$

25. $x = t + \sin t + C \cos t$ 27. $y = \frac{1 + 5e^{-x^3}}{3}$
29. $y = (x^2 + 1)(\arctan x + 10)$ 31. $y = \frac{e^x \ln(x+1)}{x+1}$
33. $x = \frac{5}{y^2} - 1 + 2\ln y$ 35. $y' + \frac{1}{x}y = \frac{\sin x}{x} + \cos x$
37. $y' + \frac{1}{x \ln x}y = \frac{1+x}{x \ln x}$ 39. $y = \frac{1}{x(C - \ln x)}$
41. $y = \frac{1}{x(-2\sqrt{x} + C)^2}$ 43. Approx. 7 min 47.4 s
45. Approx. 15.315 lb
47. $\frac{dy}{dt} + \frac{r_2 y}{V_0 + (r_1 - r_2)t} = a_1 r_1$; $y(0) = a_0$
49. $I(t) = \frac{1}{2} - \frac{1}{2}e^{-50t}$
51. $q(t) = 1.5 \times 10^5 - 1.5 \times 10^5 e^{-t/50,000}$; As $t \rightarrow \infty$, the charge stabilizes at 1.5×10^5 coulombs.
53. a. $\frac{dA}{dt} = d - kA$
b. $A = \frac{d}{k} - \frac{d}{k}e^{-kt}$; $A \rightarrow \frac{d}{k}$ as $t \rightarrow \infty$
55. False; the equation $xy' = y$ is both separable and linear.
57. False; for example, equations that are, say, both linear and separable can readily be solved by different methods.

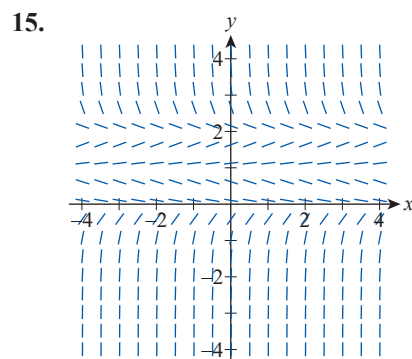
Section 8.3

1. C 3. A 5. D
7. $y = 0$: unstable; $y = 2$: stable
9. $y = -\pi/2$: unstable; $y = \pi/2$: stable
11. a.

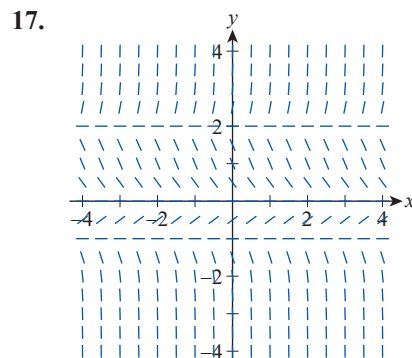




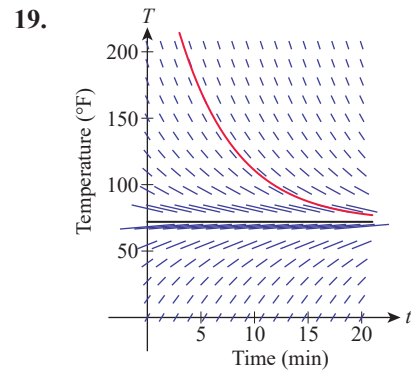
$y = 0$: unstable



$y = 0, y = 2$: stable; $y = 1$: unstable



$y = 2, y = -1$: unstable; $y = 0$: stable



21. a. $P(t) = \frac{500}{1 + 4e^{-0.3t}}$ b. Just under 6 years

23. $v = (mg/k)^2$

27. a. (i) 5.578 (ii) 6.990

b. Exact value: $\frac{5e^2 - 3}{4} \approx 8.486$

29. a. (i) 1.168 (ii) 1.326

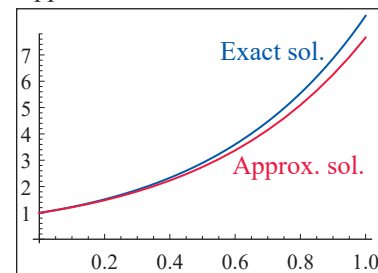
b. Exact value: $\frac{5}{4} + e^{-3/2} \approx 1.473$

31. a. (i) 1.345 (ii) 1.482

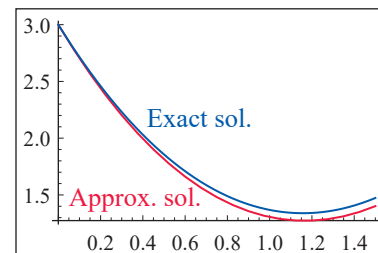
b. Exact value: $\sqrt{3} \approx 1.732$

33. 7.966

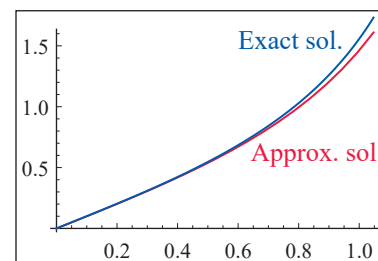
39. Approx. 7.659



41. Approx. 1.401



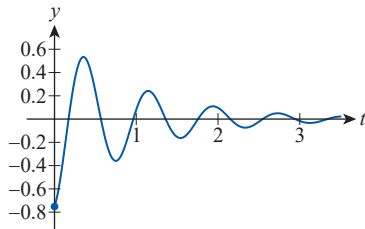
43. Approx. 1.610



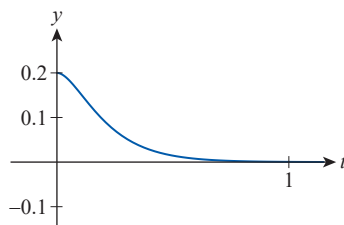
Section 8.4

1. $y = c_1 e^x + c_2 e^{-2x}$ 3. $y = c_1 e^{-x/3} + c_2 e^{2x}$
5. $y = c_1 e^{-x} + c_2 x e^{-x}$ 7. $y = c_1 e^{-x/2} + c_2 x e^{-x/2}$
9. $y = e^{2x} (c_1 \cos 3x + c_2 \sin 3x)$
11. $y = e^{x/5} (c_1 + x c_2)$
13. $y = c_1 e^{(\sqrt{13}+1)x/6} + c_2 e^{(\sqrt{13}-1)x/6}$
15. $w = e^{-t/2} [c_1 \cos(t/2) + c_2 \sin(t/2)]$
17. $y = c_1 e^{kx} + c_2 e^{-kx}$ 19. $y = \frac{5}{2} (e^{\sqrt{3}x} + e^{-\sqrt{3}x})$
21. $y = e^{-x} + 3e^{2x}$ 23. $y = e^x (\cos 2x - 2 \sin 2x)$
25. $y = 2x e^{3x/2} - e^{3x/2}$ 27. $y = 5(e^{5x} - e^{-5x})$
29. $y = e^x$ 31. No solution
33. $y = e^x (2 \cos 4x + e^{-\pi/8} \sin 4x)$
35. $y = c_1 e^{-x} + c_2 e^{3x} + c_3$
37. $y = c_1 e^x + c_2 \cos \sqrt{2}x + c_3 \sin \sqrt{2}x$
39. $y = c_1 e^x + c_2 e^{-2x} - \frac{x}{2} - \frac{1}{4}$
41. $y = c_1 e^x + c_2 e^{-5x} - \frac{1}{13} \cos x - \frac{3}{26} \sin x$
43. $y = \cos x + \sin x + x^2 - 2$
47. a. Underdamped: $c < 2\sqrt{10}$; oscillating motion, amplitude goes to 0 (Example 5)
 b. Critically damped: $c = 2\sqrt{10}$; damped motion with no oscillation, but the smallest change in c will cause it to be underdamped or overdamped
 c. Overdamped: $c > 2\sqrt{10}$; solution is a linear combination of exponential functions, no oscillation, displacement goes to 0
49. $c = 2\sqrt{g/5} \approx 2.8$

$$51. y(t) = \frac{-e^{-t}}{84} [63 \cos(3\sqrt{7}t) - 5\sqrt{7} \sin(3\sqrt{7}t)]$$



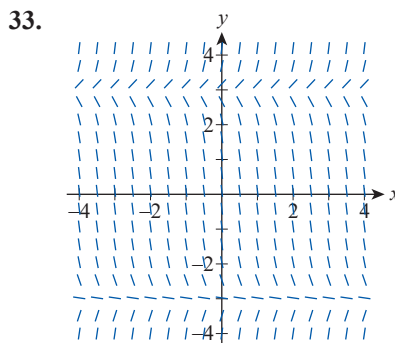
$$53. y(t) \approx -0.278e^{-11.85t} + 0.478e^{-6.9t}$$



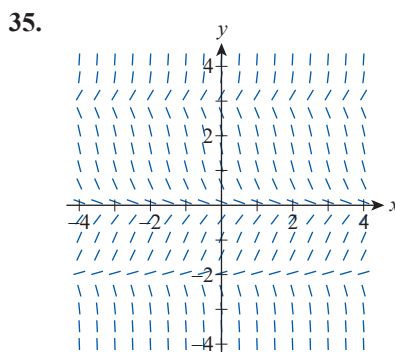
57. Approx. 8.2 s; approx. 16.66 m

Chapter 8 Review

1. Autonomous, separable, first-order
3. First-order, linear
5. $y = \ln \sqrt{x^2 + 1} + 1$ 7. $y = Cx^2$
9. $y = \left(\ln x - \frac{1}{x} + C \right)^2$ or $y = 0$
11. $u = \frac{1 - Cx^2}{1 + Cx^2}$ or $u = -1$
13. $y = \sqrt[3]{3x^3 + x^2 + 8}$
15. $y = (t^2 + 1)^2 + 2$ or $y = t^4 - 6t^2 + 11$
17. $y = Cx$
19. $y = \frac{C}{x^6} + \frac{1}{3}$ 21. $x = \frac{2y^2 + C}{y^2 + 1}$
23. $y' - 3x^2 y = x^2$ 25. $y = x(e^{1-x} + x - 1)$
27. $y = \frac{x}{x^2 + 1} - \frac{3}{\sqrt{x^2 + 1}}$
29. B; $y = 0$: unstable; $y = -2$: stable
31. A; $y = 2$: unstable; $y = -1$: stable



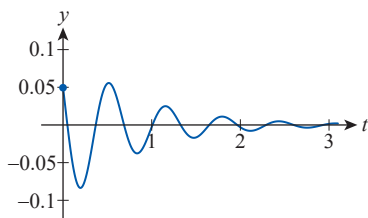
$y = 3$: unstable; $y = -3$: stable



$y = 3, y = -2$: unstable; $y = 0$: stable

37. a. (i) 8.063 (ii) 8.960
 b. Exact value: $3 + e^2 \approx 10.389$

39. $y = c_1 e^{4x} + c_2 e^{-4x}$ 41. $y = c_1 e^{5x} + c_2$
 43. $y = c_1 e^{x/2} + c_2 x e^{x/2}$ 45. $y = 2e^{-2x} + e^{3x}$
 47. $y = 2e^{(3x-6)/2} - 7$ 49. No solution
 51. $y = c_1 e^{x/2} + c_2 e^{3x} + \frac{1}{50}(\cos x - 7 \sin x)$
 53. $x^4 + 2x^2 + 2y^2 = C$ 55. Approx. 13.446 kg
 57. a. $V' = kV^{2/3}$
 b. It melts away in just under 5.5 hours.
 59. $q(t) = 250,000e^{-t/2500} + 100t - 250,000$
 61. Approx. 68.43 °C
 63. $y(t) \approx e^{-1.25t} [0.05 \cos(9.83t) - 0.095 \sin(9.83t)]$



65. False; it is linear as well as separable.
 67. False; see the slope field in Figure 1 of Section 8.3.
 69. True

Chapter 8 Conceptual Project

1. $A_0 - \frac{X(t)}{1+b}$; $B_0 - \frac{bX(t)}{1+b}$
 3. $\frac{dX}{dt} = k \left(60 - \frac{X}{4} \right) \left(40 - \frac{3X}{4} \right)$; $X(0) = 0$
 5. $\lim_{t \rightarrow \infty} X(t) = 53\frac{1}{3}$; this means that ultimately $53\frac{1}{3}$ grams of C is formed, using up all of reactant B, while leaving $46\frac{2}{3}$ grams of reactant A.

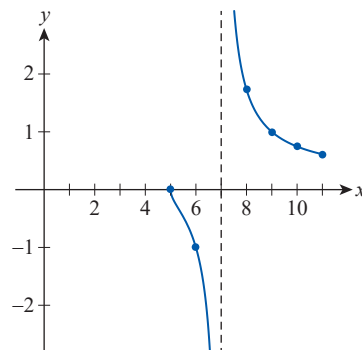
Chapter 8 Application Project

1. a. The equation here is nonhomogeneous; the nonzero term on the right-hand side indicates the presence of an additional, periodic force acting on the oscillating mass.
 b. Because of the presence of a periodic external force acting on the system, the motion is expected to exhibit some sort of periodicity, but a lot more “irregular” than that of a simple harmonic motion, such as in Example 4 of Section 8.4. No, the IVP doesn’t indicate the presence of damping forces.
 c. Period: $\frac{2\pi}{\theta}$; Frequency: $\frac{\theta}{2\pi}$
 3. $\lim_{\theta \rightarrow \omega} y(t) = Y(t) = \frac{f}{2\omega} \left(\frac{\sin(\omega t)}{\omega} - t \cos(\omega t) \right)$

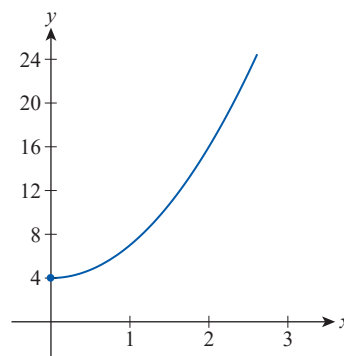
Chapter 9

Section 9.1

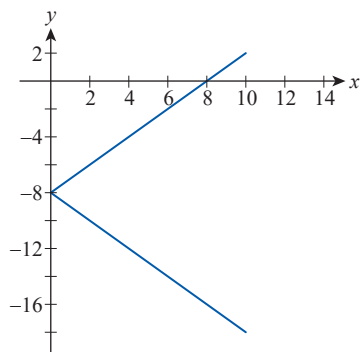
1.	t	x	y
	0	5	0
	1	6	-1
	2	7	undefined
	3	8	$\sqrt{3}$
	4	9	1
	5	10	$\sqrt{5}/3$
	6	11	$\sqrt{6}/4$



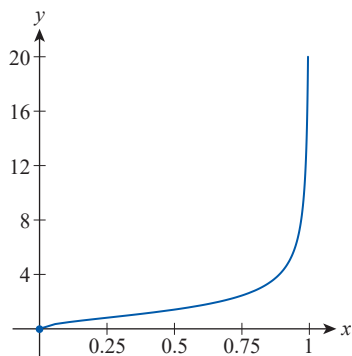
3. $x = t$, $y = -t^2 - 5$ 5. $x = t$, $y = \pm\sqrt{t-4}$
 7. $x = t$, $y = |t-1|$ 9. $x = t$, $y = \pm\sqrt{1-t^2}$
 11. $y = 3x^2 + 4$ ($t \geq 2$)



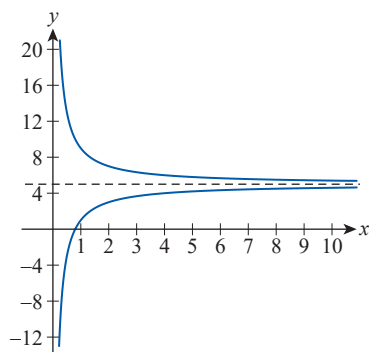
13. $x = |y + 8|$



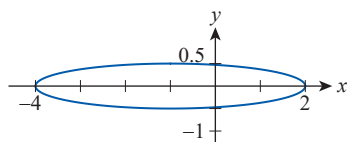
15. $y = \sqrt{\frac{2x}{1-x}} \quad (t \geq 0)$



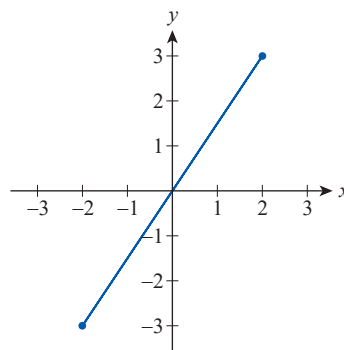
17. $x = \frac{4}{|y-5|} \quad (t \neq 3)$



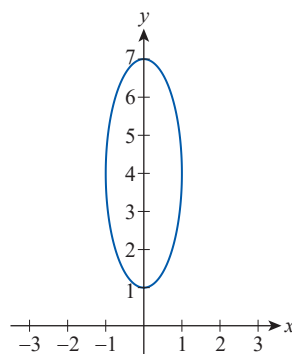
19. $y = \pm \frac{\sqrt{8-2x-x^2}}{6}$



21. $y = \frac{3}{2}x$

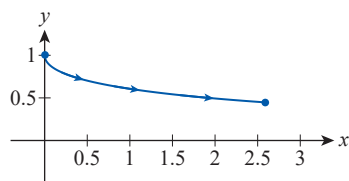


23. $y = 4 \pm 3\sqrt{1-x^2}$

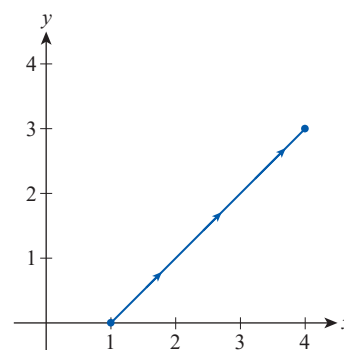


25. F 27. C 29. A

31.



33.



37. $x = h + a \cos t, y = k + b \sin t$

39. Sample answer: $x = t, y = -\frac{3}{2}t + 6$

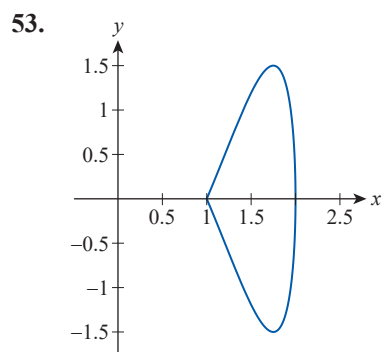
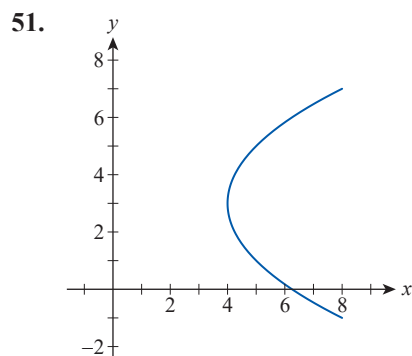
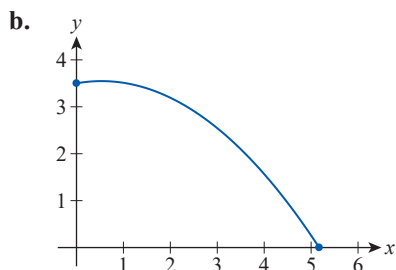
41. Sample answer: $x = t - 3, y = -\frac{3}{4}t + 1 \quad (0 \leq t \leq 8)$

43. Sample answer: $x = 6 \cos t, y = -2 + 6 \sin t$

45. Sample answer: $x = 6 \cos t, y = 1 + \sqrt{11} \sin t$

47. Yes

49. a. $x \approx 9.85t, y \approx -16t^2 + 1.74t + 3.5$



57. $(0, 0)$ and $(1, 3)$

59. $x = 3(\theta - \sin \theta), y = 3(1 - \cos \theta)$

65. $x = (R + r)\cos t - r\cos\left(\frac{R + r}{r}t\right),$

$$y = (R + r)\sin t - r\sin\left(\frac{R + r}{r}t\right)$$

67. $y = \frac{8r^3}{x^2 + 4r^2}$

69. True

71. False; consider the graph of a circle centered at the origin for which neither statement is true.

Section 9.2

1. Neither differentiable nor smooth
3. Differentiable but not smooth
7. Horizontal: $y = 1$; vertical: $x = -1/4$
9. Horizontal: none; vertical: $x = 1$
11. Horizontal: none; vertical: none
13. Horizontal: $y = \pm 5$; vertical: $x = \pm 2$

15. $\frac{dy}{dx}\bigg|_{t=1/2} = 1; \frac{d^2y}{dx^2}\bigg|_{t=1/2} = 1$

17. $\frac{dy}{dx}\bigg|_{t=0} = 0; \frac{d^2y}{dx^2}\bigg|_{t=0} = 2$

19. $\frac{dy}{dx}\bigg|_{t=0} = -1; \frac{d^2y}{dx^2}\bigg|_{t=0} = 3$

21. $\frac{dy}{dx}\bigg|_{t=\pi/4} = 0; \frac{d^2y}{dx^2}\bigg|_{t=\pi/4} = -8$

23. $\frac{dy}{dx}\bigg|_{t=1} = 1; \frac{d^2y}{dx^2}\bigg|_{t=1} = \frac{1}{2}$

25. $t = 10/3$ 27. $t = -1/2, 1/\sqrt[3]{4}$

29. $4/3$ 31. $\pi^3/6$ 33. $\frac{2}{5}(1 + e^{-\pi/2})$

39. π 41. $92/9$ 43. $59/24$ 45. $2e^{\pi/2} - 2$

47. $\frac{2e^2 + 3e - 2}{4e}$

49. a. $16\sqrt{2}\pi$ b. $24\sqrt{2}\pi$

51. a. $\frac{12,289\pi}{192}$ b. $\frac{(135 + \ln 2)\pi}{6}$

53. a. $\frac{(16e^4 + e^2 - 1)\pi}{16e^2}$ b. $\frac{\pi}{2}\left(5 - \frac{2}{e}\right)$

55. $48\pi/5$

57. a. $\frac{\pi}{2}(4 + \sinh 4)$ b. $2\pi(3 \sinh 2 - \cosh 2 + 1)$

59. Yes 61. $2\sqrt{5}$ m/s 63. 9 m/s

65. $t > 0$ 67. $32\pi/15$ 69. $(18/5, 9/8)$

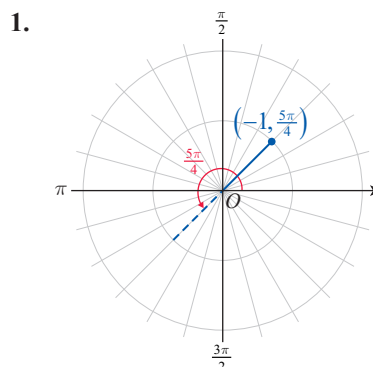
75. False; consider Example 1.

77. True

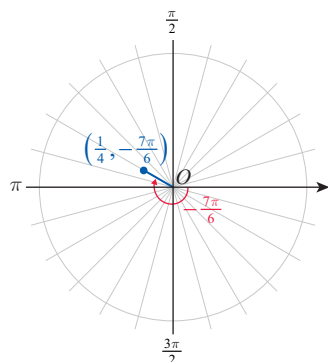
79. 28.362 81. 3.820

83. a. 81.870 b. 258.882

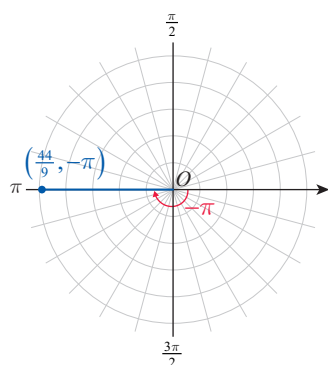
Section 9.3



3.



5.



7. $\left(\frac{5}{\sqrt{2}}, -\frac{5}{\sqrt{2}}\right)$

9. $\left(-\frac{25}{4\sqrt{2}}, -\frac{25}{4\sqrt{2}}\right)$

11. $\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

13. $(-3, 0)$ and $(3, \pi)$

15. $(\sqrt{145}, -0.08)$ and $(-\sqrt{145}, 3.06)$

17. $(2\sqrt{21}, 1.76)$ and $(-2\sqrt{21}, -1.38)$

19. $r^2 = 25$

21. $r \cos \theta = 12$

23. $\sin \theta = \cos \theta$

25. $r \cos \theta = 16a$

27. $r^2 - 4a r \cos \theta = 0$

29. $r^2 \sin^2 \theta - 4r \cos \theta - 4 = 0$

31. $x^2 + y^2 = 5x$

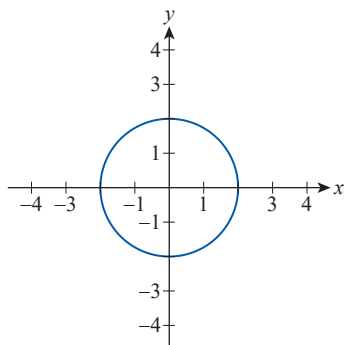
33. $x^2 + y^2 = 49$

35. $y = 1/2$

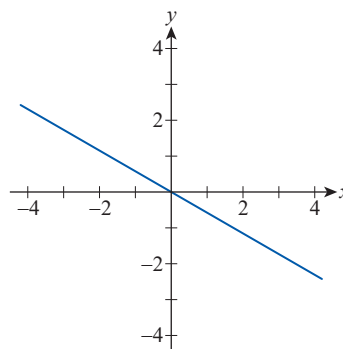
37. $x^4 + y^4 + 2x^2y^2 = 2xy$

39. $4y + 7x = 12$

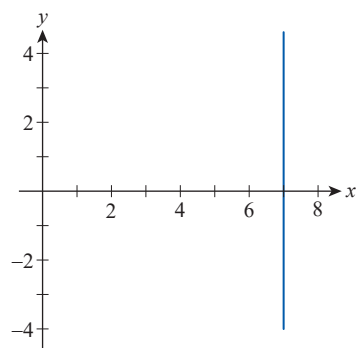
41. $x^2 + y^2 = 4$



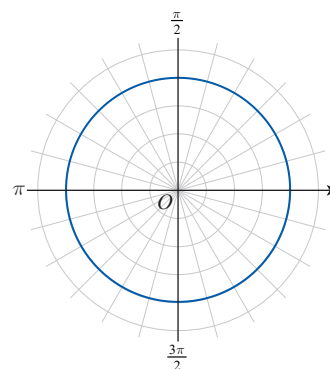
43. $y = -x/\sqrt{3}$



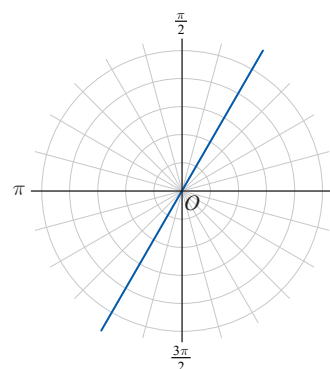
45. $x = 7$



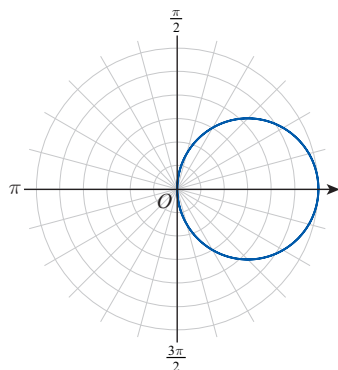
47.



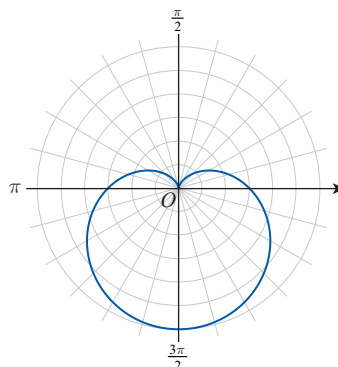
49.



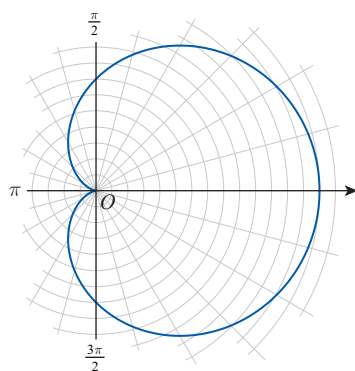
51.



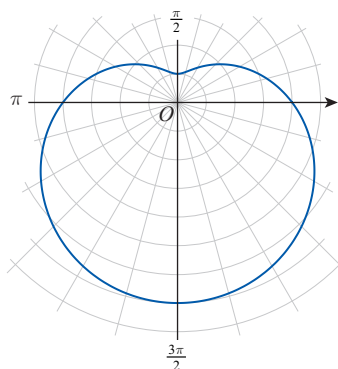
53.



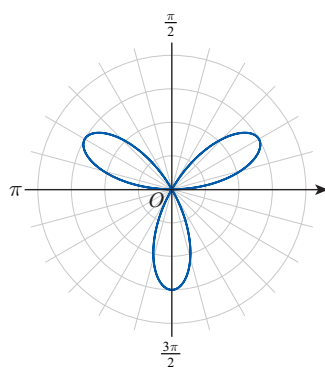
55.



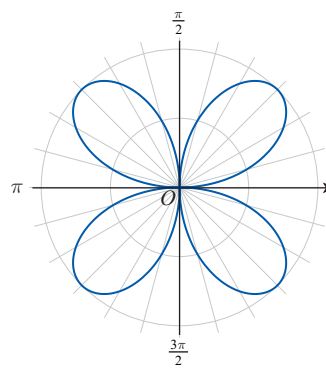
57.



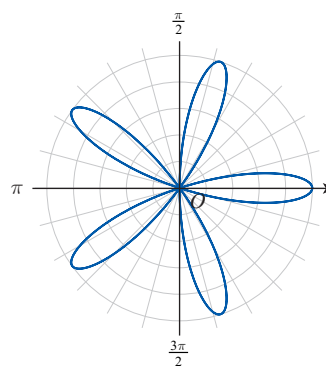
59.



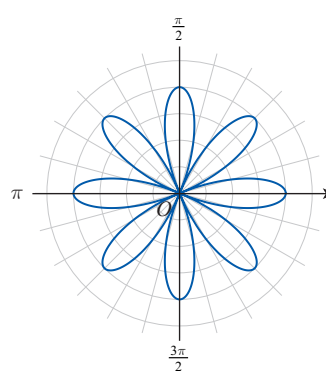
61.



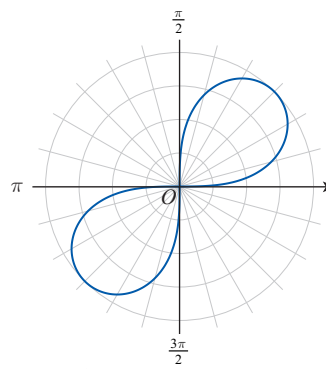
63.



65.



67.


 69. $(0,0), (\sqrt{2}/2, \pi/4)$

 71. $(0,0), (1 + \sqrt{2}/2, 3\pi/4), (1 - \sqrt{2}/2, -\pi/4)$

 73. The graph of $f(\theta - a)$ is that of $f(\theta)$ rotated about the origin by a radians.

Section 9.4

 1. $-\sqrt{3}/3$ 3. $-2/\pi$ 5. 3 7. $-\sqrt{3}/5$

9. $\frac{2\pi+3\sqrt{3}}{2\sqrt{3}\pi-3}$ 11. $\tan 1$ 13. 4

15. Horizontal: $(2, \pi/2)$, $(1/2, 7\pi/6)$, $(1/2, 11\pi/6)$;
vertical: the pole, $(3/2, \pi/6)$, $(3/2, 5\pi/6)$

17. Horizontal: the pole, $(3a/2, \pm\pi/3)$;
vertical: $(2a, 0)$, $(a/2, \pm 2\pi/3)$

21. $\theta = 0$ 23. $\theta = \pm\pi/4$ 25. $\theta = 0$ 27. $9\pi/4$

29. $3\pi/2$ 31. $11\pi/4$ 33. $19\pi/2$ 35. 5π

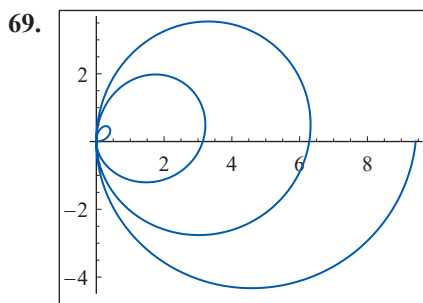
37. $\pi/2$ 39. 8π 41. 4 43. $\frac{6\sqrt{3}-2\pi}{3}$

45. $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$ 47. $\pi - \frac{3\sqrt{3}}{2}$ 49. $\frac{11\sqrt{3}}{2} + \frac{14\pi}{3}$

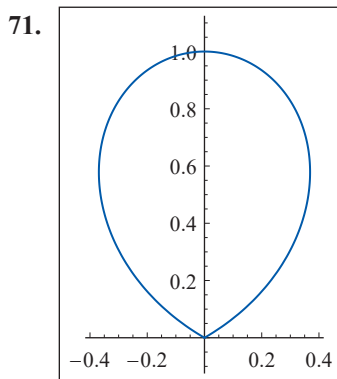
51. $6\pi\sqrt{16\pi^2+1} + \frac{3\sinh^{-1}4\pi}{2}$

53. $2/\sqrt{3}$ 55. $\frac{8(\pi^2+1)^{3/2}-8}{3}$

57. $4\sqrt{2}$ 59. π^2 61. $\frac{18\sqrt{2}\pi(e^{2\pi}+1)}{5}$



$L \approx 45.3582$



$L \approx 2.6825$

73. $A \approx 8580.85$ 75. $A \approx 37.1940$

Section 9.5

1. Parabola 3. Ellipse 5. Parabola

7. Ellipse; H 9. Hyperbola; M 11. Hyperbola; J

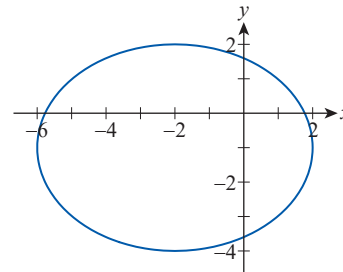
13. Hyperbola; O 15. Ellipse; G 17. Parabola; N

19. Parabola; E 21. Hyperbola; I

23. $e = \frac{\sqrt{6}}{4}$; major = $4\sqrt{2}$; minor = $2\sqrt{5}$

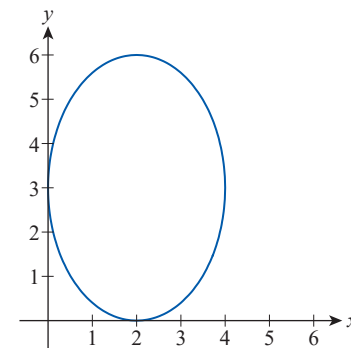
25. $e = \frac{\sqrt{6}}{3}$; major = $2\sqrt{6}$; minor = $2\sqrt{2}$

27.



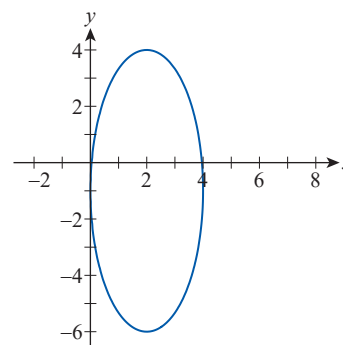
Foci: $(-2 \pm \sqrt{7}, -1)$

29.



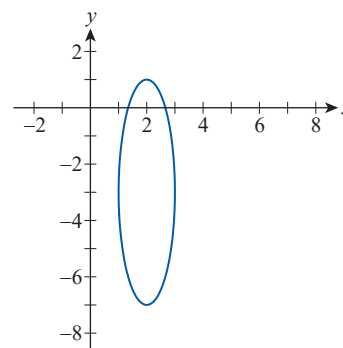
Foci: $(2, 3 \pm \sqrt{5})$

31.



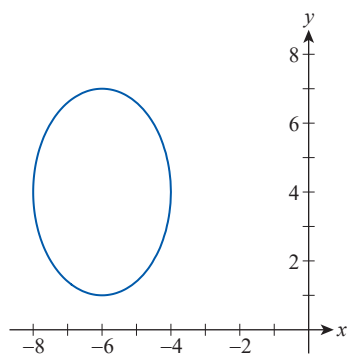
Foci: $(2, -1 \pm \sqrt{21})$

33.

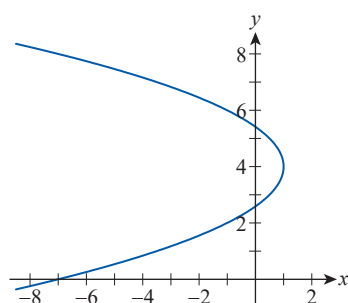


Foci: $(2, -3 \pm \sqrt{15})$

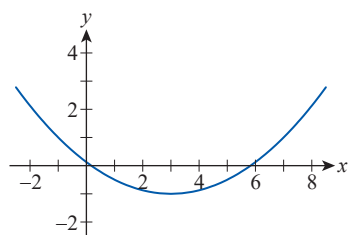
35.

Foci: $(-6, 4 \pm \sqrt{5})$

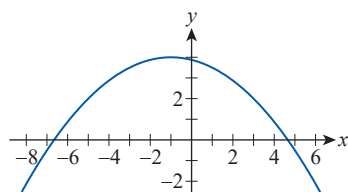
37.

Focus: $(1/2, 4)$; Directrix: $x = 3/2$

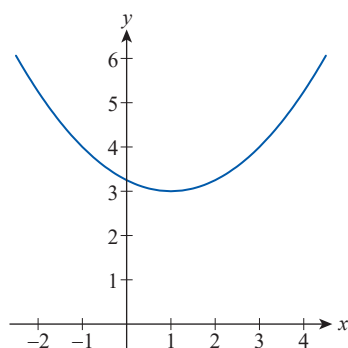
39.

Focus: $(3, 1)$; Directrix: $y = -3$

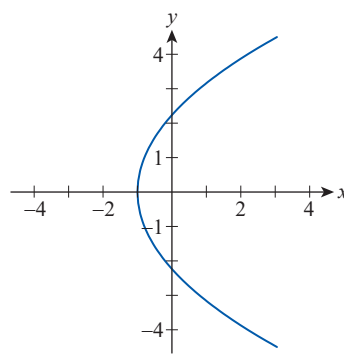
41.

Focus: $(-1, 2)$; Directrix: $y = 6$

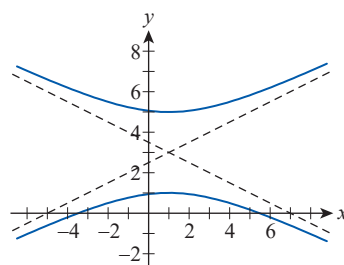
43.

Focus: $(1, 4)$; Directrix: $y = 2$

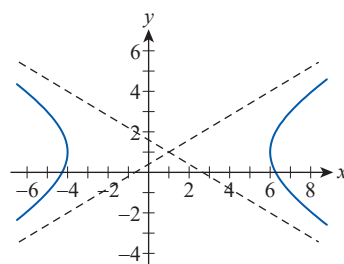
45.

Focus: $(1/4, 0)$; Directrix: $x = -9/4$

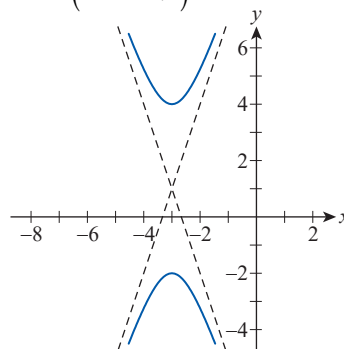
47.

Foci: $(1, 3 \pm 2\sqrt{5})$

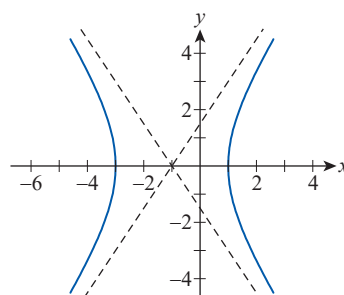
49.

Foci: $(1 \pm \sqrt{34}, 1)$

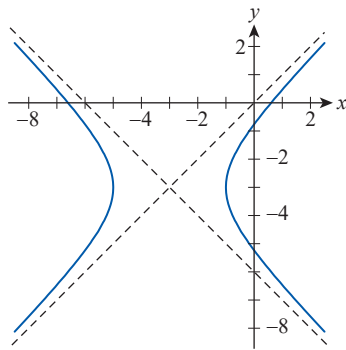
51.

Foci: $(-3, 1 \pm \sqrt{10})$

53.

Foci: $(-1 \pm \sqrt{13}, 0)$

55.

Foci: $(-3 \pm 2\sqrt{2}, -3)$

57. $(x+2)^2 = 2\left(y - \frac{1}{2}\right)$ 59. $(y-2)^2 - \frac{(x-1)^2}{8} = 1$

61. $\frac{(y-1)^2}{2} - \frac{(x+1)^2}{2} = 1$ 63. $\frac{(x-3)^2}{36} + \frac{y^2}{27} = 1$

65. $\frac{(x+9)^2}{25} + \frac{(y-6)^2}{21} = 1$ 67. $(y-1)^2 = -12(x+1)$

69. $\frac{(y+4)^2}{16} - \frac{(x-3)^2}{25} = 1$ 71. $\frac{(x+1)^2}{16} + \frac{(y+1)^2}{4} = 1$

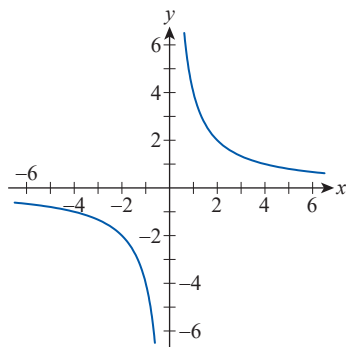
73. $(x-2)^2 = 2(y-1)$ 75. $e \approx 0.249$

77. 185.93 million miles 79. 2 ft 81. 1.5 in.

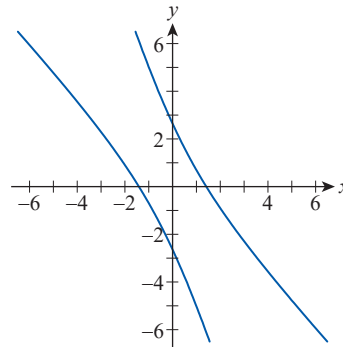
85. $\frac{y^2}{144} - \frac{x^2}{25} = 1$

87. $(4\sqrt{3}+3, -4+3\sqrt{3})$ 89. $\left(\frac{-5\sqrt{2}}{16}, \frac{3\sqrt{2}}{16}\right)$

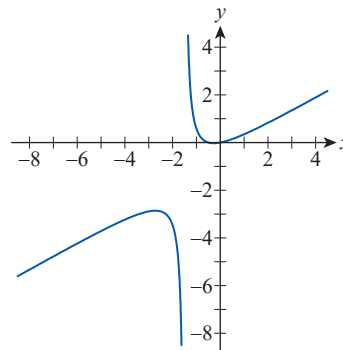
91. Hyperbola, $\theta = \pi/4$; $x'^2 - y'^2 = 8$



93. Hyperbola, $\theta = \pi/6$; $19x'^2 - y'^2 = 28$



95. Hyperbola, $\theta = \frac{\pi}{3}$; $\frac{(x'+2)^2}{3} - \left(y' - \frac{1}{\sqrt{3}}\right)^2 = 1$



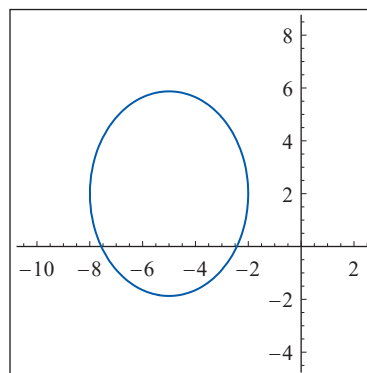
97. C 99. D

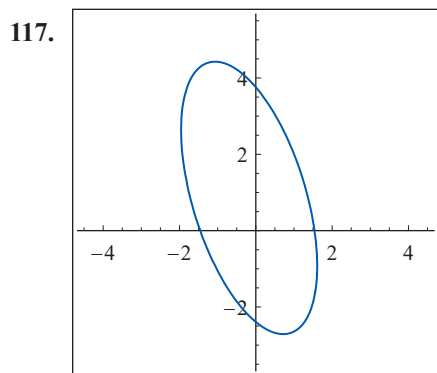
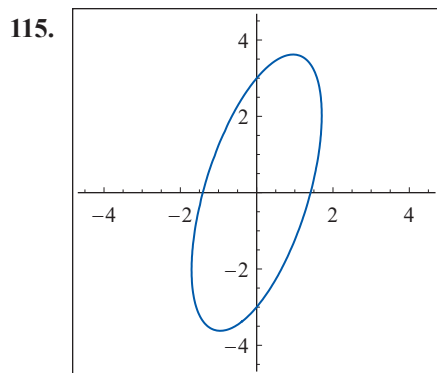
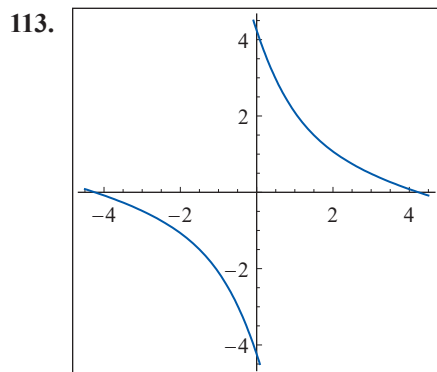
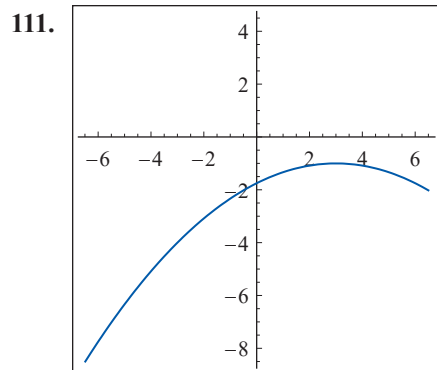
101. The objective of the rotation of axes is to eliminate the $x'y'$ -term. If your final equation contains an $x'y'$ -term, you know that a mistake has occurred.

105. False; every point on the parabola has to be the same distance from the directrix and the focus (which is at a positive distance from the directrix.) This would not be true of the point of tangency.

107. True

109.



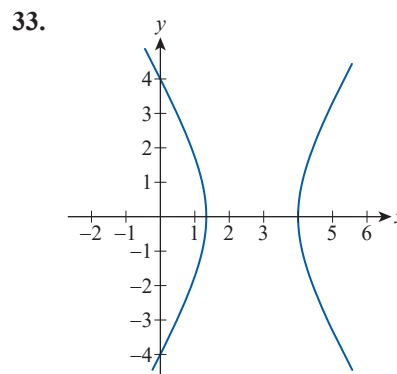
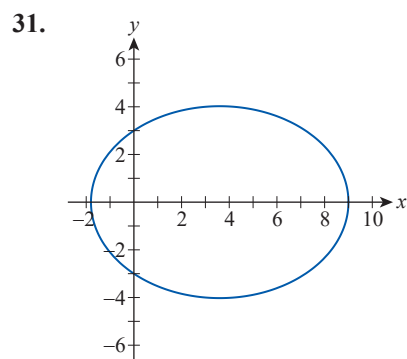
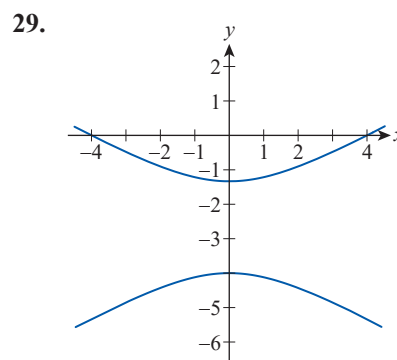
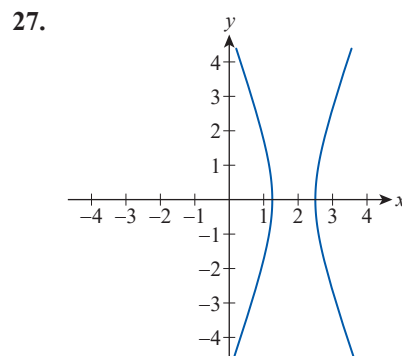


Section 9.6

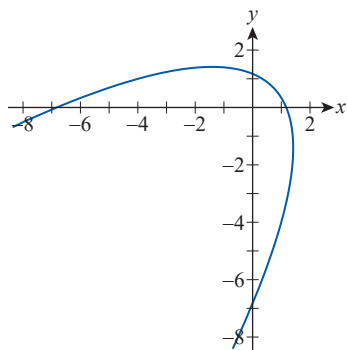
1. C 3. F 5. B 7. Hyperbola; $y = 7/6$
 9. Ellipse; $x = -3$ 11. Hyperbola; $x = 1/3$
 13. Ellipse; $x = 5$ 15. Hyperbola; $x = -6/5$
 17. Parabola; $y = 3/2$ 19. Hyperbola; $x = -4/7$

21. $r = \frac{2}{1 - \cos \theta}$ 23. $r = \frac{3}{1 - 4 \sin \theta}$

25. $r = \frac{3}{1 + \frac{1}{4} \cos \theta}$



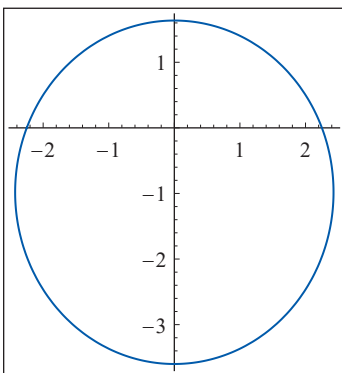
35.



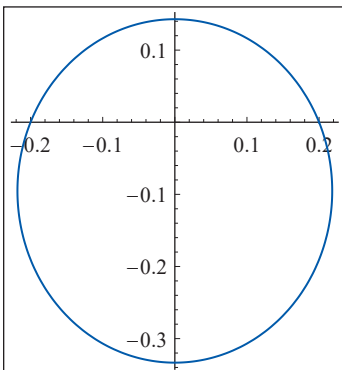
41. $2ed$ 43. $\left(\frac{ed}{e+1}, 0\right)$ and $\left(\frac{ed}{e-1}, 0\right)$

45. $\frac{(x-3.5)^2}{6.25} - \frac{y^2}{6} = 1$

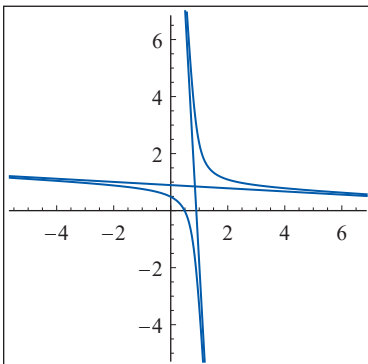
47.



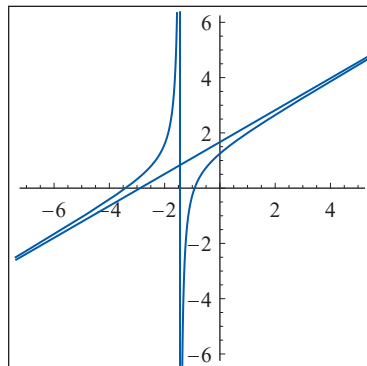
49.



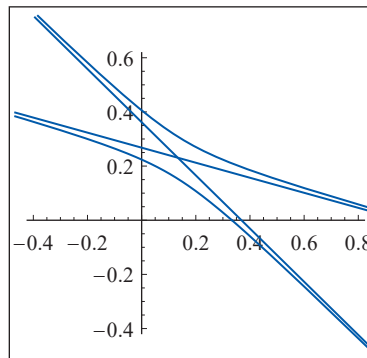
51.



53.

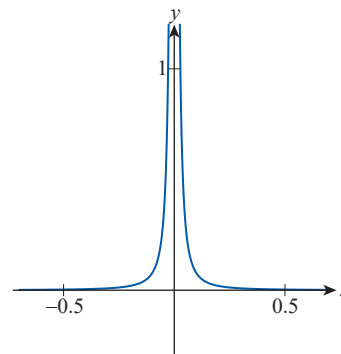


55.

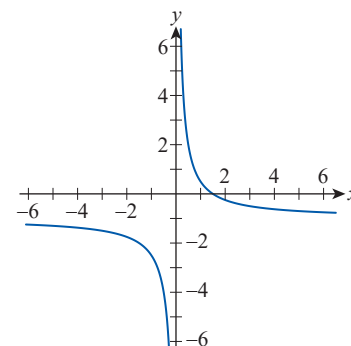


Chapter 9 Review

1. $y = \frac{1}{1296x^2}$



3. $y = \frac{3}{2x} - 1$



5. $x = t, y^2 = t^2 + 4$

7. Sample answer: $x = t, y = \frac{12}{17}t - \frac{100}{17}$

9. Horizontal: $y = -4$; vertical: none

11. $\left. \frac{dy}{dx} \right|_{t=2} = \frac{1}{16}; \left. \frac{d^2y}{dx^2} \right|_{t=2} = -\frac{1}{128}$

13. $\left. \frac{dy}{dx} \right|_{t=1} = -3; \left. \frac{d^2y}{dx^2} \right|_{t=1} = 8$ 15. $t = 0, t = \pm 1/\sqrt{3}$

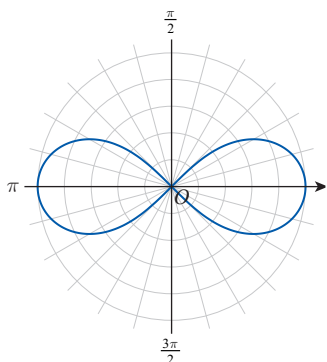
17. $6\sqrt{3}/5$ 19. $76/3$ 21. $61\pi/108$ 23. 81π

25. $(-6.06, -3.5)$

27. $(2\sqrt{61}, 0.88)$ and $(-2\sqrt{61}, 4.02)$

29. $r^2 - 9a r \cos \theta = 0$ 31. $x + y = 4$

33.

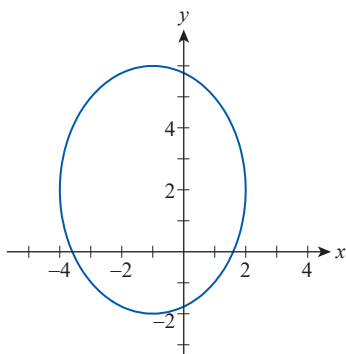


35. $\frac{4\pi}{4 + \pi^2}$

37. Horizontal: $(4/3, \cos^{-1} \sqrt{2/3}), (4/3, \cos^{-1} (-\sqrt{2/3}))$;
vertical: $(2, 0), (0, \pi/2), (2, \pi)$

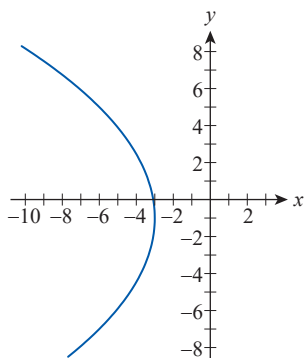
39. $\frac{33\sqrt{3} + 28\pi}{6}$ 41. $4\sqrt{3} + \frac{8\pi}{3}$ 43. 2 45. 16

47.



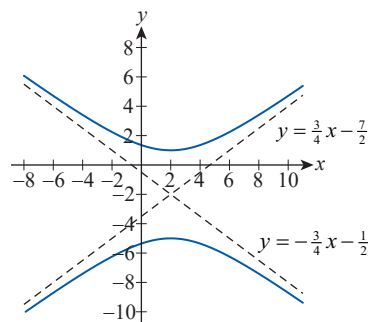
Foci: $(-1, 2 \pm \sqrt{7})$

49.



Focus: $(-6, -1)$; Directrix: $x = 0$

51.



Foci: $(2, 3), (2, -7)$

53. $\frac{(x+1)^2}{9} + \frac{(y-4)^2}{16} = 1$ 55. $\frac{(x-2)^2}{9/4} + \frac{(y+1)^2}{9/4} = 1$

57. $(x+2)^2 = 4(y-3)$ 59. $(y+1)^2 = 2\left(x - \frac{5}{2}\right)$

61. $\frac{(x+1)^2}{4} - \frac{(y+2)^2}{25} = 1$ 63. $\frac{(y-2)^2}{9} - (x-2)^2 = 1$

65. Hyperbola; $y = -5/8$ 67. Parabola; $x = -2/3$

69. Ellipse; $x = -4/3$

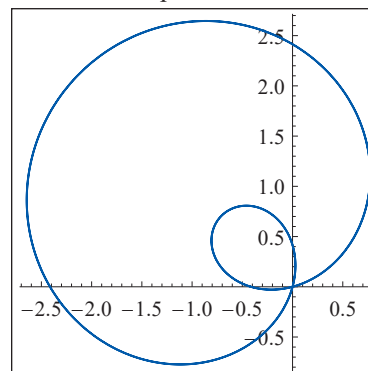
71. $r = \frac{3}{1 - \cos \theta}$ 73. $r = \frac{3}{1 - \frac{1}{5} \sin \theta}$

75. $r = \frac{7}{1 - \sin \theta}$ 77. $1\frac{1}{4}$ inches 81. True

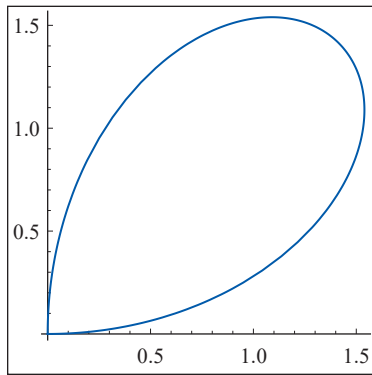
83. False; see Example 1 in Section 9.2.

85. True 87. True 89. True

91. $r = 1 - 2 \cos\left(\theta + \frac{\pi}{4}\right)$; the curve is a rotated limaçon with inner loop.

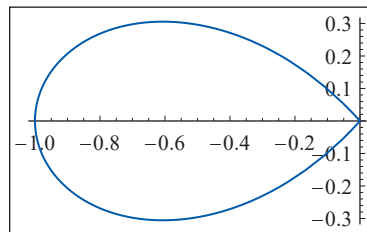


95.


 Horizontal: $\theta = 0$, $\theta = \cos^{-1}(1/\sqrt{3})$;

 vertical: $\theta = \tan^{-1}(\sqrt{2}/2)$, $\theta = \pi/2$

97.



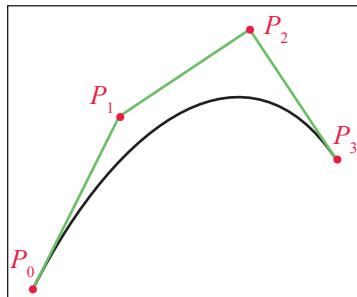
$$A = \frac{17 \cos^{-1}(2/3) - 6\sqrt{5}}{2} \approx 0.441$$

 99. $L \approx 6.8694$ 101. $A \approx 34.3518$

Chapter 9 Conceptual Project

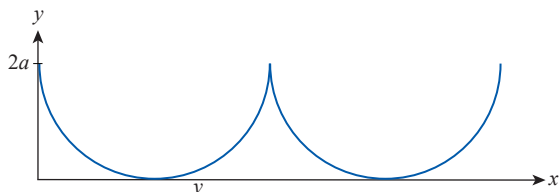
1. $x(t) = (a_1 - a_0)t + a_0$, $y(t) = (b_1 - b_0)t + b_0$, $t \in [0, 1]$

7.

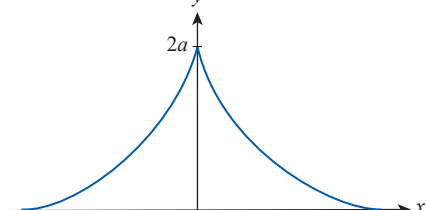


Chapter 9 Application Project

1. a.



b.



c. Period: $\frac{2\pi}{\theta}$; Frequency: $\frac{\theta}{2\pi}$

3. a. At the lowest points of the cycloid (these points are called vertices of the cycloid).

b. $s = 4a \left(1 - \cos \frac{\theta_0}{2} \right)$

Chapter 10

Section 10.1

1. $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}$

3. $3, 3, \frac{3}{2}, \frac{1}{2}, \frac{1}{8}, \frac{1}{40}$ 5. 1, 3, 7, 15, 31, 63

7. 4, 8, 24, 96, 480, 2880 9. $a_n = n^2 + n$

11. $a_n = 10 - n^2$ 15. B 17. E 19. D

 25. Converges to $1/2$ 27. Converges to 0

29. Converges to 0 31. Converges to 0

33. Diverges 35. Converges to 0

37. Converges to 0.5 39. Converges to 0

 41. Converges to 0 43. Converges to $\ln 3$

 45. Converges to 1 47. Converges to $\pi/6$

49. Diverges 51. Converges to 1

53. Diverges 55. Converges to 1

57. Converges to 1 59. Converges to 0

61. Converges to 1 63. Converges to 0

 65. Converges to e^2 67. Converges to e^3

69. Converges to 2 75. The limit is 1.

 77. The limit is 2. 83. $T_n = 2T_{n-1} + 1$, $T_1 = 1$

 99. No; consider $a_n = -1/n$ and $b_n = 1/n$.

101. $a_n = \frac{5}{3} + \frac{1}{3} \left(-\frac{1}{2} \right)^{n-2}$; the limit is $5/3$.

103. True 105. True 107. True

109. False; consider the constant sequence

 $(-2)^2, (-2)^2, (-2)^2, \dots$ that converges to 4, but $-2, -2, -2, \dots$ does not converge to 2.

111. True 113. True

Section 10.2

 1. No. The convergence of a_n to 0 does not imply the convergence of the series (see Example 6).

3. 1, 0, 1, 0, 1 5. $\frac{\sqrt{3}}{2}, \sqrt{3}, \sqrt{3}, \frac{\sqrt{3}}{2}, 0$

9. $25/6$ 11. Diverges 13. $10/17$

15. $40/27$ 17. $25/36$ 19. $x < 2$ or $x > 3$
 21. $41/333$ 23. $169/50$ 25. 2 27. 1
 29. $1/3$ 31. Diverges
 33. Diverges 35. $14/3$ 37. Diverges
 39. $1/6$ 41. Diverges 43. $33/2$
 45. Diverges 47. Diverges 49. Diverges
 55. Answers will vary. $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ are an example.
 77. 150 km
 79. $\sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1}{3}$ 81. $\sum_{n=3}^{\infty} \frac{1}{2^n} = \frac{1}{4}$

Section 10.3

1. Diverges 3. Diverges 5. Converges
 7. Converges 9. Diverges 11. Converges
 13. Converges 15. Converges 17. Diverges
 19. Converges 21. Converges 23. Converges
 25. Converges 27. Diverges 29. Converges
 31. Diverges
 33. The terms of the series are not monotonically decreasing.
 35. The terms of the series are not positive nor monotonically decreasing.
 39. $1.19955 \leq s \leq 1.20566$ 41. $1.06621 \leq s \leq 1.08946$
 43. $2.08997 \leq s \leq 2.13419$ 45. $n = 5; s \approx 1.2026$
 47. $n = 4; s \approx 0.404881426$ 49. $n = 5; s \approx 3.2357$
 51. $n = 40; s \approx 1.0369277553$
 53. $n = 20; s \approx 1.0020083928262$

Section 10.4

1. Converges 3. Diverges 5. Converges
 7. Diverges 9. Converges 11. Converges
 13. Converges 15. Diverges 17. Converges
 19. Converges 21. Converges 23. Converges
 25. Converges 27. Diverges 29. Converges
 31. Converges 33. Diverges 35. Diverges
 37. Converges 39. Converges 41. Converges
 43. Diverges 45. Converges 47. Diverges
 49. Diverges 51. Converges 53. Converges
 55. Converges 57. Diverges 59. Converges
 61. Diverges 63. Diverges 65. Converges

67. Converges 69. Converges 83. True
 85. False; consider $a_n = 1/n$ and $b_n = 1/n^2$.
 87. False; consider $b_n = c_n = 1/n$.
 89. True 91. True

Section 10.5

1. Converges 3. Converges 5. Converges
 7. Converges 9. Converges 11. Converges
 13. Diverges 15. Converges 17. Diverges
 19. Converges 21. Diverges 23. Converges
 25. Converges 27. Diverges 29. Converges
 31. Diverges 33. Diverges 35. Diverges
 37. Diverges 39. Converges 41. Converges
 43. Converges 45. Converges 47. Converges
 49. Diverges 51. Converges 53. Converges
 55. Converges 57. Diverges 59. Converges
 61. Converges 63. Diverges 65. Diverges
 67. Diverges 69. Converges 73. $0 < p < 1$
 75. True 77. False
 79. False; consider Example 3.

Section 10.6

1. Converges by the Alternating Series Test
 3. Diverges by the n^{th} -Term Divergence Test
 5. Converges by the Alternating Series Test
 7. Diverges by the n^{th} -Term Divergence Test
 9. Diverges by the n^{th} -Term Divergence Test
 11. Converges by the Alternating Series Test
 13. Diverges by the n^{th} -Term Divergence Test
 15. Diverges by the n^{th} -Term Divergence Test
 17. Converges by the Alternating Series Test
 19. Diverges by the n^{th} -Term Divergence Test
 21. Converges by the theorem "Absolute Convergence Implies Convergence"
 23. Diverges by the n^{th} -Term Divergence Test
 25. 0.90 27. 0.36788 29. 0.057
 31. Converges conditionally
 33. Diverges 35. Converges conditionally
 37. Converges absolutely
 39. Converges conditionally
 41. Converges absolutely 43. Diverges

45. Converges conditionally
 47. Converges absolutely
 49. Converges conditionally
 51. Converges absolutely
 53. Diverges 55. Diverges 57. Diverges
 59. No; a_n is not monotonic, so the test doesn't apply. The series is actually divergent.
 65. An example is $\sum \frac{(-1)^n}{\sqrt{n}}$.
 67. At least 1000 terms are needed.

Section 10.7

1. $(-1/3, 1/3)$; $R = 1/3$ 3. $[-1/4, 1/4]$; $R = 1/4$
 5. $[-1, 1]$; $R = 1$ 7. $\{0\}$; $R = 0$
 9. $(-\infty, \infty)$; $R = \infty$ 11. $(-5, -1)$; $R = 2$
 13. $\{0\}$; $R = 0$ 15. $[3, 5]$; $R = 1$
 17. $\{5\}$; $R = 0$ 19. $[-4, 0]$; $R = 2$
 21. $[0, 2]$; $R = 1$ 23. $(-\infty, \infty)$; $R = \infty$
 25. $(2/3, 4/3)$; $R = 1/3$ 27. $(-1, 1)$; $R = 1$
 29. $[-7/3, -5/3]$; $R = 1/3$
 31. $(2, 4)$; $R = 1$ 33. $(0, 2/3)$; $f(x) = \frac{1}{2-3x}$
 35. $(-3, 3)$; $f(x) = \frac{3}{3+x}$ 37. $(1, 5)$; $f(x) = \frac{2}{5-x}$
 39. $(3/5, 9/5)$; $f(x) = \frac{9}{9-(5x-6)^2}$
 41. $\sum_{n=0}^{\infty} 2^n x^n$; $R = 1/2$ 43. $\sum_{n=0}^{\infty} (-1)^n x^{2n}$; $R = 1$
 45. $\sum_{n=0}^{\infty} \frac{4^n x^n}{3^n}$; $R = 3/4$ 47. $\sum_{n=0}^{\infty} [(-1)^n + 2] x^n$; $R = 1$
 49. $\sum_{n=0}^{\infty} (-1)^n (x-1)^n$; $R = 1$
 51. $\sum_{n=1}^{\infty} (-1)^n n(x-1)^{n-1}$; $R = 1$ 53. 10 terms
 55. $\sum_{n=2}^{\infty} n(n-1)x^{n-2} = \frac{2}{(1-x)^3}$;
 $\sum_{n=3}^{\infty} n(n-1)(n-2)x^{n-3} = \frac{6}{(1-x)^4}$
 57. $\frac{18}{(2-3x)^3}$ on $(0, 2/3)$
 59. $\sum_{n=1}^{\infty} (-1)^{n+1} (2n)x^{2n-1}$; $R = 1$

65. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$; this is the series expansion of $\cos 2x$ about 0.
 67. False; consider Exercise 16.
 69. False; consider Exercise 62.

Section 10.8

1. $4 + 2x - \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{24}$
 3. $f(x) = -7 + 3x + 5x^2 - x^3 + 2x^4$; $R = \infty$
 5. $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$; $R = \infty$ 7. $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^n}{5^{n+1}}$; $R = 5$
 9. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n(x-2)^{n-1}}{2^{n+1}}$; $R = 2$
 11. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$; $R = \infty$
 13. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n+1}}{(2n+1)!}$; $R = \infty$
 15. $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$; $R = \infty$ 17. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$; $R = \infty$
 19. $-x - x^2 + \sum_{n=0}^{\infty} \frac{(2n^2 + 6n + 3)x^{n+3}}{(n+2)!}$; $R = \infty$
 21. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{n+1}$; $R = 1$
 23. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n(x-3)^{n-1}}{3^{n+1}}$; $R = 3$
 25. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} - 3^{n+1}}{6^{n+1}} x^n$; $R = 2$
 27. $\sum_{n=0}^{\infty} x^{2n+2}$; $R = 1$ 29. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+1}}{(2n)!}$; $R = \infty$
 31. $\frac{\pi}{4} + \frac{x-1}{2} - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3 - \frac{1}{40}(x-1)^5 + \dots$
 33. $e - \frac{ex^2}{2} + \frac{ex^4}{6} - \frac{31ex^6}{720} + \frac{379ex^8}{40,320} - \dots$
 35. $2 + (\ln 2)(x-1) + (\ln 2)^2(x-1)^2 + \frac{(\ln 2)^3(x-1)^3}{3} + \frac{(\ln 2)^4(x-1)^4}{12} + \dots$
 37. $1 + 2x + \frac{5x^2}{2} + \frac{8x^3}{3} + \frac{65x^4}{24} + \dots$
 39. $x - 2x^2 + \frac{11x^3}{6} - x^4 + \frac{41x^5}{120} - \dots$

$$41. x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \frac{62x^9}{2835} - \dots$$

$$43. x^2 - \frac{x^3}{2} + \frac{x^4}{6} - \frac{x^5}{6} + \frac{11x^6}{72} - \dots$$

$$45. -x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{3x^5}{40} - \frac{x^6}{16} - \dots$$

$$47. 1 - 2x + \left(1 - \frac{\pi^2}{2}\right)x^2 + \pi^2 x^3 + \frac{\pi^4 - 12\pi^2}{24}x^4 - \dots$$

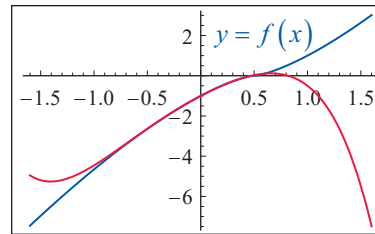
$$49. 0.36786; 8 \text{ terms} \quad 51. 0.8452; 5 \text{ terms}$$

$$53. 0.3102682; 4 \text{ terms} \quad 55. 0.304; 3 \text{ terms}$$

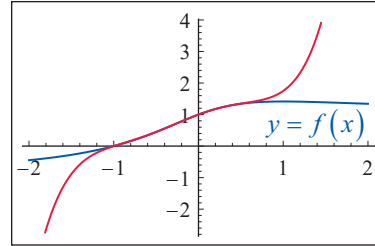
$$57. 0.922; 6 \text{ terms} \quad 61. 2 \quad 63. 1$$

$$67. 4 \quad 71. 0 \text{ and } -162!, \text{ respectively}$$

47.



49.



Section 10.9

$$1. i \quad 3. \frac{5e^2}{2} + i\frac{5\sqrt{3}e^2}{2}$$

$$5. \frac{2}{3}\sin 2 \quad 7. -4e^{i\pi/3} \quad 9. -4$$

$$13. 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x^{10}$$

$$15. 1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \frac{5x^4}{128} - \frac{7x^5}{256} - \dots$$

$$17. 1 + \frac{x^2}{3} - \frac{x^4}{9} + \frac{5x^6}{81} - \frac{10x^8}{243} + \frac{22x^{10}}{729} + \dots$$

$$19. 1 + \frac{4x^2}{3} + \frac{20x^4}{9} + \frac{320x^6}{81} + \frac{1760x^8}{243} + \frac{9856x^{10}}{729} + \dots$$

$$21. 5 - \frac{5x}{2} + \frac{25x^2}{8} - \frac{75x^3}{16} + \frac{975x^4}{128} - \frac{3315x^5}{256} + \dots$$

$$23. \sin^{-1} x = \sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2 4^n (2n+1)} x^{2n+1}; R=1$$

$$25. \sinh^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(n!)^2 4^n (2n+1)} x^{2n+1}; R=1$$

$$31. \text{Approx. } 5.777 \text{ miles}$$

$$35. \text{Approx. } 4.211 \times 10^7 \text{ m/s}$$

$$39. y = a \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n}}{(2n)!} + \frac{1}{3} b \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!}$$

$$= a \cos 3x + \frac{1}{3} b \sin 3x$$

$$41. y = a \sum_{n=0}^{\infty} x^{2n} + b \sum_{n=0}^{\infty} x^{2n+1} = \frac{a}{1-x^2} + \frac{bx}{1-x^2}$$

$$45. a_0 = 1/2; a_k = 0; b_k = \frac{(-1)^k - 1}{k\pi} \quad (k \geq 1)$$

Chapter 10 Review

$$1. 1, 0, \frac{1}{25}, -\frac{1}{125}, \frac{81}{83,521} \quad 3. 0, 2, 8, 26, 80$$

$$7. \text{Diverges} \quad 9. \text{Converges to } e^6$$

$$13. 219/999 \quad 15. \text{The limit is } 3.$$

$$19. \text{a. } d_1 = d, d_n = \frac{1}{2}d_{n-1} + d, n \geq 2 \quad \text{b. } \lim_{n \rightarrow \infty} d_n = 2d$$

$$23. 3 \quad 25. 85/4 \quad 27. \text{Diverges}$$

$$29. \text{Converges} \quad 31. \text{Converges} \quad 33. n=5; s \approx 2.616$$

$$35. \text{Diverges} \quad 37. \text{Converges} \quad 39. \text{Converges}$$

$$41. \text{Diverges} \quad 43. \text{Converges} \quad 45. \text{Converges}$$

$$47. \text{Converges} \quad 49. \text{Converges}$$

$$51. \text{Converges by Alternating Series Test}$$

$$53. \text{Diverges by } n^{\text{th}}\text{-Term Divergence Test}$$

$$55. 0.9498; \text{three terms were needed. According to the } n^{\text{th}}\text{-remainder estimate, five terms are sufficient.}$$

$$57. \text{Converges absolutely} \quad 59. \text{Converges absolutely}$$

$$61. \text{Converges absolutely}$$

$$63. (-1/2, 1/2) \quad 65. (-\infty, \infty) \quad 67. [-4, 2)$$

$$69. (1, 2); f(x) = \frac{1}{4-2x} \quad 71. \sum_{n=0}^{\infty} (-1)^n (3x)^n; R=1/3$$

$$73. \frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n \frac{(x-2)^n}{2^{n+1}},$$

$$\frac{1}{x^3} = \sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)(x-2)^n}{2^{n+4}}; R=2$$

$$75. \sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{6n}}{(2n)!}; R=\infty$$

$$77. \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x-3)^n}{n3^n}; R=3$$

79. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}; R = \infty$

81. $\frac{1}{3} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2(x-1)^n}{3^{n+1}}; R = 3$

83. $\sqrt{2} - \sqrt{2} \left(x - \frac{\pi}{4} \right) + \frac{3}{\sqrt{2}} \left(x - \frac{\pi}{4} \right)^2 - \frac{11}{3\sqrt{2}} \left(x - \frac{\pi}{4} \right)^3 + \dots$

85. $1 - x + \frac{x^2}{2} - \frac{x^3}{2} + \frac{13x^4}{24} - \dots$

87. 0.540302; 5 terms

89. $\sqrt[4]{1.2} \approx 1.04663$; 5 terms 91. -1

93. The sum is 2.

95. $1 - \frac{x^2}{2} + \frac{3x^4}{8} - \frac{5x^6}{16} + \frac{35x^8}{128} - \frac{63x^{10}}{256} + \dots$

97. $1 + x^3 - \frac{x^6}{2} + \frac{x^9}{2} - \frac{5x^{12}}{8} + \frac{7x^{15}}{8} - \dots$

99. $y = a \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!} + \frac{1}{2} b \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$
 $= a \cos 2x + \frac{1}{2} b \sin 2x$

101. $\sum_{n=0}^{\infty} \frac{(-1)^n 2x^n}{n!} = 2e^{-x}$, this is the Maclaurin series of $2e^{-x}$.

113. False; consider $a_n = 1/n$.

115. False; consider $a_n = 1$ (for all n).

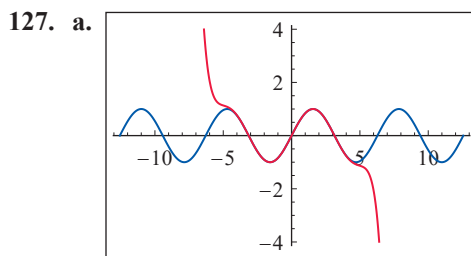
117. False; consider $a_n = 1/n$.

119. False; consider $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

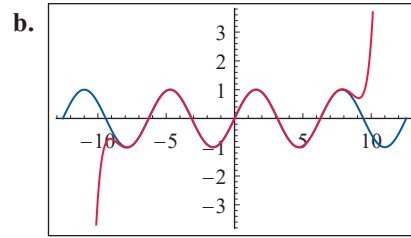
121. False; consider $a_n = \frac{(-1)^n}{\sqrt{n}}$.

123. False; the series $\sum_{n=0}^{\infty} (n!) x^n$ converges only at $x = 0$.

125. a. 91,380 terms b. Approx. 15.0859



The approximation seems acceptable on the interval $(-3\pi/2, 3\pi/2)$.



The approximation seems acceptable on the interval $(-5\pi/2, 5\pi/2)$.

Chapter 10 Conceptual Project

5. $\gamma \approx 0.5772156649$

Chapter 10 Application Project

1. a. $C'(t) = -kC(t); C(0) = D$

b. $C(t) = De^{-kt}$

3. $C_n = D(e^{-kp} + e^{-2kp} + \dots + e^{-nkp}) = De^{-kp} \cdot \frac{1 - e^{-nkp}}{1 - e^{-kp}}$

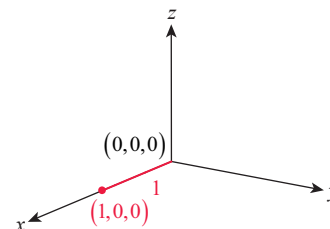
5. $p = \frac{1}{k} \ln \frac{m_t}{m_e}$

Chapter 11

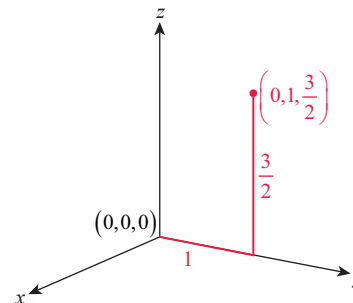
Section 11.1

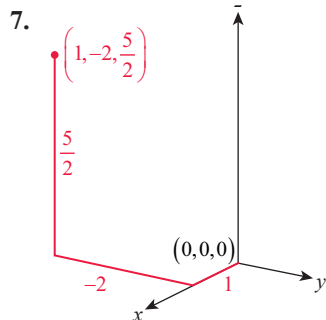
1. Out of the page

3.



5.





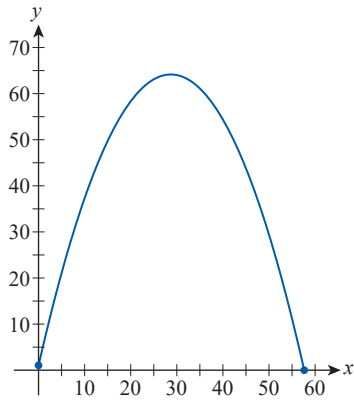
9. The third coordinate is 0.
 11. The x - and z -coordinates are 0.
 13. The x -coordinate is negative, the y - and z -coordinates are 0.
 15. The y -coordinate is -1 .
 17. 3 19. 0 21. $(5, 3.2, 0)$ 23. $(0, 0, 0)$
 25. $(1.3, 4.5, -10)$
 27. A plane parallel to the xz -plane, intersecting the y -axis at $(0, 1, 0)$
 29. The line $y = 1 - x$ in the xy -plane
 31. The union of the three coordinate planes
 33. A line in the plane $x = 5$ and through the point $(5, 0, -1)$, having a slope of 2 in the y -direction
 35. A “doubly infinite staircase,” any cross-section perpendicular to the y -axis being the graph of the greatest integer function
 37. $(2.5, 2.5, 2.5)$ 39. $(-1, -2, 5)$ 41. $(\sqrt{3}, 1, 4)$
 43. A 45. B
 47. The exterior (i.e., the “outside”) of the circular cylinder $x^2 + y^2 = 1$
 49. A sphere of radius 3, centered at $(3, 0, 0)$ (note that the yz -plane is tangent to this sphere)
 51. An infinite parabolic surface resulting from all vertical translations of the parabola $y = 1 - x^2$ (note that the z -coordinate is irrelevant)
 53. The parabola $x = z^2 + 2$, in the plane $y = 1$
 55. The union of the three coordinate planes
 57. The upper hemisphere of radius $3\sqrt{5}$, centered at $(-2, 4, 5)$
 59. The point $(0, -4, -2)$ 61. $x = 1$
 63. $z = e$ 65. $y = -1, z = 2$
 67. $x^2 + (y - 2)^2 + z^2 = 4$
 69. $(x - 1)^2 + y^2 + (z + 2)^2 = 49$
 71. $x^2 + (y + 4)^2 + (z - 1)^2 = 9, x = 0$

73. $(x + 1)^2 + (y - 4)^2 + (z + 3)^2 = 1$
 75. $(x - 3)^2 + (y - 2)^2 + (z + 1)^2 = 9, x, y, z > 0$
 77. $x = 3$ 79. $\sqrt{129}$ 81. 5 83. a 85. d

Section 11.2

1. Scalar 3. Vector 5. Vector 7. Scalar
 9. Not a vector 11. Vector 13. Vector
 15. $2\overrightarrow{CM}$ 17. $\overrightarrow{MC}$ 19. $\overrightarrow{CM}$
 21.
 23.
 25. $\langle 12, -3 \rangle$ 27. $\langle 16, 33 \rangle$
 29. $\langle \frac{3}{2}, -5 \rangle$ 31. $(10, -2, 3)$ 33. $(2, 4, -6)$
 35. $\mathbf{v} = \langle -1, 5, 0 \rangle$ 37. Collinear
 39. $\mathbf{w} = \frac{1}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}$ 41. $\mathbf{w} = 4\mathbf{u} - \frac{1}{4}\mathbf{v}$ 43. $2/5$
 45. $3/2$ 47. $17\sqrt{2}$ 49. $\sqrt{293}/10$
 51. Not equal 53. Equal 55. Not equal
 57. $\overrightarrow{PQ} = \langle 0, 0, 0 \rangle; |\overrightarrow{PQ}| = 0$
 59. $\overrightarrow{PQ} = \langle 3, -3, -2 \rangle; |\overrightarrow{PQ}| = \sqrt{22}$
 61. $\overrightarrow{PQ} = \langle -\frac{2}{3}, -3, -\frac{\sqrt{5}}{3} \rangle; |\overrightarrow{PQ}| = \sqrt{10}$
 63. Not a parallelogram 65. Parallelogram
 67. Not a parallelogram 69. $(1, 2, -9)$
 71. $(0, 4, 4)$ and $(3, 7, 1)$ 73. $\mathbf{u} = \frac{2}{\sqrt{85}}\mathbf{i} + \frac{9}{\sqrt{85}}\mathbf{j}$
 75. $\mathbf{u} = -\frac{4}{3\sqrt{5}}\mathbf{i} - \frac{\sqrt{5}}{3}\mathbf{j} + \frac{2}{3\sqrt{5}}\mathbf{k}$
 77. $\mathbf{u} = \frac{2}{\sqrt{14}}\mathbf{i} - \frac{3}{\sqrt{14}}\mathbf{j} - \frac{1}{\sqrt{14}}\mathbf{k}$
 81. $a = \pm \frac{5\sqrt{11}}{22}$

85. $\mathbf{v}(t) = \langle 8, -9.8t + 35.2 \rangle$



87. Approx. 544.45 mph

89. Ground speed is approx. 259.07 mph in the direction of $\langle 1, 3.31, 1.61 \rangle$.

95. $\langle -1, -2, 6, 11 \rangle$ 97. $\langle 2, 1, -7, 10, -\frac{25}{3}, -2 \rangle$

Section 11.3

1. 4 3. -1 5. -10

7. -3 9. 12 11. 2 13. 0

15. The vectors are perpendicular.

17. 61° 19. Approx. 1.68 rad or 96.50°

21. Approx. 1.54 rad or 87.95°

29. Parallel 31. Neither 33. Orthogonal

35. Orthogonal 37. Orthogonal 39. Orthogonal

41. b and c 43. a

45. Approx. 1.75 rad $\approx 100.4^\circ$ and 1.39 rad $\approx 79.6^\circ$, respectively

47. Any $s \in \mathbb{R}$ 49. $s = 0$ or $t = 0$

55. $(y-b) = m(x-a)$ 57. $\left\langle \frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}} \right\rangle$

59. $n_1(x-a) + n_2(y-b) + n_3(z-c) = 0$

61. $\langle 1, -4, -2 \rangle$ 63. Approx. 71.41°

65. $\alpha \approx 64.12^\circ$, $\beta \approx 102.6^\circ$, $\gamma \approx 29.21^\circ$

67. $\alpha \approx 83.42^\circ$, $\beta \approx 156.49^\circ$, $\gamma \approx 112.46^\circ$

69. Precisely when $\theta = 0$ or $\theta = \pi$, or when at least one of $\mathbf{u}$ or $\mathbf{v}$ is the zero vector

71. $\mathbf{v} = 2\mathbf{u}$ or $\mathbf{v} = -2\mathbf{u}$

75. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{40}{61}, -\frac{48}{61} \right\rangle$; $\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{82}{61}, \frac{205}{183} \right\rangle$

77. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{36}{179}, -\frac{216}{179}, \frac{60}{179} \right\rangle$;

$$\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{608}{537}, \frac{37}{179}, \frac{58}{895} \right\rangle$$

79. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{-12st^2 - 4t}{20t^2 + 1}, \frac{-24st^2 - 8t}{20t^2 + 1}, \frac{6st + 2}{20t^2 + 1} \right\rangle$;

$$\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle -5s + \frac{12st^2 + 4t}{20t^2 + 1}, s + \frac{24st^2 + 8t}{20t^2 + 1}, 2 - \frac{6st + 2}{20t^2 + 1} \right\rangle$$

81. $\text{proj}_{\mathbf{u}}\mathbf{v} = -\frac{64}{85}\mathbf{i} + \frac{8}{17}\mathbf{j} + \frac{72}{85}\mathbf{k}$;

$$\mathbf{u} - \text{proj}_{\mathbf{u}}\mathbf{v} = \frac{213}{170}\mathbf{i} + \frac{9}{17}\mathbf{j} + \frac{209}{255}\mathbf{k}$$

83. $\text{proj}_{\mathbf{u}}\mathbf{v} = -\frac{1}{4}\mathbf{v}$; $\mathbf{u} - \text{proj}_{\mathbf{u}}\mathbf{v} = \mathbf{u} + \frac{1}{4}\mathbf{v}$

85. Answers will vary.

91. $7/\sqrt{13}$ 93. $6/\sqrt{30}$ 95. 44 ft·lb 97. $1000\sqrt{2}$ kJ

99. a. Approx. 1856.7 J b. Approx. 107.2 N

101. Approx. 0.955 rad $\approx 54.74^\circ$

103. $\mathbf{p} \cdot \mathbf{s}$ is the total revenue during a given cycle.

105. True

107. False; the dot product of a vector and a scalar is undefined.

109. True 111. True 113. True

115. True 117. True

119. False; let $\mathbf{u} \perp \mathbf{w}$ and $\mathbf{v} = 2\mathbf{u}$.

Section 11.4

1. $\langle 4, 3, 7 \rangle$ 3. $\langle 3, 11, 20 \rangle$ 5. $\left\langle -\frac{3}{4}, \frac{1}{12}, -\frac{5}{9} \right\rangle$

15. $\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$ 17. $\langle 7, 26, 24 \rangle$

19. $\pm \left\langle \frac{2}{\sqrt{17}}, \frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}} \right\rangle$ 21. $\pm \left\langle \frac{3}{7}, \frac{6}{7}, \frac{2}{7} \right\rangle$

23. $\langle -7, -5, 9 \rangle$ 25. $\left\langle \frac{11}{4}, -\frac{19}{4}, 3 \right\rangle$

29. Collinear 31. Collinear

33. $19/2$ 35. 45 37. $\sqrt{70}$ 39. $\sqrt{13/2}$

41. $\langle 0, 5, 4 \rangle$ 43. $\langle 22, -4, 5 \rangle$ 45. Not possible

47. $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$ 49. 1 51. 20

53. Three vectors are coplanar if and only if their triple scalar product is zero.

55. Coplanar 57. Coplanar 59. $15\sqrt{3}/2$ lb·ft

61. 0.024 N
 67. $11x - 19y + 12z = 25/2$
 71. $7\sqrt{13}/13$ 73. $\sqrt{6/5}$
 79. Nonsense 81. Vector 83. True
 85. False; the cross product of a vector and a scalar is undefined.
 87. False; the cross product of a vector and a scalar is undefined.
 89. False; $\mathbf{u} \times \mathbf{u} = \mathbf{0}$. 91. True 93. True

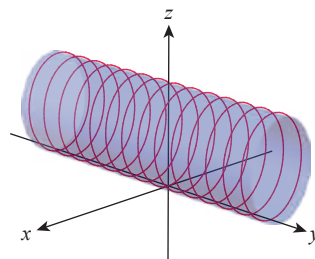
Section 11.5

1. A and C 3. A and B
 5. $\mathbf{r}(t) = \langle 1+3t, 2+2t, 3+t \rangle$;
 $x = 1+3t, y = 2+2t, z = 3+t$
 7. $\mathbf{r}(t) = \langle 4t, -5+15t, 8+19t \rangle$;
 $x = 4t, y = -5+15t, z = 8+19t$
 9. $\mathbf{r}(t) = \langle -9+4t, 11, -6+3t \rangle$;
 $x = -9+4t, y = 11, z = -6+3t$
 11. $\mathbf{r}(t) = \langle 16-53t, 21+22t, 28-32t \rangle$;
 $x = 16-53t, y = 21+22t, z = 28-32t$
 13. Answers will vary. 15. Answers will vary.
 17. Answers will vary. 19. Answers will vary.
 21. $x = 1+2t, y = 2, z = 3-2t, 0 \leq t \leq 1$
 23. $x = -3+8t, y = \frac{2}{3} - \frac{8}{3}t, z = -11+4t, 0 \leq t \leq 1$
 25. $(3, 6, 0), (-3, 0, -18)$, and $(0, 3, -9)$
 27. $\left(\frac{17}{4}, \frac{39}{8}, 0\right), \left(14, 0, -\frac{13}{4}\right)$, and $\left(0, 7, \frac{17}{12}\right)$
 29. $(0, -1, 0), \left(2, 0, \frac{1}{2}\right)$, and $(0, -1, 0)$
 31. $(0, 2, 7), \left(\frac{6}{7}, 0, \frac{37}{7}\right)$, and $\left(\frac{7}{2}, -\frac{37}{6}, 0\right)$
 33. $(1, -2, 1)$ 35. $(2, -3, -1)$ 37. Not possible
 39. $5x - 4y - z = 1$;
 $\{(x, y, -1+5x-4y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$
 41. $6x + 9y + 10z = 54$;
 $\{(x, y, 5.4 - 0.6x - 0.9y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$
 43. $3x + 6y - 2z = 6$ 45. $6x + 4y - z = 14$
 47. $8x - 82y + 15z = 23$
 49. $x = t - 3, y = 4t - 1, z = 4 - 3t$
 51. $x = -1 + t, y = -1 + t, z = -5 + 3t$

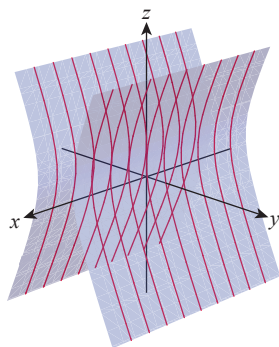
53. $\frac{x-1}{3} = \frac{y-2}{2} = z-3$ 55. $\frac{x}{4} = \frac{y+5}{15} = \frac{z-8}{19}$
 57. $3x - 2y + 5z = 17$ 59. $x - 4y - z = 8$
 61. $x + 2y - 3z = 19$ 63. $8x + 19y + z = 15$
 65. $7x + y + 4z = 2$ 67. $\left(30, \frac{74}{5}, \frac{48}{5}\right)$
 69. $(-3, 4, -8)$
 71. $x = \frac{5}{7} + 2t, y = \frac{3}{7} - 3t, z = -7t$
 73. $x = -3 - 2t, y = 8 + 10t, z = 2t$
 75. $\frac{1}{a}x + \frac{1}{b}y + \frac{1}{c}z = 1$
 77. $1/\sqrt{42}$ 79. 4 81. $5/3$ 83. $6/\sqrt{29}$
 85. $2\sqrt{6/11}$ 87. $2\sqrt{3/7}$ 89. True
 91. False; the dot product of the normal vectors has to be zero, which is not equivalent to the condition described in this statement.
 93. False; they could be skew lines.
 95. False; they could be skew lines.
 97. True 99. True 101. True
 103. False; the line could lie in the plane.
 105. False; any vector normal to P has to be perpendicular to the xz -plane, while the given vector actually lies in the xz -plane.
 107. False; in fact, y is the only variable with a (potentially) nonzero coefficient.
 109. False; $y = 0$ is the equation of the xz -plane. The only point in common with the y -axis is the origin.
 111. True 113. True 115. True

Section 11.6

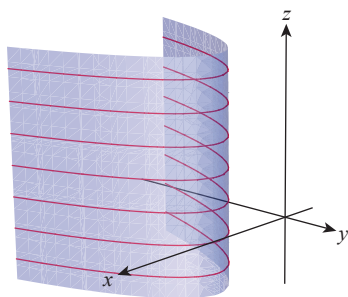
1. Parabolic cylinder; C 3. Hyperbolic paraboloid; A
 5. Ellipsoid; B 7. Hyperboloid of two sheets; F
 9.



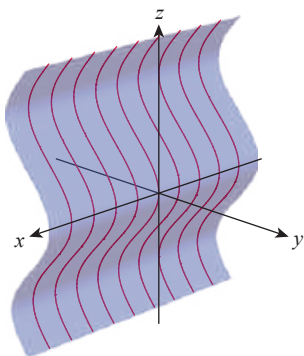
11.



13.



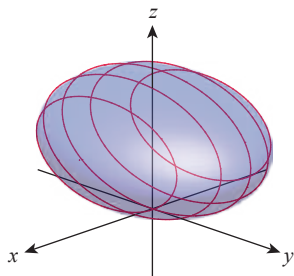
15.



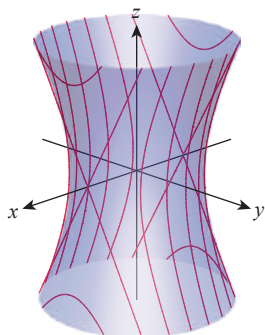
17. Length of major axis: $4\sqrt{2}$;
length of minor axis: 4; foci: $(\pm 2, 0, 4)$

19. Vertex: $(1, 0, 1/2)$; focus: $(1, 0, 3/4)$

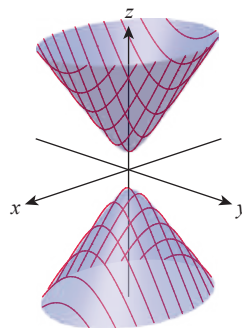
21. Ellipsoid



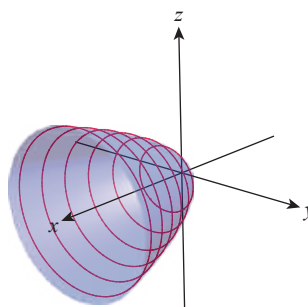
23. Hyperboloid of one sheet



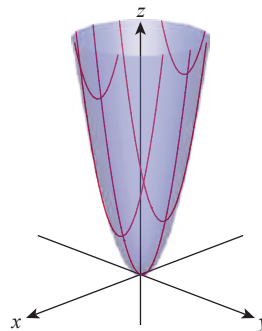
25. Hyperboloid of two sheets



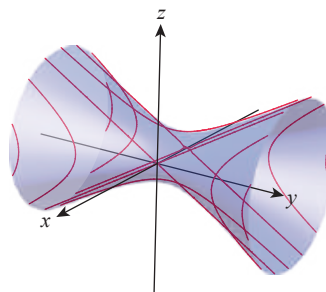
27. Elliptic paraboloid

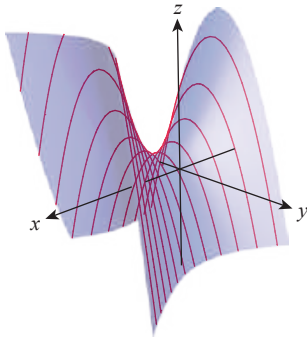


29. Elliptic paraboloid (horizontal cross-sections are circles)



31. Hyperboloid of one sheet



33. Hyperbolic paraboloid

35. $2x^2 = z$ **37.** $y^2 = 5z$

39. $x^2 + z^2 = 2y$ **41.** $z = x^2 + y^2 + 2$

43. $\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2} = 1$ **45.** $x^2 + y^2 = \frac{9}{4z^2}$

47. $\frac{x^2}{6378^2} + \frac{y^2}{6378^2} + \frac{z^2}{6357^2} = 1$

51. False; $a = b = c$ is possible in the equation of an ellipsoid.**53.** True **55.** False; consider a sphere.**Chapter 11 Review**

1. 2 **3.** $\sqrt{2}/2$ **5.** $4\sqrt{2}/3$

7. The exterior of the sphere of radius 1, centered at $(0, 2, 0)$ **9.** The interior of the elliptic cylinder $x^2 + 4z^2 = 2$ **11.** The interior of the hyperboloid (of one sheet)
 $\frac{x^2}{8} + \frac{y^2}{4} - \frac{z^2}{2} = 1$ (including the hyperboloid itself)

13. $(9, -9, 1)$ **15.** $\mathbf{v} = \left\langle -\frac{7}{2}, 1, \frac{1}{2} \right\rangle$

17. Yes, they are collinear.

19. $\overrightarrow{PQ} = \left\langle 3\sqrt{3}/2, -1, 2 \right\rangle$; $|\overrightarrow{PQ}| = \sqrt{47}/2$

21. $\left\langle -6, -\frac{5}{2}, \frac{23}{4} \right\rangle$ **23.** $\mathbf{u} = \left\langle \frac{12}{13}, -\frac{3}{13}, \frac{4}{13} \right\rangle$

25. -32 **27.** $\pi/3 = 60^\circ$

29. **a.** $s = \sqrt{2}$ **b.** $s = -\frac{2\sqrt{2}}{9}$

31. Approx. 78.91°

33. $\alpha \approx 52.24^\circ$, $\beta \approx 45^\circ$, $\gamma \approx 69.3^\circ$

35. $\text{proj}_{\mathbf{v}} \mathbf{u} = \left\langle \frac{12}{11}, -\frac{30}{11}, \frac{10}{11} \right\rangle$; $\mathbf{u} - \text{proj}_{\mathbf{v}} \mathbf{u} = \left\langle \frac{-25}{33}, -\frac{3}{11}, \frac{1}{11} \right\rangle$

37. $\text{proj}_{\mathbf{v}} \mathbf{u} = -\frac{4}{9}\mathbf{i} - \frac{8}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$; $\mathbf{u} - \text{proj}_{\mathbf{v}} \mathbf{u} = \frac{13}{9}\mathbf{i} - \frac{10}{9}\mathbf{j} - \frac{7}{18}\mathbf{k}$

39. 19.7 J **41.** $\left\langle 2, \frac{7}{2}, \frac{15}{2} \right\rangle$

43. $\pm \left\langle 0, \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$ **45.** $\langle 29, 2, 36 \rangle$

47. Collinear **49.** $11/\sqrt{2}$

51. Not possible **53.** $\left\langle -\frac{13}{2}, -\frac{17}{2}, -4 \right\rangle$ **55.** 4

57. $x = \frac{5}{2} + 8t$, $y = 9 + 2t$, $z = -3t$

59. $x = 4 + 4t$, $y = -\frac{3}{2} + 7t$, $z = 7 - 4t$, $0 \leq t \leq \frac{1}{2}$

61. $(-4, 0, 1)$ **63.** Not possible

65. $\left\{ \left(x, y, \frac{37}{6} - \frac{3}{4}x + \frac{1}{6}y \right) \mid x \in \mathbb{R}, y \in \mathbb{R} \right\}$

67. $2x - 5y - 14z + 11 = 0$

69. $x = t - 2$, $y = 2 + 3t$, $z = \frac{1}{2} - 2t$

71. $z - \sqrt{3}y = \sqrt{3}$ **73.** $(2, 3, 2)$

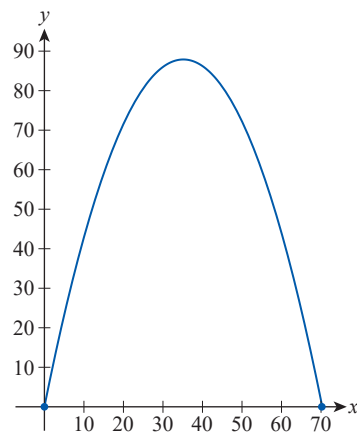
75. $x = 1 + 2t$, $y = 6 - 11t$, $z = -7t$

77. $3x - 2y - 2z + 1 = 0$

79. $56/\sqrt{59}$ **81.** Elliptic cylinder; G

83. Plane; D**85.** Hyperboloid of two sheets; A**87.** Hyperboloid of one sheet; F**89.** Hyperbolic paraboloid**91.** Hyperboloid of one sheet**93.** Hyperboloid of two sheets

95. $\mathbf{v}(t) = \langle 15, 75 - 32t \rangle$



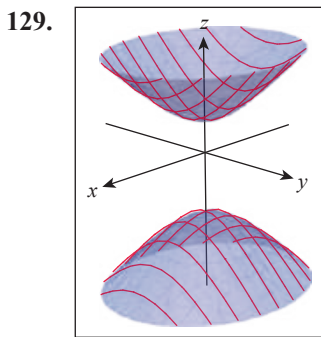
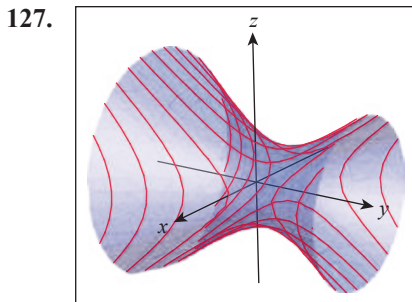
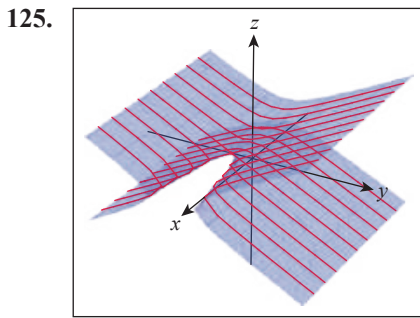
97. Approx. 166.96 mph **99.** $1080\sqrt{3} \approx 1870.61$ ft · lb

101. Approx. 15 N · m **103.** Approx. $6.72 \cdot 10^{-16}$ N

105. True **107.** True**109.** False; torque is perpendicular to both the force vector and the radius from the axis of rotation.**111.** True **113.** True

115. False; $\mathbf{u}$ is parallel with $\mathbf{v} \times \mathbf{w}$.

117. True 119. True



Chapter 11 Application Project

1. $y^2 + z^2 = R^2$

3. b. $v(r) = \frac{1}{2\eta} \cdot \frac{dP}{dx} \cdot \frac{r^2}{2} + C$ 5. $x = -\frac{10}{9}y^2$

Chapter 12

Section 12.1

1. $\text{Dom} = (-\infty, 0) \cup (0, \infty)$

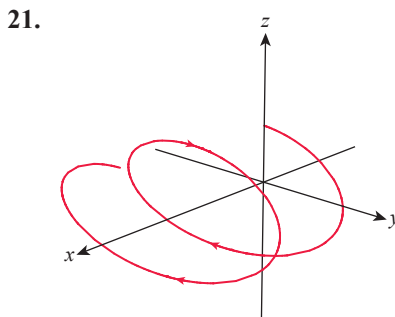
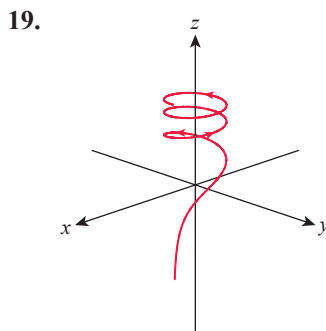
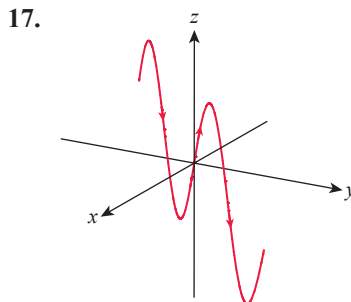
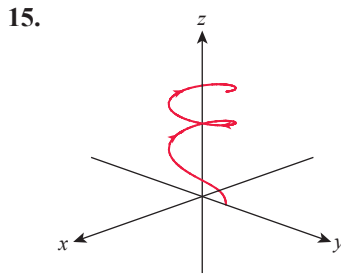
a. $\frac{1}{4}\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ b. $\frac{1}{25}\mathbf{i} - 10\mathbf{j} + 5\mathbf{k}$

3. $\text{Dom} = (0, 3)$

a. Undefined b. $\frac{\sqrt{2}}{4}\mathbf{i} - \mathbf{j}$

5. $\mathbf{u}(t) = -2t$; no, it is a scalar-valued function.

7. C 9. A 11. E 13. D



23. $\mathbf{r}(t) = \left\langle \sqrt{3} \cos t, \sqrt{2} \sin t, 1 - \frac{3\sqrt{3}}{2} \cos t \right\rangle, t \in [0, 2\pi]$

25. $\mathbf{r}(t) = \langle \cos t, \sin t, 2 \cos^2 t - \sin^2 t \rangle, t \in [0, 2\pi]$

27. $\mathbf{r}(t) = \left\langle t^2, t, \frac{2t^4 + t^2}{2} \right\rangle$

29. $\mathbf{r}(t) = \left\langle \frac{t^2}{2}, t, \frac{\sqrt{-3t^4 - 16t^2 + 192}}{2\sqrt{6}} \right\rangle$

31. The limit does not exist.

33. Yes, the limit is $\langle 0, 0, 1 \rangle$.

35. The limit does not exist. 37. No discontinuities

39. $t = 0$ 47. $\mathbf{r}'(t) = -3t^2\mathbf{i} + t^4\mathbf{k}$

49. $\mathbf{r}'(t) = \frac{1}{t}\mathbf{i} + \csc t \cot t \mathbf{j} - \frac{t}{\sqrt{4-t^2}}\mathbf{k}$

$$51. \mathbf{r}'(t) = \left\langle -2t \sin(t^2 + 1), -\frac{2}{(1-t)^2}, \frac{3}{\sqrt{1-t^2}} \right\rangle$$

$$53. \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle \quad 55. \left\langle \frac{4}{5}, \frac{3}{5}, 0 \right\rangle$$

$$57. \langle 1+3t, -1-2t, 1+t \rangle$$

$$59. \left\langle \frac{\pi}{6} + \frac{2t}{\sqrt{3}}, \frac{\pi}{3} - \frac{2t}{\sqrt{3}}, 2t - \ln 2 \right\rangle$$

$$69. \left\langle t^3, \frac{t^4 - 2t^2}{4}, \frac{-2t^{3/2}}{3} \right\rangle + \mathbf{C}$$

$$71. \frac{t^2}{2} \mathbf{i} + 3t \mathbf{j} - t^4 \mathbf{k} + \mathbf{C}$$

$$73. 2t \mathbf{i} - \ln|t| \mathbf{j} + \frac{2t^{5/2}}{5} \mathbf{k} + \mathbf{C}$$

$$75. \frac{3}{2} \mathbf{i} - 12 \mathbf{j} + 9 \mathbf{k} \quad 77. \left\langle 0, 0, \frac{2}{5} \right\rangle$$

$$79. \langle 2(e^e - e), -1, 1 \rangle$$

$$81. \text{Max. altitude: 78.48 m; range: 543.725 m}$$

$$83. \mathbf{v}(t) = \langle t^2 + 3, t + 1, -1 \rangle;$$

$$\mathbf{r}(t) = \left\langle \frac{t^3}{3} + 3t - 5, \frac{t^2}{2} + t, 2 - t \right\rangle$$

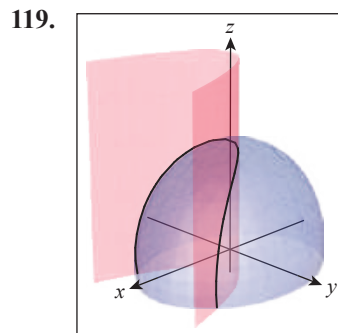
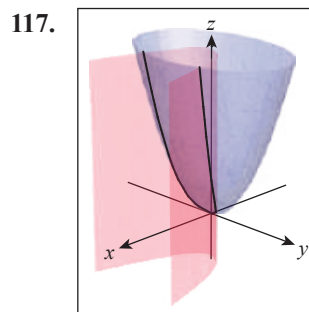
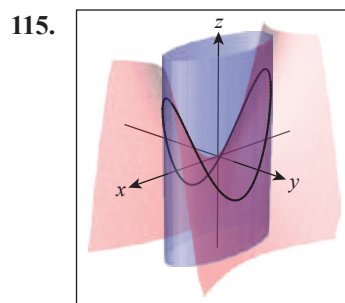
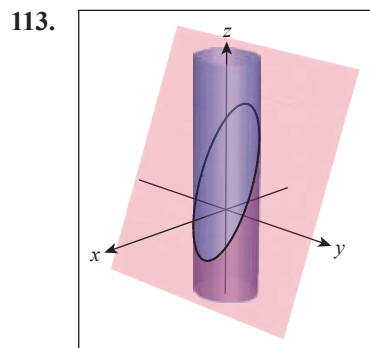
$$85. \varphi = \operatorname{arccot} b$$

105. See the answer given for Exercise 15.

107. See the answer given for Exercise 17.

109. See the answer given for Exercise 19.

111. See the answer given for Exercise 21.



Section 12.2

$$1. 12\sqrt{5}\pi \quad 3. 18 \quad 5. e^2 \quad 7. 145 \quad 9. \ln(2 + \sqrt{3})$$

$$11. \mathbf{r}(t(s)) = \left\langle 1 + \frac{2s}{3\sqrt{5}}, 3 - \frac{\sqrt{5}s}{3}, 4 + \frac{4s}{3\sqrt{5}} \right\rangle$$

$$13. \mathbf{r}(t(s)) = \left\langle 2 \cos \frac{s}{\sqrt{13}}, 2 \sin \frac{s}{\sqrt{13}}, \frac{3s}{\sqrt{13}} \right\rangle$$

$$15. \mathbf{r}(t(s)) = \left\langle b \frac{s}{\sqrt{b^2 + a^2 \omega^2}}, a \cos \frac{\omega s}{\sqrt{b^2 + a^2 \omega^2}}, a \sin \frac{\omega s}{\sqrt{b^2 + a^2 \omega^2}} \right\rangle$$

$$17. \mathbf{T}(t) = \frac{1}{\sqrt{9t^4 + 4t^2 + 1}} \langle 1, 3t^2, -2t \rangle$$

$$19. \mathbf{T}(t) = \frac{1}{\sqrt{3}} \langle 1, \cos t - \sin t, \cos t + \sin t \rangle$$

$$21. \mathbf{T}(t) = \frac{1}{\sqrt{t^2 + 1}} \langle -t \sin t, 1, t \cos t \rangle$$

$$23. \mathbf{T}(t) = \frac{1}{e^t + e^{-t}} \langle e^t, -e^{-t}, \sqrt{2} \rangle$$

$$25. \mathbf{r}(t(s)) = \left\langle \frac{s}{\sqrt{m^2 + 1}}, \frac{ms}{\sqrt{m^2 + 1}} + b \right\rangle$$

$$27. c$$

$$29. x(t(s)) = a \left(4 \sin^{-1} \sqrt{\frac{s}{8a}} - \sin \left(4 \sin^{-1} \sqrt{\frac{s}{8a}} \right) \right),$$

$$y(t(s)) = a \left(1 - \cos \left(4 \sin^{-1} \sqrt{\frac{s}{8a}} \right) \right)$$

$$31. L = \sqrt{(2\pi nr)^2 + h^2} \quad 33. \text{Range} = \frac{v_0^2 \sin 2\theta}{g}$$

$$35. \theta = 45^\circ \quad 37. \theta = \frac{90^\circ + \varphi}{2}$$

$$39. \text{a. } \mathbf{r}(t) = \left\langle 15t, 15\sqrt{3}t - \frac{gt^2}{2} + 2 \right\rangle$$

- b. Max. height: approx. 36.4 m;
range: approx. 80.59 m;
impact speed: approx. 30.65 m/s

$$41. \text{a. } \mathbf{r}(t) = \left\langle 64\sqrt{3}t, -16t^2 + 64t + 4 \right\rangle$$

- b. Max. height: 68 ft
c. Horizontal distance traveled: approx. 450.23 ft;
impact speed: approx. 129 ft/s

$$43. x \approx 26^\circ \text{ or } x \approx 64^\circ$$

$$45. \mathbf{r}(t) = \left\langle 333.3t, 8000 - \frac{gt^2}{2} \right\rangle;$$

- horizontal distance: approx. 13,461.8 m;
impact speed: approx. 517.76 m/s

$$47. \mathbf{r}(t) = \left\langle \frac{v_0 \cos \theta}{C} (1 - e^{-Ct}), \frac{v_0 \sin \theta}{C} (1 - e^{-Ct}) + \frac{g}{C^2} (1 - Ct - e^{-Ct}) \right\rangle$$

$$49. \text{b. } \mathbf{r}(t(s)) = a \left\langle \cos \sqrt{2s/a} + \sqrt{2s/a} \sin \sqrt{2s/a}, \sin \sqrt{2s/a} - \sqrt{2s/a} \cos \sqrt{2s/a} \right\rangle$$

51. False; they are perpendicular only for $t = 0$.

53. False; its range quadruples. (The vertical component of its initial velocity doubled, the projectile is aloft twice as long, and the horizontal component of its velocity is also doubled.)

Section 12.3

$$1. \mathbf{T}(t) = \langle -\sin t, \cos t, 0 \rangle; \mathbf{N}(t) = \langle -\cos t, -\sin t, 0 \rangle; \mathbf{B}(t) = \langle 0, 0, 1 \rangle$$

$$3. \mathbf{T}(t) = \langle \cos 2t, -\sin 2t, 0 \rangle; \mathbf{N}(t) = \langle -\sin 2t, -\cos 2t, 0 \rangle; \mathbf{B}(t) = \langle 0, 0, -1 \rangle$$

$$5. \mathbf{T}(t) = \left\langle \frac{\cos \pi t}{\sqrt{2}}, -\frac{\sin \pi t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle; \mathbf{N}(t) = \langle -\sin \pi t, -\cos \pi t, 0 \rangle; \mathbf{B}(t) = \left\langle \frac{\cos \pi t}{\sqrt{2}}, -\frac{\sin \pi t}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$$

$$7. \mathbf{T}(t) = \left\langle \frac{2}{\sqrt{13}}, -\frac{3 \sin 3t}{\sqrt{13}}, \frac{3 \cos 3t}{\sqrt{13}} \right\rangle; \mathbf{N}(t) = \langle 0, -\cos 3t, -\sin 3t \rangle;$$

$$\mathbf{B}(t) = \left\langle \frac{3}{\sqrt{13}}, \frac{2 \sin 3t}{\sqrt{13}}, -\frac{2 \cos 3t}{\sqrt{13}} \right\rangle$$

$$9. \mathbf{T}(t) = \left\langle \frac{\cos t + \sin t}{\sqrt{e^2 + 2}}, \frac{\cos t - \sin t}{\sqrt{e^2 + 2}}, \frac{e}{\sqrt{e^2 + 2}} \right\rangle;$$

$$\mathbf{N}(t) = \left\langle \frac{\cos t - \sin t}{\sqrt{2}}, \frac{-\cos t - \sin t}{\sqrt{2}}, 0 \right\rangle;$$

$$\mathbf{B}(t) = \left\langle \frac{e(\cos t + \sin t)}{\sqrt{2e^2 + 4}}, \frac{e(\cos t - \sin t)}{\sqrt{2e^2 + 4}}, -\frac{\sqrt{2}}{\sqrt{e^2 + 2}} \right\rangle$$

11. Osculating plane: $z = 0$;

normal plane: $y = 0$;

rectifying plane: $x = 1$

13. Osculating plane: $z = 3$;

normal plane: $x = 0$;

rectifying plane: $y = 3$

15. Osculating plane: $2y + 2z = \pi$;

normal plane: $2y - 2z = -\pi$;

rectifying plane: $x = 1$

17. Osculating plane: $3x + 2z = 6\pi$;

normal plane: $2x - 3z = 4\pi$;

rectifying plane: $y = -1$

19. Osculating plane: $e(y - x) + 2z = e^{1+(\pi/2)}$;

normal plane: $x - y + ez = e^{\pi/2} (e^2 + 1)$;

rectifying plane: $x + y = e^{\pi/2}$

21. $\mathbf{r}(t) + u \langle -\sin 2t, -\cos 2t, 0 \rangle, u \in [-1/4, 1/4]$

23. $\mathbf{r}(t) + \cos u \langle -\sin \pi t, -\cos \pi t, 0 \rangle$

$$+ \sin u \left\langle \frac{\cos \pi t}{\sqrt{2}}, -\frac{\sin \pi t}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle, u \in [0, 2\pi]$$

25. $\mathbf{r}(t) + 0.3 \cos u \langle 0, -\cos 3t, -\sin 3t \rangle$

$$+ 0.2 \sin u \left\langle \frac{3}{\sqrt{13}}, \frac{2 \sin 3t}{\sqrt{13}}, -\frac{2 \cos 3t}{\sqrt{13}} \right\rangle, u \in [0, 2\pi]$$

$$27. \mathbf{r}(t) + \left(\cos t + 2 \cos \frac{t}{2} \right) \cdot \left\langle \frac{\cos t - \sin t}{\sqrt{2}}, \frac{-\cos t - \sin t}{\sqrt{2}}, 0 \right\rangle + \left(\sin t - 2 \sin \frac{t}{2} \right) \cdot \left\langle \frac{e(\cos t + \sin t)}{\sqrt{2e^2 + 4}}, \frac{e(\cos t - \sin t)}{\sqrt{2e^2 + 4}}, -\frac{\sqrt{2}}{\sqrt{e^2 + 2}} \right\rangle, u \in [0, 2\pi]$$

31. $\kappa = 0$; $\tau = 0$

$$33. \kappa = \frac{1}{2 \cosh^2 t}; \tau = \frac{1}{2 \cosh^2 t}$$

35. If $b = 0$, the “helix” is actually a circle, and we obtain the result of Example 4b.

37. $\kappa = \frac{a\omega^2}{(a\omega)^2 + b^2}; \tau = \frac{b\omega}{(a\omega)^2 + b^2}$

39. $\kappa = 2/5; \tau = -1/5$

41. $\kappa = \frac{2t^3}{(t^4 + 1)^{3/2}}; \tau = 0$ 43. $\kappa = 1/2; \tau = 0$

45. $\kappa = 9/25; \tau = -12/25$

47. $\kappa = \frac{\sqrt{2}}{3e^t}; \tau = \frac{1}{3e^t}$

51. $\frac{3}{5\sqrt{10}}$ 53. $\sqrt{3}/2$

55. $(2k\pi, 1), ((2k+1)\pi, -1), k \in \mathbb{Z}$

59. $1/4$ 61. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = 2$

63. $x^2 + y^2 = 1$

65. $(x-2)^2 + (y-2)^2 = 2$

67. $\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{5}{6}\right)^2 = \frac{125}{36}$

69. $\mathbf{r}(t) = \langle \pi + 4\sin t, -2 - 4\cos t \rangle$

71. $\mathbf{r}(t) = \left\langle \frac{\cos t + 1}{2}, \frac{\sin t}{2}, 0 \right\rangle$

75. $\mathbf{r}(t) = \left\langle \frac{t - 9t^5}{2}, \frac{1 + 15t^4}{6t} \right\rangle$

77. $a_T = 0; a_N = 2$

79. $a_T = \frac{4t}{\sqrt{4t^2 + 5}}; a_N = \frac{2\sqrt{5}}{\sqrt{4t^2 + 5}}$

81. $a_T = \frac{\sin t}{\sqrt{2 - 2\cos t}}; a_N = \frac{1 - \cos t}{\sqrt{2 - 2\cos t}}$

83. $a_T = 2t; a_N = 2$ 85. $a_T = e^t - e^{-t}; a_N = \sqrt{2}$

87. $a_T = \sqrt{3}e^t; a_N = \sqrt{2}e^t$ 89. $F_c = \frac{mv^2}{r}$

91. A slow entering speed decreases the normal component of the acceleration vector, while gently accelerating in the curve adds a component parallel to motion, making the (resultant) acceleration vector more in line with the direction of motion, thereby decreasing the chances of skidding.

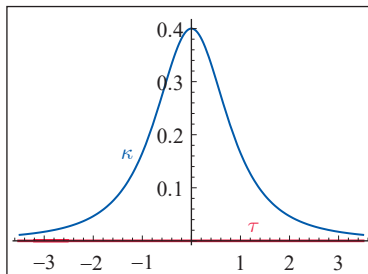
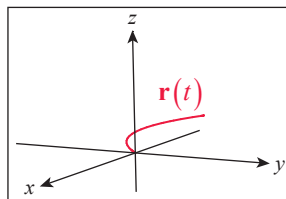
99. False; $(\mathbf{T} \times \mathbf{N}) \cdot \mathbf{B} = 1$.

101. False; the osculating circle is undefined.

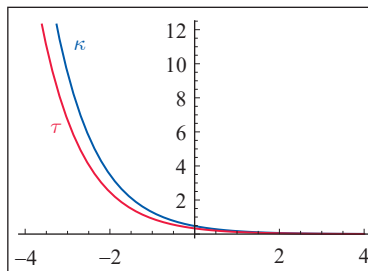
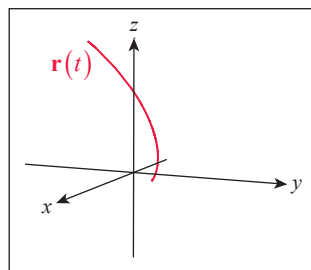
103. False; the force is quadrupled.

105. True

107.



109.



Section 12.4

1. $\mathbf{v} = 2\mathbf{u} + 4t\mathbf{u}_\perp; \mathbf{a} = -8t\mathbf{u} + 8\mathbf{u}_\perp$

3. $\mathbf{v} = \frac{1}{2\sqrt{t}}\mathbf{u} + 4\sqrt{t}\mathbf{u}_\perp; \mathbf{a} = \frac{-64t^2 - 1}{4t^{3/2}}\mathbf{u} + \frac{4}{\sqrt{t}}\mathbf{u}_\perp$

5. $\mathbf{v} = 4\cos 2t\mathbf{u} + 6t^2\sin 2t\mathbf{u}_\perp;$
 $\mathbf{a} = -(18t^4 + 8)\sin 2t\mathbf{u} + 12(2t^2\cos 2t + t\sin 2t)\mathbf{u}_\perp$

9. 28,000 km/h 11. 5.973×10^{24} kg

13. 227,940,000 km

17. The eccentricity is approx. 0.0167. An equation is

$$r(\theta) \approx \frac{1.49557 \times 10^8}{1 + 0.0167 \cos \theta} \text{ km.}$$

19. Approx. 76 years

23. $v_p \approx 30.287 \text{ km/s} \approx 67,764.6 \text{ mph};$

$v_a \approx 29.291 \text{ km/s} \approx 65,536.1 \text{ mph}$

25. $v_p \approx 1.08 \text{ km/s} \approx 2416 \text{ mph};$

$v_a \approx 0.963 \text{ km/s} \approx 2155 \text{ mph}$

27. $v_p \approx 0.03135 \text{ AU/day}$
 $\approx 54.281 \text{ km/s} \approx 121,449 \text{ mph}$
 $v_a \approx 0.000526 \text{ AU/day}$
 $\approx 0.911 \text{ km/s} \approx 2038 \text{ mph}$
29. Mission control is right. This way, she will move into an orbit closer to Earth, and by Kepler's Third Law, the orbital period will decrease.

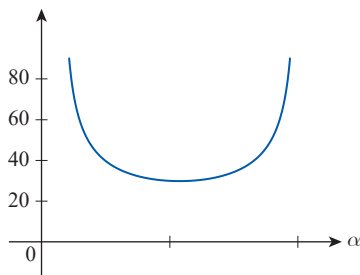
Chapter 12 Review

1. C 3. D 5. A 7. H
9. $\mathbf{r}(t) = \langle \sqrt{2} \cos t, \sqrt{5} \sin t, 10 \cos 2t \rangle$
11. Yes, the limit is $\langle 0, 1, 1/2 \rangle$. 13. $t = \pm 1$
15. $\mathbf{r}'(t) = \left\langle \sec t \tan t, \frac{1}{2\sqrt{t(1-t)}}, 2 \sec^2 t \tan t \right\rangle$
17. $\langle 8t, 1, 1 \rangle$ 19. $\left\langle 3+t, -\ln 19 - \frac{12t}{19}, -3 - \frac{t}{2} \right\rangle$
21. $\left\langle e^{t+1}, \sqrt{t^2+2}, -\frac{\cos 2t}{2} \right\rangle + \mathbf{C}$ 23. $\left\langle \frac{14}{3}, \frac{27}{2}, 9 \right\rangle$
25. 20
27. $\mathbf{r}(t(s)) = \left\langle 2 - \frac{s}{\sqrt{21}}, 1 + \frac{4s}{\sqrt{21}}, 3 + \frac{2s}{\sqrt{21}} \right\rangle$
29. $\mathbf{T}(t) = \left\langle \frac{1}{2 \sec t}, -\sin t, \frac{\sqrt{3}}{2 \sec t} \right\rangle$
31. $\mathbf{T}(t) = \frac{1}{\sqrt{4t^4 + t^2 + 1}} \langle 1, 2t^2, t \rangle$
33. $\mathbf{N}(t) = \langle 0, -\cos t, -\sin t \rangle$;
 $\mathbf{B}(t) = \frac{\sqrt{2}}{2} \langle 1, \sin t, -\cos t \rangle$
35. $\mathbf{N}(t) = \left\langle \frac{\sqrt{2}e^t}{1+e^{2t}}, \frac{\sqrt{2}e^t}{1+e^{2t}}, \frac{1-e^{2t}}{1+e^{2t}} \right\rangle$;
 $\mathbf{B}(t) = \left\langle \frac{-1}{1+e^{2t}}, \frac{e^{2t}}{1+e^{2t}}, \frac{\sqrt{2}e^t}{1+e^{2t}} \right\rangle$
37. Osculating plane: $z = x$;
normal plane: $z = -x$;
rectifying plane: $y = 2$
39. Osculating plane: $z = \frac{\sqrt{2}}{2e}x - \frac{\sqrt{2}e}{2}y + \sqrt{2}$;
normal plane: $\sqrt{2}z = e^2 - \frac{1}{e^2} + 2 - ex + \frac{y}{e}$;
rectifying plane: $y = \frac{1}{e} + e^3 - e^2x$
41. $\mathbf{r}(t) + u \langle 0, -\cos t, -\sin t \rangle$, $u \in [-1/4, 1/4]$
43. $\mathbf{r}(t) + 2 \cos u \left\langle \frac{\sqrt{2}e^t}{1+e^{2t}}, \frac{\sqrt{2}e^t}{1+e^{2t}}, \frac{1-e^{2t}}{1+e^{2t}} \right\rangle$
 $+ 2 \sin u \left\langle \frac{-1}{1+e^{2t}}, \frac{e^{2t}}{1+e^{2t}}, \frac{\sqrt{2}e^t}{1+e^{2t}} \right\rangle$, $u \in [0, 2\pi]$
45. $\kappa = 2/3$; $\tau = \sqrt{2}/3$
47. $\kappa = \frac{t^2 \sqrt{t^4 + 4t^2 + 1}}{(t^4 + t^2 + 1)^{3/2}}$; $\tau = \frac{-2t^2}{t^4 + 4t^2 + 1}$
49. a. $\kappa = a/b^2$ b. $\kappa = b/a^2$
51. $(1, 1), (-1, -1)$
53. $(x-4)^2 + \left(y + \frac{8}{3}\right)^2 = \frac{250}{9}$
55. $a_T = 0$; $a_N = 4$
57. $a_T = \frac{t}{\sqrt{t^2+1}}$; $a_N = \frac{t^2+2}{\sqrt{t^2+1}}$
59. $\mathbf{v} = \mathbf{u} + \frac{t}{2}\mathbf{u}_\perp$; $\mathbf{a} = -\frac{t}{4}\mathbf{u} + \mathbf{u}_\perp$
61. Max. altitude: approx. 196 ft;
range: approx. 452.66 ft
63. Approx. 243,576.6 ft 65. Approx. 13.89 m/s²
67. a. $\mathbf{r}(t) \approx \left\langle 83.33t, 500 - 144.34t - \frac{gt^2}{2} \right\rangle$
b. Approx. 231.13 m c. Approx. 759.71 km/h
73. $\kappa = |b|$; The curvature at $t = b$ is equal to the arc length over the interval $[0, b]$.
75. Approx. 27,830 km/h 77. Approx. 90,107.7 km
79. a. $v_p \approx 13.711 \text{ km/s} \approx 30,849.8 \text{ mph}$
b. $v_a \approx 12.434 \text{ km/s} \approx 27,976.9 \text{ mph}$
81. False; consider the graph of $\mathbf{r}(t) = \langle t^3, t^3, t^3 \rangle$.
83. True 85. False; the curvature is zero.
87. False; κ and τ do not depend on parametrization; this is evident from the formulas (given for κ and τ) in the summary table at the end of Section 12.3.
89. True
91. $\mathbf{r}(t) = \left\langle \left(8 - \frac{8t}{5\pi}\right) \cos 5t, \left(8 - \frac{8t}{5\pi}\right) \sin 5t, \frac{t}{4} \right\rangle$

Chapter 12 Application Project

1. $\mathbf{r}(t) = \left\langle 0, \frac{\sqrt{2}}{2}v_0t, -16t^2 + \frac{\sqrt{2}}{2}v_0t + 7 \right\rangle$
3. $v_0 \approx 30 \text{ ft/s}$

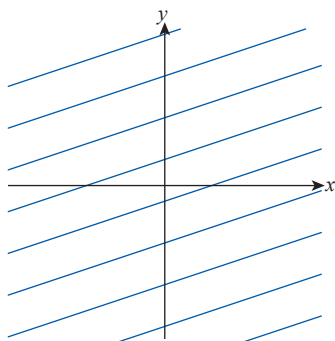
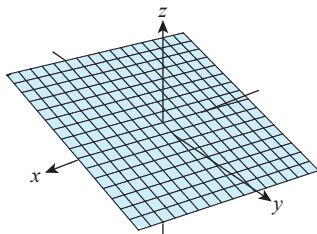
$$5. v_0(\alpha) \approx \frac{98.8}{\cos \alpha \sqrt{24.7 \tan \alpha - 3}}$$



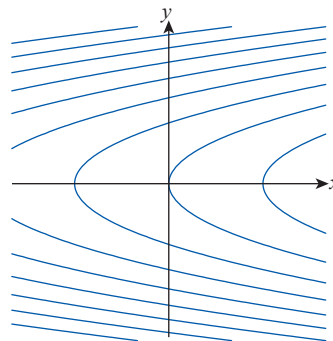
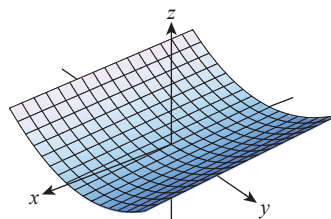
Chapter 13

Section 13.1

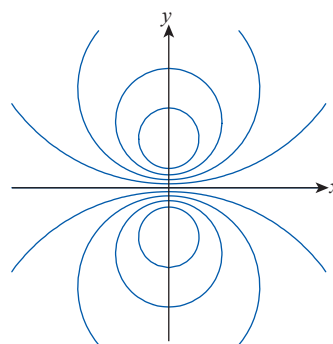
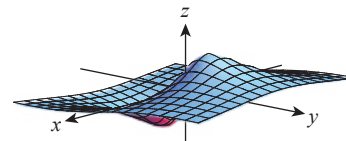
1. $f(0,0) = 0$, $f(2,1) = -2$
3. $f(5,0,-1) = -5$, $f(-4,2,1) = -4/9$
5. $\text{Dom} = \mathbb{R}^2$; $\text{Ran} = \mathbb{R}$; $f(3,-1) = -30$
7. $\text{Dom} = \{(x,y) | y \geq 0\}$; $\text{Ran} = [0, \infty)$; $f(2,9) = 6$
9. $\text{Dom} = \{(x,y) | x \neq 0 \text{ and } y \neq 0\}$;
 $\text{Ran} = \mathbb{R}$; $f(e,-1) = 1$
11. $\text{Dom} = \mathbb{R}^3$; $\text{Ran} = (-\pi/2, \pi/2)$; $f(5,3,2) = \pi/4$
13. Plane 15. Elliptic paraboloid
17. Elliptic cone ($z \geq 0$)
19. Elliptic paraboloid 21. Boundary point
23. Boundary point 25. Boundary point
27. Closed 29. Open 31. Open 33. Neither
35. Answers will vary. 37. Answers will vary.
39. Answers will vary. 41. Answers will vary.
43. C 45. B 47. E 49. E 51. A 53. D
- 55.



57.



59.



61. Ellipsoids centered at the origin
63. a. The direction perpendicular to the contour through A, approx. 30° south of west
b. Answers will vary, but the path should run perpendicularly to the contour lines.
c. Approx. 4044 ft
d. Answers will vary.

Section 13.2

1. 6 3. 3 5. 0 7. $\sqrt{5}/10$ 9. 0
11. The limit does not exist. 13. The limit does not exist.
15. 0 17. The limit does not exist.
19. The limit does not exist. 21. The limit does not exist.
23. Domain in Exercise 5:
 $\{(x,y) \in \mathbb{R}^2 - \{(0,0)\} | x, y \geq 0\}$;
domain in Exercise 6:

- $\{(x, y) \in \mathbb{R}^2 \mid x > 0, y \geq 1, y \neq x + 1\}$
 37. -3 39. 0 45. $\mathbb{R}^2$ 47. $\mathbb{R}^2$ 49. $\mathbb{R}^2$
 51. $\{(x, y, z) \mid z > \frac{1}{3}(x^2 + 2y^2)\}$
 53. $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 9\}$
 55. (0, 0); nonremovable discontinuity
 57. (0, 0); nonremovable discontinuity
 59. (0, 0, 0); nonremovable discontinuity
 61. False; we can redefine $f(a, b)$ without changing the limit.
 63. True
 65. False; consider the function

$$f(x, y) = \begin{cases} \frac{1}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ L & \text{if } (x, y) = (0, 0). \end{cases}$$

Section 13.3

1. $z_x(2, 1) = 4$; $z_y(2, 1) = 18$;
 $L_1: \langle 2 + t, 1, 8 + 4t \rangle$; $L_2: \langle 2, 1 + t, 8 + 18t \rangle$
 3. $z_x(-1, 0) = 4$; $z_y(-1, 0) = -4$;
 $L_1: \langle -1 + t, 0, -4 + 4t \rangle$; $L_2: \langle -1, t, -4 - 4t \rangle$
 5. $\frac{\partial f}{\partial x} = 3x^2$, $\frac{\partial f}{\partial y} = 3y^2$
 7. $\frac{\partial h}{\partial x} = 15x^2y$, $\frac{\partial h}{\partial y} = 5x^3 + 4y^3$
 9. $\frac{\partial V}{\partial r} = \frac{2\pi rh}{3}$, $\frac{\partial V}{\partial h} = \frac{\pi r^2}{3}$
 11. $\frac{\partial k}{\partial x} = y^2 + \frac{y}{\sqrt{2xy}}$, $\frac{\partial k}{\partial y} = 2xy + \frac{x}{\sqrt{2xy}}$
 13. $\frac{\partial m}{\partial x} = (4x^3y - 24x^4)(y - 3x)^3$,
 $\frac{\partial m}{\partial y} = 4x^4(y - 3x)^3$
 15. $\frac{\partial p}{\partial x} = \frac{x^2y + 2xy^2}{(x + y)^2}$, $\frac{\partial p}{\partial y} = \frac{x^3}{(x + y)^2}$
 17. $\frac{\partial G}{\partial x} = -\frac{xy}{(x^2 + y^2)^{3/2}}$, $\frac{\partial G}{\partial y} = \frac{x^2}{(x^2 + y^2)^{3/2}}$
 19. $\frac{\partial I}{\partial x} = \frac{1}{y} \cos \frac{x}{y}$, $\frac{\partial I}{\partial y} = -\frac{x}{y^2} \cos \frac{x}{y}$
 21. $\frac{\partial K}{\partial x} = -\frac{x}{y} - \frac{2y^2}{x^2}$, $\frac{\partial K}{\partial y} = \frac{x^2}{2y^2} + \frac{4y}{x}$
 23. $\frac{\partial M}{\partial x} = \frac{\sqrt{xy}}{2x + 2x^2y}$, $\frac{\partial M}{\partial y} = \frac{\sqrt{xy}}{2y + 2xy^2}$
 25. $\frac{\partial g}{\partial x} = x^{(y-z)/z} \frac{y}{z}$, $\frac{\partial g}{\partial y} = x^{y/z} \frac{\ln x}{z}$,
 $\frac{\partial g}{\partial z} = -x^{y/z} \frac{y \ln x}{z^2}$
 27. $\frac{\partial h}{\partial x} = \cos(y + z^2)$, $\frac{\partial h}{\partial y} = -x \sin(y + z^2)$,
 $\frac{\partial h}{\partial z} = -2xz \sin(y + z^2)$
 29. $L: \left\langle 1 + t, 1, 3 + \frac{4t}{3} \right\rangle$ 31. $L: \left\langle 2 + t, \frac{3}{\sqrt{5}}, 5 + \frac{8t}{5} \right\rangle$
 33. $L: \left\langle -1 + t, 1, 3 - \frac{4t}{3} \right\rangle$
 35. $z_x = \frac{2y}{y - 2z}$, $z_y = \frac{z - 2x}{2z - y}$
 37. $z_x = \frac{z(2xy - z)}{xz + 1}$, $z_y = \frac{x^2z}{xz + 1}$
 39. $y_x = \frac{y^2}{2y^2 - xy - z}$, $y_z = \frac{y \ln y}{2y^2 - xy - z}$
 55. $f_{jyx} = 2y - \frac{1}{y^2}$ 57. $h_{urs} = -\frac{2r}{u^2}$
 59. $f_{jyx} = 4x^2 \sin(x^2 - y) - 2 \cos(x^2 - y)$
 61. $f_{zyx} = 6xz$
 63. $f_{xyzw} = -\frac{15wxyz}{(w^2 + x^2 + y^2 + z^2)^{7/2}}$
 65. $\frac{\partial f}{\partial x} = -\frac{\sqrt{y}}{x^2}$, $\frac{\partial f}{\partial y} = \frac{1}{2x\sqrt{y}}$
 67. $f(x, y) = x^2y - \sin y$
 69. Not possible; if such a function existed, it wouldn't satisfy Clairaut's Theorem.
 71. $\frac{\partial i}{\partial o} = -\left(\frac{f}{o - f}\right)^2$; image distance increases when the object distance decreases, and vice versa.
 73. Approx. 2% 93. 0 and $-\sqrt{2}$, respectively
 97. From the first formula, $\frac{\partial A}{\partial r} = r\alpha = C - 2r$, while from the second formula $\frac{\partial A}{\partial r} = \frac{1}{2}C - 2r$.
 The answers are not equivalent because in the first case α is held constant while the radius changes, in the second case C is held constant while the radius changes. The rates of change for area are different in the two cases.
 99. Inflation has a greater effect.

$$101. \text{ b. } \frac{\partial R}{\partial R_k} = \frac{\prod_{i \neq k} R_i^2}{\left(\sum_{i=1}^n \prod_{j \neq i} R_j \right)^2}$$

105. No, the partials don't exist.

107. No, the origin is a discontinuity (f is undefined at the origin).

$$119. \{(x, y) | x \neq y\}$$

Section 13.4

$$3. \frac{dg}{dt} = \frac{\partial g}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial g}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial g}{\partial z} \cdot \frac{dz}{dt}$$

$$5. \frac{\partial k}{\partial z} = \frac{\partial k}{\partial u} \cdot \frac{\partial u}{\partial z} + \frac{\partial k}{\partial v} \cdot \frac{\partial v}{\partial z} \quad 7. \frac{dy}{dx} = 6x^2 + 4x \sin 2x^2$$

$$9. \frac{dy}{dx} = \cot x + \tan x \quad 11. \frac{dy}{dx} = \sec^2 x + \sin 2x$$

$$13. \frac{dy}{dx} = -\frac{4}{x\sqrt{x^4-4}} \quad 15. \frac{dy}{dx} = 2(x \sin 2x + \sin^2 x)$$

$$17. \frac{dy}{dx} = \sin x(10x-2) + x \cos x(5x-2)$$

$$19. f'\left(\frac{\pi}{2}\right) = 0 \quad 21. f'(0) = 36$$

$$23. z_x = 2(u-v), \quad z_y = 2(2u+v)$$

$$25. z_x = (3v^2 - 2u^2) \cos x, \quad z_y = 4uv \sin y$$

$$27. z_x = -3\sqrt{v}e^x, \quad z_y = \left(8v^3 - \frac{3u}{2\sqrt{v}}\right)e^y$$

$$29. z_x = 2u(ux+v-3w^2y) - w^3, \\ z_y = u(2v-u-6w^2x) - w^3$$

$$31. z_x = (9vy+2v-2)ve^{2u+3w}, \\ z_y = (9vx-2v+4)ve^{2u+3w}$$

$$33. z_x = (vw^2 + 4uxyv) \cos(uw^2) - \sin(uw^2), \\ z_y = vw(2ux^2 + 3wy^2) \cos(uw^2) + 3 \sin(uw^2)$$

$$39. \frac{dy}{dx} = \frac{2x-y}{x-2y} \quad 41. \frac{dy}{dx} = -\frac{2xy}{4+x^2} \quad 43. \frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$$

$$45. \frac{dy}{dx} = \frac{2(2x-3)(x^2+y^2-3x)-2x}{2y-4y(x^2+y^2-3x)}$$

$$47. \frac{dy}{dx} = \frac{2x(x^2+y^2)^2 + 2xy}{x^2 - y^2}$$

$$49. z_x = \frac{yz - e^{x+y+z}}{e^{x+y+z} - xy}, \quad z_y = \frac{xz - e^{x+y+z}}{e^{x+y+z} - xy}$$

$$51. z_x = \frac{y+2xz^3}{y \cos(yz) - 3x^2z^2}, \quad z_y = \frac{x-z \cos(yz)}{y \cos(yz) - 3x^2z^2}$$

$$53. z_x = \frac{e^x \sin y}{z(4-y)}, \quad z_y = \frac{z^2 + 2e^x \cos y}{2z(4-y)}$$

$$55. z_x = -\frac{yz(x^2+y^2+z^2)+2x}{xy(x^2+y^2+z^2)+2z},$$

$$z_y = -\frac{xz(x^2+y^2+z^2)+2y}{xy(x^2+y^2+z^2)+2z}$$

$$57. f_x = f_t \cdot \frac{\partial t}{\partial x} + f_u \cdot \frac{\partial u}{\partial x} + f_v \cdot \frac{\partial v}{\partial x} + f_w \cdot \frac{\partial w}{\partial x},$$

$$f_y = f_t \cdot \frac{\partial t}{\partial y} + f_u \cdot \frac{\partial u}{\partial y} + f_v \cdot \frac{\partial v}{\partial y} + f_w \cdot \frac{\partial w}{\partial y}$$

61. Homogeneous of degree 1

$$65. 8.25 \text{ in.}^2/\text{min} \quad 67. 2\pi \text{ in.}^3/\text{s}$$

$$69. \text{Approx. } 1.9 \times 10^4 \text{ kJ/s}$$

$$71. \text{Approx. } -0.466 \text{ degrees/min}$$

Section 13.5

$$1. \nabla f(1,2) = \langle 15, -8 \rangle \quad 3. \nabla h(-1,1) = \langle -2e^2, 2e^2 \rangle$$

$$5. \nabla F(-2,3,3) = \langle 3, -3, 1 \rangle \quad 7. 2/5 \quad 9. \frac{1}{4\sqrt{2}} \quad 11. 0$$

$$13. \text{ a. } -3 \quad \text{ b. } -6;$$

If we multiply the unit direction vector by a constant, the limit (the "directional derivative") will be multiplied by the same factor.

$$15. 0 \quad 17. 6/5 \quad 19. 2 \quad 21. -31/26$$

$$23. \sqrt{3} \quad 25. 2/e^6 \quad 27. \frac{\pi}{4\sqrt{3}} \quad 29. 8/5$$

$$31. \text{Direction: } \langle 2, 2 \rangle; \text{ value: } 2\sqrt{2}$$

$$33. \text{Direction: } \left\langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\rangle; \text{ value: } \frac{\sqrt{3}}{2}$$

$$35. \text{Direction: } \left\langle \frac{\pi}{2\sqrt{2}}, \frac{\pi}{2\sqrt{2}} \right\rangle; \text{ value: } -\frac{\pi}{2}$$

$$37. \text{Direction: } \langle -1, 0, -3 \rangle; \text{ value: } -\sqrt{10}$$

$$39. \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle \text{ and } \langle -1/\sqrt{2}, -1/\sqrt{2} \rangle$$

$$41. 5y - x = 14 \quad 43. 5y - 4x = 14$$

$$45. y + 2x = 3 \quad 47. y + x = 2$$

$$49. 2x + 3y - 4z = 11 \quad 51. 6x - 8y - z = 1$$

$$57. \nabla f(x, y) = \left\langle \frac{2x^4 + 4x^3y + y^{3/2} + 2x\sqrt{y}}{x^2(x+y)^2}, \frac{y-x-4x^3\sqrt{y}}{2x\sqrt{y}(x+y)^2} \right\rangle$$

59. $\nabla f(x, y, z)$

$$= \left\langle \frac{yz(y^2 + z^2 - x^2)}{(x^2 + y^2 + z^2)^2}, \frac{xz(x^2 - y^2 + z^2)}{(x^2 + y^2 + z^2)^2}, \frac{xy(x^2 + y^2 - z^2)}{(x^2 + y^2 + z^2)^2} \right\rangle$$

61. $f(x, y) = 2xy$

63. Not possible; if such a function existed, it wouldn't satisfy Clairaut's Theorem.

65. $f(x, y, z) = 3x^2yz^2$

67. a. $-10/\sqrt{5}$ b. $10/\sqrt{2}$

69. $160\sqrt{5}/243$ 73. $y^2 = \frac{x^3}{1600}$

Section 13.6

1. $z = 2x - 4y + 1$; $\langle 1 - 2t, 1 + 4t, -1 + t \rangle$

3. $2z = 5x - y$; $\langle 1 - 5t, -3 + t, 4 + 2t \rangle$

5. $z = 32x - 56y - 172$; $\langle 2 + 4t, -2 - 7t, 4 - \frac{1}{8}t \rangle$

7. $6x + 6y + z = 0$; $\langle 1 + 6t, -3 + 6t, 12 + t \rangle$

9. $-2x + 3y - z = 2$ 11. $x - 6y + 18z = 9$

13. $2x - 6y + z = -6$ 15. $3x + 4y + 125z = 50$

19. $x = 1 - 3t, y = 2 + 4t, z = 1 + 2t$

21. $x = -2 - 6t, y = 1 - 2t, z = -1 + t$

23. One possible parametrization is

$$\mathbf{r}(t) = \left\langle t, \sqrt{\frac{8-3t^2}{5}}, 4-t^2 - 2\left(\frac{8-3t^2}{5}\right) \right\rangle,$$

yielding the parametric equations $x = 1 + t$,
 $y = 1 - (3t/5)$, $z = 1 + (2t/5)$.

25. Two points: $(1/2, 1/2, 1)$ and $(-1/2, -1/2, -1)$

27. a. $df = 3x^2 dx - (3 + 2y) dy$

b. $L(x, y) = 3x - 9y + 3$

29. a. $df = \frac{x}{\sqrt{x^2 + y^2}} dx + \frac{y}{\sqrt{x^2 + y^2}} dy$

b. $L(x, y) = \frac{\sqrt{3}}{2}x - \frac{1}{2}y$

31. a. $df = e^{x^2 - y^2} (x dx - y dy)$ b. $L(x, y) = x - y + \frac{1}{2}$

33. a. $df = \frac{dx}{\sqrt{x}} - \frac{dy}{\sqrt{y}}$ b. $L(x, y) = x - \frac{y}{2} - 1$

35. $L(w, x, y, z) = 26x + 12y - 21z + 40$

37. $L(w, x, y, z) = \frac{e^3}{3}(-2w + 2x - z - 6)$

39. $L(w, x, y, z) = \frac{1}{2}(2ex + y - z - e)$

41. Approx. 0.9625 43. Approx. 0.204

45. Approx. 8.855 47. 7% 49. 5%

51. Approx. 2.5% 55. -485.6 Pa

59. False; $f(x, y)$ has to be differentiable at (a, b) .61. False; $\nabla f(a, b)$ is not contained in a tangent plane to the graph of f . It is a two-dimensional vector.

Section 13.7

1. Abs. min.: $f(-2, 3) = -3$

3. Abs. max.: $f(5, -2) = 29$

5. No extrema; saddle point at $(4, -1)$

7. Abs. min.: $f(0, \pm\sqrt{2}) = 1$; saddle point at $(0, 0)$

9. Abs. max.: $f(0, 0) = 1/2$

11. Abs. min.: $f(-1, 0) = 0$

13. Rel. min. at $(-3, 2/3)$;
saddle point at $(-3, -2/3)$

15. Rel. (global) min. at $(0, -1)$ and $(0, 1)$;
saddle point at $(0, 0)$

17. Rel. max. at $(0, 0)$; rel. min. at $(2, 4/3)$;
saddle points at $(0, 4/3)$ and $(2, 0)$

19. Rel. max. at $(1, 1)$; saddle point at $(0, 0)$

21. Rel. max. at $(0, 0)$

23. Rel. (global) max. at $(\sqrt{2}/2, \sqrt{2}/2)$ and
 $(-\sqrt{2}/2, -\sqrt{2}/2)$; saddle point at $(0, 0)$

25. Rel. (global) min. at $(-1/2, 0)$

27. Rel. max. at $(-2, 0)$; saddle point at $(0, 0)$

29. Saddle points at $\left(0, \frac{2k+1}{2}\pi\right)$, $k \in \mathbb{Z}$

31. Abs. min.: $f(0, 0) = 0$

33. Abs. max.: $f(1, 2) = 7$; abs. min.: $f(-1, 0) = -5$

35. Abs. max.: $f(3/2, 3/2) = 11/2$;
abs. min.: $f(3, 0) = f(0, 3) = 1$

37. Abs. max.: $f(12, 0) = 144$;
abs. min.: $f(0, 0) = 0$ (also assumed along the line
segment $y = x/2$ within D)

39. Abs. max.: $f(-2, 1) = 9$;
abs. min.: $f(-1, -1) = -2$

41. Abs. max.: $f(1,1)=1/4$;
abs. min.: $f(0,0)=f(2,0)=f(0,2)=0$

43. Abs. max.: $f(0,\pm 1)=2/e$;
abs. min.: $f(0,0)=0$

45. No critical points

47. Abs. min. at $(-1,1)$

49. Rel. max. at $(0,y)$, $y \neq 0$;
saddle point at $(0,0)$

51. $m = \frac{3}{2}$, $b = \frac{3}{2}$ 53. $y = 2x + \frac{2}{3}$

57. $\sqrt[3]{2} \cdot \sqrt[4]{3}$ 59. $\sqrt{21}$

61. It is a cube of side length $\sqrt[3]{V}$.

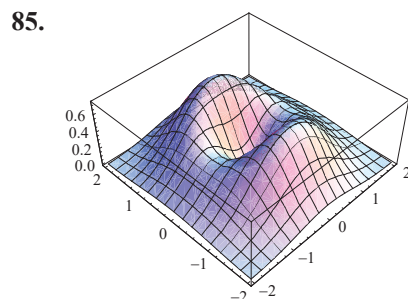
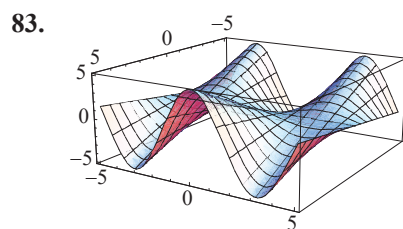
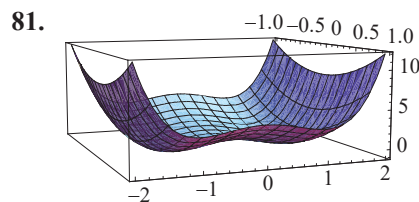
63. The base is a square of side length $\sqrt[3]{V/2}$,
the height is $\sqrt[3]{4V}$.

65. The side length of its base is $2\sqrt{5}$, the height of the
pyramid is 2, and the height of the prism is 1.

67. $\frac{8abc}{3\sqrt{3}}$ 69. 30,000 71. $9abc/2$

75. False; see Example 1 of Section 13.2 or Exercise 9 in
this section.

77. False; there may be a saddle point between them. See
Exercise 7 of this section.



3. Abs. min.: $f(\sqrt{2},\sqrt{2})=f(-\sqrt{2},-\sqrt{2})=4$

5. Abs. max.: $f(-1,1)=f(1,-1)=5$;
abs. min.: $f(-1,-1)=f(1,1)=-1$

7. Abs. max.: $f(\pm 2,0)=2$;
abs. min.: $f(0,\pm\sqrt{2})=1$

9. Abs. max.: $f(4,-2)=12$

11. Abs. max.: $f(3/\sqrt{2},\sqrt{2})=f(-3/\sqrt{2},-\sqrt{2})=6$;
abs. min.: $f(3/\sqrt{2},-\sqrt{2})=f(-3/\sqrt{2},\sqrt{2})=-6$

13. Abs. min.: $f(-2,9,31/2)=-31/2$

15. Abs. max.: $f(36/7,8/7,18/7)=28$;
abs. min.: $f(-36/7,-8/7,-18/7)=-28$

17. Abs. max.: $f(\pm 3,0,0)=9$;
abs. min.: $f(0,\pm 3\sqrt{2},0)=-36$

19. $(12/5,9/5)$ 21. $(1,1)$ 23. $(1/2,3/4)$

25. $(0,-1/\sqrt[3]{2})$ 27. $(-3,2,-\sqrt{3})$

29. $(8/3,-5/3,-5/3)$ 31. $(0,\pm\sqrt{3/8},3/4)$ 33. $(1,0,0)$

37. a. $P(x,y,z)=4x+8y+6z$ (in thousands of dollars)
b. \$252,000

39. It is the cube of side length $\sqrt[3]{V}$.

41. The base is a square of side length $\sqrt[3]{V/2}$, and the
height is $\sqrt[3]{4V}$.

43. The length of its square base is $2r/\sqrt{3}$, and its height
is $r/\sqrt{3}$.

45. 2 47. 2

51. Abs. min.: $f(2/3,1/3,1/3)=2/3$

53. Abs. max.: $f(2,0,-1/2)=5$;
abs. min.: $f(-2,0,3/2)=-7$

57. Closest point: $(2,2,16)$;
farthest point: $(-5/2,-5/2,25)$

59. $(d^{2/3}+h^{2/3})^{3/2}$

Chapter 13 Review

1. E 3. B 5. C 7. C 9. F 11. B

13. 0 15. $1/4$

17. The limit does not exist.

19. For example, $y=x$ and $y=0$.

21. $(0,0)$; nonremovable discontinuity

23. The lines $x=\pm 1$, $y=\pm 2$; nonremovable discontinuity

Section 13.8

1. Abs. max.: $f(0,\pm 2)=8$;
abs. min.: $f(\pm 2,0)=-4$

25. $z_x(-1,1) = 1$; $z_y(-1,1) = -1$;
 $L_1: \langle t-1, 1, t \rangle$; $L_2: \langle -1, 1+t, -t \rangle$
27. $\frac{\partial f}{\partial x} = \frac{y^2 z}{2\sqrt{x-z^2 x^2}}$, $\frac{\partial f}{\partial y} = 2y \sin^{-1}(z\sqrt{x})$,
 $\frac{\partial f}{\partial z} = \frac{y^2 \sqrt{x}}{\sqrt{1-z^2 x}}$
29. $z_x = -\frac{2xz + yz^3}{x^2 + 3xyz^2}$, $z_y = \frac{6y^2 - xz^3}{x^2 + 3xyz^2}$
35. $\frac{\partial f}{\partial x} = \frac{y^2}{2\sqrt{x}}$, $\frac{\partial f}{\partial y} = 2\sqrt{x}y$
37. $f(x, y) = xy^3 + \ln x$
39. Yes, the partials are continuous.
43. $\frac{dy}{dx} = 54x^2(5x^2 - 6)$
45. $\frac{dy}{dx} = 4x^3 \cos x^2 - 2x^5 \sin x^2$
47. $f'(\pi/2) = \pi(2\pi + 1)$
49. $f'(\pi) = 2\pi^3$
51. $f_x = \frac{2u\sqrt{y} + 4vxy}{u^2 + v^2}$, $f_y = \frac{x(2vx\sqrt{y} + u)}{\sqrt{y}(u^2 + v^2)}$
53. $f_x = 6x^2(v - w^2) - 6uw + u$, $f_y = -2u(w + y)$
55. $\frac{dy}{dx} = \frac{4x - 3x^2 y - y^3}{x^3 + 3xy^2}$
57. $z_x = \frac{3x^2 z^2 - y^2}{3z^2 y - 2x^3 z}$, $z_y = \frac{z^3 + 2xy}{2x^3 z - 3z^2 y}$
61. $\nabla f(1,3) = \langle 17, 3 \rangle$ 63. $\nabla f(-1,0,2) = \langle -2, 2, 0 \rangle$
65. 0 67. $\sqrt{2}/4$ 69. 0
71. Direction: $\langle -2, 1, -2 \rangle$; value: 3
73. a. Direction: $-\frac{450}{e^2} \langle 2, 1, 1 \rangle$; magnitude: $\frac{450\sqrt{6}}{e^2}$
b. $\frac{-270\sqrt{5}}{e^2}$
75. $\langle 1, 2 \rangle$ and $\langle -1, -2 \rangle$ 77. $y = 4x - 2$
79. $z = 3y + 6x - 15$
81. Not possible; if such a function existed, it wouldn't satisfy Clairaut's Theorem.
83. $23x + 4y - 6z = 13$; $\langle 1 + 23t, 2 + 4t, 3 - 6t \rangle$
85. $y = 2 - \sqrt{3}x$; $\left\langle \frac{\sqrt{3}}{2} + \sqrt{3}t, \frac{1}{2} + t, 0 \right\rangle$
87. $x = -1 + 11t$, $y = 1 - 5t$, $z = 1 + 2t$
89. a. $df = \cos(x^2 - y^2)(2x dx - 2y dy)$
b. $L(x, y) = -2x - 2y$
91. $L(w, x, y, z) = -3x + 8y + 4z - 2w + 10$
93. Abs. min.: $f(2, -1) = 1$
95. Rel. max.: $f(0, 0) = 0$;
rel. min.: $f(2/3, 8/3) = -260/27$;
saddle points at $(2/3, 0)$ and $(0, 8/3)$
97. Abs. max.: $f(-2, 2) = 9$; abs. min.: $f(1, 2) = 0$
99. $m = 46/107$, $b = 156/107$ 101. $\sqrt{3}/2$ 103. 7500
105. Abs. min.: $f(2, 2) = f(-2, -2) = 8$
107. Abs. max.: $f(0, 0) = 0$;
abs. min.: $f\left(\frac{25 + 3\sqrt{41}}{32}, -\frac{3 + \sqrt{41}}{8}\right) \approx -1.0967$
109. $Q(1/2, 0, 1/2)$
111. Abs. min.: $f(1/6, 7/6, 2/3) = 11/3$
113. Closest points: $\left(\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $\left(-\frac{2}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$;
farthest points: $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right)$
115. True 117. True 119. True
121. False; consider $f(x, y) = \sqrt{x^2 + y^2}$ at $(0, 0)$.
123. False; consider Example 9 in Section 13.3.
125. False; the directional derivatives may not exist.
Consider $f(x, y) = \sqrt{x^2 + y^2}$ at $(0, 0)$.
- 135.

Chapter 13 Conceptual Project

7. $x^2 - 2xy^2 + 3y^3 = C$

Chapter 13 Application Project

1. $v_f = g(m_1, m_2) = v_e \ln \frac{m_1 + m_2 + P}{sm_1 + m_2 + P} + v_e \ln \frac{m_2 + P}{sm_2 + P}$

5. $m_1 \approx 421$ metric tons; $m_2 \approx 88$ metric tons

Chapter 14

Section 14.1

1. 84 3. $297/16$ 5. $21/2$ 7. -36
 9. 2 11. $-3\pi^3/8$ 13. $3/2$ 15. 2
 17. 11 19. 26 21. 12 23. $3/35$
 25. $5/2$ 27. $\frac{19-e^2}{2}$ 29. $3-e$ 31. 0
 33. 8 35. $\int_0^4 \int_0^1 (3x-2y) dy dx = 20$
 37. $\int_{-1}^3 \int_2^4 (5x+xy-3y) dy dx = -8$
 39. $\int_0^{1/2} \int_0^x (x+y)^2 dy dx = \frac{7}{192}$
 41. $\int_0^1 \int_0^{3x} x^2 y dy dx + \int_1^7 \int_0^{\frac{7-x}{2}} x^2 y dy dx = \frac{693}{10}$
 43. $\int_0^3 \int_0^{\sqrt{9-y^2}} \frac{1}{\sqrt{9-x^2}} dx dy = 3$
 45. $\int_0^1 \int_{x^2}^x (x^3-x) dy dx = -\frac{1}{20}$
 47. $\int_0^3 \int_0^{2x} 2x dy dx + \int_3^5 \int_0^{15-3x} 2x dy dx = 80$
 49. $\int_0^1 \int_{\arcsin y}^{\pi/2} \frac{1}{\sqrt{1-y^2}} dx dy = \frac{\pi^2}{8}$
 51. 0 53. $\int_0^2 \int_{y+1}^{\frac{8-y}{2}} f(x,y) dx dy$
 55. $\frac{\pi}{\sqrt{2}} - 2$ 57. $27/10$ 59. -1 61. $4/3$
 63. 16 65. $16/3$ 67. 9π
 69. Using the order $dy dx$: $V = \int_0^2 \int_0^{1-x/2} 2 dy dx = 2$
 71. $1/2$ 73. $e-2$
 75. $\iint_R \rho(x,y) dA$ is the mass of the plate.
 79. a. $\iint_R f_{xy}(x,y) dA$

$$= f(b,d) - f(b,c) - f(a,d) + f(a,c)$$

 b. 104
 91. False;

$$\int_c^d \int_a^b f(x) dy dx = \int_c^d [f(x)y]_{y=a}^{y=b} dx$$

$$= \int_c^d f(x)(b-a) dx = (b-a) \int_c^d f(x) dx$$

 93. False;

$$\int_0^c \int_0^{\frac{c}{a}-\frac{b}{a}y} (c-ax-by) dx dy$$

$$= \int_0^c \left(\frac{c^2}{2a} - \frac{bc}{a}y + \frac{b^2}{2a}y^2 \right) dy = \frac{c^3}{6ab}$$

Section 14.2

1. $1/5$ 3. $1/3$ 5. 40 7. 6 9. $9/2$
 11. $\ln 4$ 13. $10/9$ 15. $-9/20$ 17. $52/35$
 19. $73/15$ 21. $296/189$ 23. 1
 25. $(2/3, 1/3)$ 27. $(3/2, 3/4)$
 29. $\left(\frac{4+\pi}{16}, \frac{1+\pi}{8}\right)$ 31. $(105/164, 485/164)$
 33. $(10, 1/2)$ 35. $\left(\frac{e^2-1}{e^2+1}, \frac{8(2e^3+1)}{27(e^2+1)}\right)$
 37. $I_x = \frac{4\rho a^4}{3}, I_y = \frac{4\rho a^4}{3}, I_0 = \frac{8\rho a^4}{3};$
 $r_x = \frac{\sqrt{3}a}{3}, r_y = \frac{\sqrt{3}a}{3}, r_0 = \frac{\sqrt{6}a}{3}$
 39. $I_x = \frac{\rho lw^3}{12}, I_y = \frac{\rho l^3 w}{12}, I_0 = \frac{\rho lw(l^2+w^2)}{12};$
 $r_x = \frac{\sqrt{3}w}{6}, r_y = \frac{\sqrt{3}l}{6}, r_0 = \frac{\sqrt{3}\sqrt{l^2+w^2}}{6}$
 41. $I_x = \frac{\sqrt{3}\rho a^4}{6}, I_y = \frac{\sqrt{3}\rho a^4}{6}, I_0 = \frac{\sqrt{3}\rho a^4}{3};$
 $r_x = \frac{\sqrt{6}a}{6}, r_y = \frac{\sqrt{6}a}{6}, r_0 = \frac{\sqrt{3}a}{3}$
 43. $I_x = I_y = \frac{4\rho(a^4-b^4)}{3}, I_0 = \frac{8\rho(a^4-b^4)}{3};$
 $r_x = \frac{\sqrt{3}\sqrt{a^2+b^2}}{3}, r_y = \frac{\sqrt{3}\sqrt{a^2+b^2}}{3}, r_0 = \frac{\sqrt{6}\sqrt{a^2+b^2}}{3}$
 45. $I_x = \frac{4\rho}{9}, I_y = \frac{\rho(\pi^2-8)}{2}, I_0 = \frac{\rho(9\pi^2-64)}{18};$
 $r_x = \frac{\sqrt{2}}{3}, r_y = \frac{\sqrt{\pi^2-8}}{2}, r_0 = \frac{\sqrt{9\pi^2-64}}{6}$
 47. $I_x = 2\rho\pi, I_y = \frac{\rho\pi}{2}, I_0 = \frac{5\rho\pi}{2};$
 $r_x = 1, r_y = \frac{1}{2}, r_0 = \frac{\sqrt{5}}{2}$
 49. $I_x = 80\rho\pi, I_y = 125\rho\pi, I_0 = 205\rho\pi;$
 $r_x = 2, r_y = \frac{5}{2}, r_0 = \frac{\sqrt{41}}{2}$
 55. $I_x = 80\rho\pi, I_y = 305\rho\pi, I_0 = 385\rho\pi;$
 $r_x = 2, r_y = \frac{\sqrt{61}}{2}, r_0 = \frac{\sqrt{77}}{2}$
 61. $I_x = 2, I_y = \frac{5}{4}, I_0 = \frac{13}{4}$

$$63. I_x = \frac{64}{315}, I_y = \frac{8}{105}, I_0 = \frac{88}{315};$$

$$r_x = \frac{2\sqrt{42}}{21}, r_y = \frac{\sqrt{7}}{7}, r_0 = \frac{\sqrt{231}}{21}$$

$$65. I_x = \frac{2\pi}{9}, I_y = \pi(\pi^2 - 6), I_0 = \frac{9\pi^3 - 52\pi}{9};$$

$$r_x = \frac{\sqrt{2}}{3}, r_y = \sqrt{\pi^2 - 6}, r_0 = \frac{\sqrt{9\pi^2 - 52}}{3}$$

$$67. I_x = \frac{\sinh 4}{8}, I_y = \frac{e^4 - 5}{8e^2}, I_0 = \frac{e^4 - 5 + e^2 \sinh 4}{8e^2};$$

$$r_x = \frac{\sqrt{\operatorname{csch} 2 \sinh 4}}{2}, r_y = \frac{\sqrt{(e^4 - 5) \operatorname{csch} 2}}{2e},$$

$$r_0 = \frac{\sqrt{(e^4 - 5 + e^2 \sinh 4) \operatorname{csch} 2}}{2e}$$

69. For the rolling objects, a portion of their initial potential energy is turned into kinetic energy, while the remaining portion will be stored in rotation. Since the object with the greater moment of inertia (I_0) will store more energy in rotation, and a smaller portion in the form of kinetic energy, it will be slower. Examining the I_0 -values for the disk and annulus, we see that more of the mass is concentrated near the outer radius in the case of the annulus, so we conclude that the disk beats the annulus, while the box always wins (with friction and rotation absent, its potential energy is turned entirely into kinetic energy.)

71. False; see the definition of center of mass. Since density is constant, it will cancel out when dividing the first moments by the mass of the object.

73. False; see the definition of radii of gyration. Since density is constant, it will cancel out when dividing the second moments by the mass of the object.

$$77. 1/2$$

Section 14.3

$$1. 1/3 \quad 3. 2 - \frac{\pi}{4} \quad 5. \frac{\sqrt{3}}{2} - \frac{\pi}{6} \quad 7. 9\pi$$

$$9. 6\pi - \frac{9\sqrt{3}}{2} \quad 11. \frac{13\sqrt{3}}{8} - \frac{5\pi}{12} \quad 13. \frac{\sqrt{3}}{4} + \frac{\pi}{6}$$

$$15. \pi - \frac{3\sqrt{3}}{2} \quad 17. 2 - \frac{\pi}{4} \quad 19. \frac{\pi \ln(3/2)}{2}$$

$$21. \pi/10 \quad 23. 2048/245 \quad 25. 4/5 \quad 27. 1/3$$

$$29. 3/16 \quad 31. 16\pi/3 \quad 33. \pi/2$$

$$37. 128\pi \quad 39. 8\pi/3 \quad 41. 53\pi/4 \quad 43. 24\pi$$

$$45. \frac{32(\sqrt{2}-1)\pi}{3} \quad 47. \frac{32(2-\sqrt{2})\pi}{3}$$

$$49. \left(0, \frac{15\pi+32}{6\pi+48}\right)$$

$$51. I_x = \left(3\sqrt{3} + \frac{4\pi}{3}\right)\rho, I_y = (11\sqrt{3} + 12\pi)\rho,$$

$$I_0 = \left(14\sqrt{3} + \frac{40\pi}{3}\right)\rho$$

$$53. I_x = \frac{5\rho\pi a^4}{4}, I_y = \frac{\rho\pi a^4}{4}, I_0 = \frac{3\rho\pi a^4}{2};$$

$$r_x = \frac{\sqrt{5}a}{2}, r_y = \frac{a}{2}, r_0 = \frac{\sqrt{3}a}{\sqrt{2}}$$

$$55. I_x = I_y = \frac{\pi}{32}, I_0 = \frac{\pi}{16}; r_x = r_y = \frac{\sqrt{\pi}}{4}, r_0 = \frac{\sqrt{\pi}}{2\sqrt{2}}$$

61. False; the correct formula is

$$\iint_R f(r, \theta) dA = \iint_R r f(r, \theta) dr d\theta.$$

63. True 65. 1/8

$$67. I_x = 49\pi/32, I_y = 21\pi/32, I_0 = 35\pi/16;$$

$$r_x = 7/(4\sqrt{3}), r_y = \sqrt{7}/4, r_0 = \sqrt{35}/24$$

$$69. I_x = \frac{15\pi - 27\sqrt{3}}{20}, I_y = \frac{29\pi}{12} - \frac{171\sqrt{3}}{40},$$

$$I_0 = \frac{76\pi - 135\sqrt{3}}{24}; r_x = \sqrt{\frac{3(5\sqrt{3}\pi - 27)}{10(2\sqrt{3}\pi - 9)}},$$

$$r_y = \frac{1}{2}\sqrt{\frac{29}{3} + \frac{78}{45 - 10\sqrt{3}\pi}}, r_0 = \frac{1}{2}\sqrt{\frac{38}{3} + \frac{21}{9 - 2\sqrt{3}\pi}}$$

Section 14.4

$$3. 1 \quad 5. 216/5 \quad 7. 18 \quad 9. 4/9$$

$$11. 1033/280 \quad 13. 16/3$$

$$15. \text{a. } \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^2 dz dy dx$$

$$\text{b. } \int_{-1}^1 \int_0^2 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy dz dx$$

$$\text{c. } \int_{-1}^1 \int_0^2 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx dz dy; \text{ Value: } 2\pi$$

$$17. \text{a. } \int_0^1 \int_{x^2}^{\sqrt{x}} \int_{-1}^1 dz dy dx$$

$$\text{b. } \int_0^1 \int_{-1}^1 \int_{x^2}^{\sqrt{x}} dy dz dx$$

$$\text{c. } \int_0^1 \int_{-1}^1 \int_{y^2}^{\sqrt{y}} dx dz dy; \text{ Value: } 2/3$$

$$19. abc/6 \quad 21. 81/20 \quad 23. 128/3$$

$$25. 4/5 \quad 27. 128/5 \quad 29. 3\sqrt{3}\pi$$

$$31. 52/105 \quad 33. 648/5 \quad 35. 4\pi$$

37. $\int_{-2}^2 \int_{x^2}^4 \int_0^z dy dz dx$, $\int_0^4 \int_y^4 \int_{-\sqrt{z}}^{\sqrt{z}} dx dz dy$,
 $\int_0^4 \int_0^z \int_{-\sqrt{z}}^{\sqrt{z}} dx dy dz$; Value: 128/5
41. $96k/5$ 43. $k\pi/4$ 45. $2/\sqrt{a}$ 47. $R^3/(8\pi)$
49. a. It is the first octant region of the unit sphere, centered at the origin.
 b. $(3/8, 3/8, 3/8)$
51. $(1/2, 5/4, 3/4)$ 53. $(0, 10/7, 20/7)$
55. $(17/14, -1/2, 9/14)$ 57. $(7a/12, 7a/12, 7a/12)$
59. $(1/5, 2/5, 4/5)$ 61. $(3/5, 0, 112/(75\pi))$
65. $(a/2, a/2, a/2)$; $r_x = r_y = r_z = a\sqrt{2/3}$
67. $(3/8, 1/8, 3/4)$; $r_x = \sqrt{37/40}$, $r_y = 3/\sqrt{2}$, $r_z = 1/2$
69. $(0, 0, -2/5)$; $r_x = \sqrt{3/7}$, $r_y = \sqrt{59/105}$, $r_z = 2\sqrt{2/15}$
71. $(4/(3\pi), 4/(3\pi), 1/2)$; $r_x = r_y = \sqrt{7/12}$, $r_z = 1/\sqrt{2}$
73. $I_x = I_y = I_z = 38a^7k/45$; $r_x = r_y = r_z = \frac{a}{3}\sqrt{\frac{38}{5}}$
75. $I_x = I_y = I_z = 512k/315$; $r_x = r_y = r_z = 8/(3\sqrt{7})$
77. $(16/35, 16/35, 16/35)$; $r_x = r_y = r_z = 1/\sqrt{2}$
79. $(7/18, 7/54, 2/3)$; $r_x = \sqrt{199/264}$, $r_y = \sqrt{85/88}$,
 $r_z = \sqrt{35/132}$
51. $\frac{2\pi(2e-5)}{3}$ 53. $56\sqrt{3}\pi/5$ 55. $(0, 0, 61/9)$
57. $(0, 0, 5/12)$ 59. $\left(0, 0, \frac{e^2-3}{e^2-1}\right)$
61. The center of mass is on the axis of the cone, one-fourth of the way from the base towards the vertex.
63. The center of mass is on the axis of the cone, two-fifths of the way from the base towards the vertex.
65. $(3/5, 0, 112/(75\pi))$
67. $I_z = \pi\rho/2$, $r_z = 1/\sqrt{2}$
69. $(0, 0, 1/2)$, $I_z = \pi/24$, $r_z = 1/2$
71. $(0, 0, 8/15)$, $I_z = \pi\rho/12$, $r_z = 1/\sqrt{3}$
73. $I_z = 99\pi\rho$, $r_z = \sqrt{22}/3$
75. $I_z = 16\pi k/3$, $r_z = 2/\sqrt{3}$
77. $(\sqrt{8}, \pi/6, \pi/4)$ 79. $(2, -\pi/2, 5\pi/6)$
81. $(1, 0, 1)$ 83. $(0, -3, 0)$
85. $\rho = 2$ 87. $\tan^2 \varphi = 1$
89. $\tan \theta = 1$ 91. $\rho \sin \varphi = 2$
93. Unit sphere centered at the origin; $x^2 + y^2 + z^2 = 1$
95. Cone; $z = \sqrt{x^2 + y^2}$
97. The xy -plane; $z = 0$
99. Cylinder; $x^2 + y^2 = 9$

Section 14.5

1. $(\sqrt{2}, \pi/4, 2)$ 3. $(12, -5\pi/6, 0)$
5. $(-\sqrt{2}, \sqrt{2}, \sqrt{2})$ 7. $(0, -1, 1)$
9. $r = 4 \cos \theta$ 11. $r = 2 \sin \theta$
13. $z = r/3$ 15. $z = e^{-r^2/2}$
17. Circular cylinder; $x^2 + y^2 = 1$
19. Ellipsoid centered at the origin; $x^2 + y^2 + \frac{z^2}{2} = 1$
21. Circular double cone; $z^2 = x^2 + y^2$
23. Upper hemisphere of radius 1, centered at the origin;
 $z = \sqrt{1 - x^2 - y^2}$
25. $\int_0^{2\pi} \int_0^2 \int_0^{4-r^2} r dz dr d\theta$ 27. $\int_0^{2\pi} \int_0^4 \int_0^{4-r} r dz dr d\theta$
29. $\int_0^{2\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r dz dr d\theta$ 31. $\int_0^{2\pi} \int_0^1 \int_{\sqrt{2+r^2}}^{\sqrt{3}} r dz dr d\theta$
33. $\int_1^5 \int_0^{2\pi} \int_0^{z/2} r dr d\theta dz$ 35. $\int_{\sqrt{2}}^{\sqrt{3}} \int_0^{2\pi} \int_0^{\sqrt{z^2-2}} r dr d\theta dz$
39. $\pi/6$ 41. $k\pi/4$ 43. $\frac{4\pi(3-e)}{3}$
45. $\pi/2$ 47. $\pi/2$ 49. $10\pi k/3$
101. $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^2 \sin \varphi d\rho d\varphi d\theta$
103. $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{6\cos \varphi} \rho^2 \sin \varphi d\rho d\varphi d\theta$
105. $\int_{\pi/6}^{\pi/3} \int_0^{\pi/2} \int_0^5 \rho^2 \sin \varphi d\rho d\varphi d\theta$
107. $125\pi/18$ 111. $\frac{32(\sqrt{2}-1)\pi}{3}$
113. $8\pi/3$ 115. $(0, 0, 3R/8)$
117. a. $8\pi\rho$ b. $(0, 0, 7/3)$
121. $I_z = 8\pi R^5 \rho/15$, $r_z = \sqrt{2/5}R$
123. $I_z = 76\pi k/9$
127. a. It is the first octant portion of the unit sphere, centered at the origin.
 b. $(3/8, 3/8, 3/8)$
 c. $I_x = I_y = I_z = \pi/15$
 d. $r_x = r_y = r_z = \sqrt{2/5}$
129. a. The cardioid $\rho = 1 + \cos \varphi$, (or in standard polar coordinates, $r = 1 + \cos \theta$) in the xz -plane, rotated about the z -axis.
 b. $8\pi/3$

131. $\pi(\sqrt{3}-1)/8$
 133. $\int_{-\pi/2}^{\pi/2} \int_{\pi/4}^{\pi/2} \int_0^{2\csc\varphi\cos\theta} \rho^3 \sin^2\varphi \cos\theta d\rho d\varphi d\theta = \frac{64}{15}$
 135. $(16/35, 16/35, 16/35)$

Section 14.6

1. 4 3. 3 5. 3
 7. $8v-4u$ 9. 2 11. v 13. -1
 15. $x = \frac{v-u}{4}, y = \frac{3u+v}{4}$
 17. $x = \frac{4(v-u)}{7}, y = \frac{u+6v}{7}$
 19. $x = \frac{v-u}{4}, y = \frac{u+v}{2}$
 21. $816/49$ 23. $\frac{4\ln 3}{7}$ 25. $\frac{(e^4-1)\ln 3}{7}$
 27. $2\ln 2$ 29. ab 33. $\frac{36}{5}-4\sqrt{2}$
 35. 70; the answers are equal.
 37. -1 43. 112
 45. $I_x = 2\rho\pi, I_y = \rho\pi/2, I_0 = 5\rho\pi/2;$
 $r_x = 1, r_y = 1/2, r_0 = \sqrt{5}/2$
 47. $4\pi abc/3$

Chapter 14 Review

1. Approximation: $315/8$; true value: $81/2$
 3. 34 5. 3 7. 2 9. 120
 11. $\int_0^2 \int_0^{3y} (xy-1) dx dy + \int_2^8 \int_0^{8-y} (xy-1) dx dy = 120$
 13. 72 15. $\frac{2}{5}(2^{5/4}-1)$ 17. $\pi/2$
 19. $4/5$ 21. $(78/35, 6/5)$
 23. $I_x = 32\rho/3, I_y = 8\rho/3, I_0 = 40\rho/3;$
 $r_x = 2\sqrt{2/3}, r_y = \sqrt{2/3}, r_0 = \sqrt{10/3}$
 25. $I_x = 2048\rho/105, I_y = 64\rho/15, I_0 = 832\rho/35;$
 $r_x = 8\sqrt{2/35}, r_y = 2/\sqrt{5}, r_0 = 2\sqrt{39/35}$
 27. $I_0 = (3\pi-4)\rho/6$
 29. $(3\sqrt{3}+2\pi)/6$ 31. $(16\pi-21\sqrt{3})/24$
 33. $\frac{(\sqrt{6}-\sqrt{2})\pi}{2}$ 35. $\frac{1}{\sqrt{3}}-\frac{1}{2}$
 37. $\pi/2$ 39. $81\pi/2$ 41. 81π 43. 375
 45. $1/8$ 47. $8/3$ 49. $32/15$ 51. $2\sqrt{2}\pi$

53. $\int_0^1 \int_0^{2-2x} \int_0^{4-4x-2z} 5xy dy dz dx = \frac{4}{3}$
 55. $(3/5, 3/5, 3/5)$
 57. $(3/2, 1/2, 3/2);$
 $I_x = 48\rho, I_y = 432\rho/5, I_z = 48\rho;$
 $r_x = 2, r_y = 6/\sqrt{5}, r_z = 2$
 59. $(2, -\pi/4, -\sqrt{2})$ 61. $(2\sqrt{2}, \pi/4, 3\pi/4)$
 63. $(1, -1, -\pi/4)$ 65. $(2, -2, -2\sqrt{6})$
 67. $z = r(r-4\sin\theta)$ 69. $\rho = 2\sin\phi\cos\theta$
 71. $z = x^2 + y^2 - 1$ 73. $z = -\sqrt{\frac{x^2+y^2}{3}}$ 75. $81\pi/2$
 77. $243\pi; (0, 0, 9/4)$ 79. 9π 81. $(0, 0, 27/16)$
 83. $\frac{9\ln(8/3)}{10}$ 85. $x = \frac{u-2v}{7}, y = \frac{4u-v}{7}$
 87. $4736/7$ 89. 14
 93. False; let R be the unit disk centered at the origin, $f(x)$ be an odd function, and consider $g(x, y) = f(x)$ on R .
 95. False; this integral cannot be evaluated exactly in rectangular coordinates (see Example 3 of Section 14.3).
 97. False; see the definition of the spherical coordinate system in Section 14.5.
 99. $\left(\frac{6}{5}(\sqrt{2}-1), \frac{6}{5}(\sqrt{3}-\sqrt{2}), 0\right)$

Chapter 14 Conceptual Project

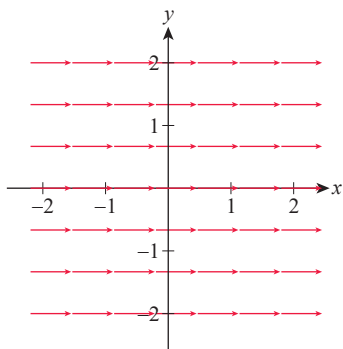
1. $\frac{\partial(x, y)}{\partial(r, \theta)} = abr$
 3. $\frac{\partial(x, y, z)}{\partial(\rho, \phi, \theta)} = -abc\rho^2 \sin\phi$
 5. $(0, 0, 3c/8)$
 7. $I_x = \frac{m}{5}(b^2+c^2), I_y = \frac{m}{5}(a^2+c^2), I_z = \frac{m}{5}(a^2+b^2);$
 $r_x = \sqrt{\frac{b^2+c^2}{5}}, r_y = \sqrt{\frac{a^2+c^2}{5}}, r_z = \sqrt{\frac{a^2+b^2}{5}}$

Chapter 15

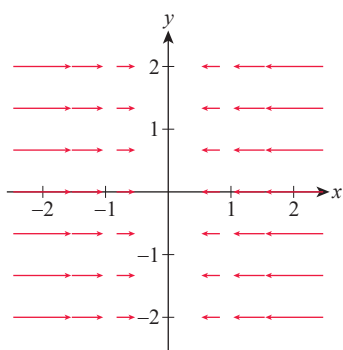
Section 15.1

1. Scalar field 3. Vector field 5. Vector field
 7. $\langle 0, -2 \rangle$ 9. $\langle 0, 1, 5 \rangle$ 11. B 13. A

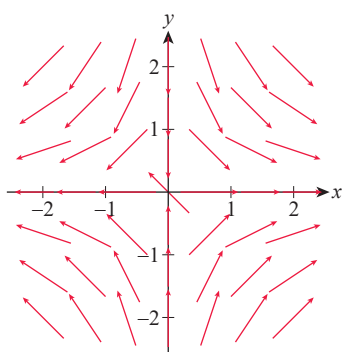
15.



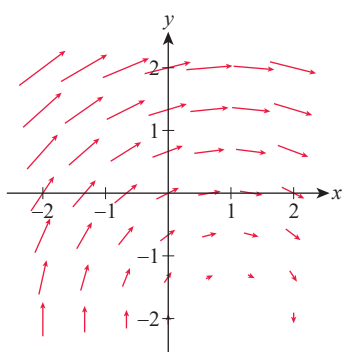
17.



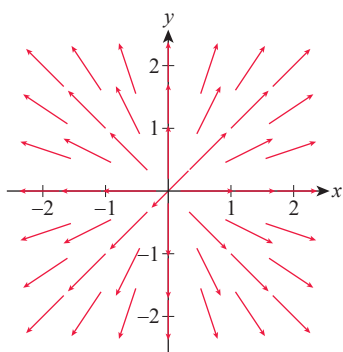
19.



21.



23.



25. A 27. B

$$29. \frac{\langle x, y \rangle}{x^2 + y^2} \quad 31. \left\langle \frac{y^2}{1+x^2}, 2y \tan^{-1} x \right\rangle$$

$$33. e^{xyz} \langle yz^2, xz^2, 1+xyz \rangle \quad 35. \frac{\langle -xz, -yz, x^2 + y^2 \rangle}{(x^2 + y^2 + z^2)^{3/2}}$$

37. The function $f(x, y, z) = \frac{-\varepsilon Q q}{\sqrt{x^2 + y^2 + z^2}}$ is a potential for $\mathbf{F}$.

39. If $\mathbf{F} = \langle F_1, F_2 \rangle$ is conservative, then $\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$.

41. Not conservative 43. Conservative

45. Conservative 47. Not conservative

$$49. f(x, y) = e^{xy^2} + C \quad 51. f(x, y) = \sqrt{x^2 + y^2} + C$$

$$53. f(x, y, z) = x^2 - y + z(x + 2y) + C$$

$$57. \nabla r^n = nr^{n-2} \mathbf{r} \text{ (or } \nabla r^n = nr^{n-1} \mathbf{e}_r \text{)}$$

59. $-1/r$ is a potential function.

61. The flow lines are the circles $x^2 + y^2 = C$, $C \in \mathbb{R}^+$.

Section 15.2

$$1. 16/3 \quad 3. 3\pi \quad 5. 8-3\sqrt{3} \quad 7. 41/5$$

9. Mass: $(\pi - 2)kR^2$, where k is the constant of proportionality; center of mass: $\left(0, \frac{(4-\pi)R}{2(\pi-2)}\right)$

$$11. \frac{13\sqrt{13}-5\sqrt{5}}{6} \quad 13. 24 + \frac{5\sqrt{5}}{2}; \text{ no}$$

$$15. \frac{16(10\sqrt{10}-1)}{3} \quad 17. \frac{2(1-e^{4\pi})}{\sqrt{5}}$$

$$19. 8\sqrt{5}\pi^2 k \quad 21. \frac{\sqrt{2} + \tanh^{-1}(1/\sqrt{2})}{2}$$

23. Mass: $\sqrt{2}k$ (where the density is $\rho = kz$); center of mass: $(0, 0, 2/3)$

$$25. 4\sqrt{10}\pi^2 \quad 27. \pi/2$$

29. a. 1 b. 1 c. 1; the answers are equal.

$$31. 0 \quad 33. 6 \quad 35. 36 \quad 37. 7/4 \quad 39. 9/20$$

$$41. 4\pi \quad 43. -k/20 \quad 45. 90 \quad 47. 7\pi \quad 49. \text{b. } 0$$

$$51. \frac{1}{4\pi\varepsilon_0} \cdot \frac{10^{-5}}{2\sqrt{2}} \text{ volts}$$

$$53. \frac{2}{4\pi\varepsilon_0} \cdot \frac{1 + \sinh^{-1}(2\pi) - \sqrt{4\pi^2 + 1}}{10^6 \sqrt{2}} \text{ volts}$$

$$55. I_x \approx 577.782\rho, I_y \approx 83.18\rho, I_z \approx 660.962\rho; r_x \approx 5.444, r_y \approx 2.066, r_z \approx 5.823$$

$$57. I_x \approx 6283.17, I_y \approx 1123.3, I_z \approx 7406.47; \\ r_x \approx 4.987, r_y \approx 2.109, r_z \approx 5.415$$

$$59. \text{Approx. } \frac{1.5077}{4\pi\epsilon_0 \times 10^5} \text{ volts}$$

Section 15.3

3. $\pi - e$ 5. $1 + e$
 7. $f(x, y) = x^3 + 2xy^2 + y^3$
 9. A potential function does not exist.
 11. $f(x, y) = y \sin x + e^x + \ln|y|$
 13. $f(x, y) = x^4 y - xy^5$ 15. $f(x, y, z) = x^2 yz$
 17. A potential function does not exist.
 19. $f(x, y, z) = x \tan z + y^2 z$
 21. A potential function does not exist.
 27. 36 29. $4e + 16$ 31. 3
 35. a. No, because the domain of $\mathbf{F}$ is not simply connected.
 b. The equality does not hold on the domain of $\mathbf{F}$, since $-\arctan(x/y)$ is not defined for $y = 0$.
 37. Yes 39. Yes 41. Yes
 51. False; only a simply connected region guarantees the statement.
 53. True

Section 15.4

1. $\text{div } \mathbf{F} = 15x^2 + 8xy - 2y$;
 $\text{div } \mathbf{F}(2, 1) = 74$; $\text{div } \mathbf{F}(1, -3) = -3$
 3. $\text{div } \mathbf{F} = 2(x - y)$;
 $\text{div } \mathbf{F}(2, -1, 0) = 6$; $\text{div } \mathbf{F}(1, 1, 4) = 0$
 5. $\text{div } \mathbf{F} = 0$;
 $\text{div } \mathbf{F}(1, 1, 1) = 0$; $\text{div } \mathbf{F}(2, -3, 4) = 0$
 7. $\text{div } \mathbf{F} = e^y + 4z^3 + 1$;
 $\text{div } \mathbf{F}(1, 0, -3) = -106$; $\text{div } \mathbf{F}(2, 1, 4) = 257 + e$
 9. $\text{div } \mathbf{F} = e^{xyz}(yz + xy + xz)$;
 $\text{div } \mathbf{F}(1, 0, 1) = 1$; $\text{div } \mathbf{F}(e, 1, -1) = -e^{-e}$
 11. $\text{curl } \mathbf{F} = 2(x + 2y^2)\mathbf{k}$
 13. $\text{curl } \mathbf{F} = -2z(3x + 1)\mathbf{i} + 3z^2\mathbf{k}$
 15. $\text{curl } \mathbf{F} = \mathbf{0}$ 17. $\text{curl } \mathbf{F} = -3y^2\mathbf{k}$
 19. $\text{curl } \mathbf{F} = e^{xyz}[x(z - y)\mathbf{i} + y(x - z)\mathbf{j} + z(y - x)\mathbf{k}]$
 21. At A , divergence is 0. At B , divergence is positive.
 At C , divergence is negative.
 23. $\text{curl } \mathbf{F} = -\mathbf{k}$

25. $\text{curl } \mathbf{F} = -2y\mathbf{k}$; negative spin above the x -axis, positive below
 27. $\text{curl}(\text{curl } \mathbf{F}) = (-2x - 8z)\mathbf{i} + 2y\mathbf{j}$
 29. $\text{curl}(\text{curl } \mathbf{F}) = -2x\mathbf{i} + 2y\mathbf{j}$
 31. No meaning 33. Vector field
 35. Scalar field 37. No meaning
 49. Both integrals are equal to 8.
 51. Both integrals are equal to 2π .
 53. $8/3$ 55. $3/4$ 57. $11/15$ 59. 0
 61. -2 63. $\pi/2$ 65. 0 67. 0 69. 4
 71. 18π 73. $352/7$ 75. $e^5 - 5e - 6$ 77. πr^2
 79. 6 81. 48 83. 4 85. 9 87. 12
 89. $5\pi ab$ 93. 0 97. $\left(0, \frac{4R}{3\pi}\right)$
 101. True 103. True

Section 15.5

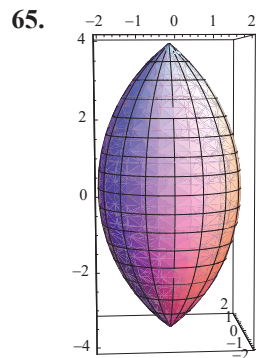
1. Possible parametrization:
 $\mathbf{r}(s, t) = \left\langle s, \frac{1}{3}s \cos t, \frac{1}{3}s \sin t \right\rangle$,
 $0 \leq s \leq 6, 0 \leq t \leq 2\pi$
 3. Possible parametrization:
 $\mathbf{r}(s, t) = \left\langle (1 - s^2) \cos t, (1 - s^2) \sin t, s \right\rangle$,
 $-1 \leq s \leq 1, 0 \leq t \leq 2\pi$
 5. C 7. H 9. F 11. D 13. G
 15. The grid curves are quarter circles of radius R . The graph is the first octant portion of the sphere of radius R , centered at the origin.
 17. The grid curves are parabolas. The graph is an elliptic paraboloid.
 19. The grid curves are parabolas if t is held constant and circles if s is held constant. The graph is an elliptic paraboloid.
 21. $\mathbf{r}(s, t) = \langle s, t, s + t \rangle$, $-\infty < s < \infty, -\infty < t < \infty$
 23. $\mathbf{r}(s, t) = \langle \cos s, \sin s, t \rangle$, $0 \leq s \leq 2\pi, 0 \leq t \leq 1$
 25. $\mathbf{r}(s, t) = \langle 2 \sin s \cos t, 2 \sin s \sin t, 2 \cos s \rangle$,
 $0 \leq s \leq \pi, 0 \leq t \leq 2\pi$
 27. $\mathbf{r}(s, t) = \langle \sin s \cos t, \sin s \sin t, \cos s \rangle$,
 $\pi/2 \leq s \leq \pi, 0 \leq t \leq 2\pi$
 29. $\mathbf{r}(s, t) = \langle 2 \cos s, \sin s, t \rangle$, $0 \leq s \leq 2\pi, -\infty < t < \infty$
 31. $\mathbf{r}(s, t) = \langle 2 \sin s \cos t, 2 \sin s \sin t, 2 \cos s \rangle$,
 $\pi/6 \leq s \leq 5\pi/6, 0 \leq t \leq 2\pi$
 33. $4x + 4y - z = 6$ 35. $\sqrt{2}x + 2\sqrt{2}y - z = 4$

37. $2\sqrt{3}x - y + 2z = 3$ 41. 12π 43. $\pi/2$
 45. $\frac{(5\sqrt{5}-1)\pi}{6}$ 47. $\frac{(37\sqrt{37}-17\sqrt{17})\pi}{6}$
 49. $\frac{(17\sqrt{17}-1)\pi}{24}$ 51. $\frac{2(5\sqrt{5}-2\sqrt{2})\pi}{3}$

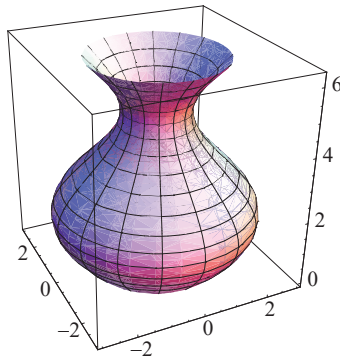
53. $\sqrt{5} + \frac{\sinh^{-1} 2}{2}$ 55. 10π 57. $4ab\pi^2$

61. False; consider the formula for the area of a surface generated by revolving a (single-variable) function graph about a coordinate axis.

63. True



67. For $a = 2$:



The area is approx. 96.012.

Section 15.6

5. $4\sqrt{6}$ 7. $\frac{a^3}{2\sqrt{3}}$ 9. $7\sqrt{2/3}$ 11. $\frac{23}{2\sqrt{6}}$
 13. $\frac{149}{60\sqrt{2}}$ 15. $\frac{(25\sqrt{5}+1)\pi}{120}$ 17. $149\pi/30$
 19. $\frac{(2\sqrt{2}-1)\pi}{12}$ 21. 8π 23. $152/3$ 25. 0
 27. $4(\pi+4)$ 29. $\left(0, \frac{2(2+\pi)}{4+\pi}, \frac{4(3+\pi)}{3(4+\pi)}\right)$
 31. $35k\pi$, where the density function is $\rho(x, y, z) = kz$
 33. $I_x = I_y = I_z = 8\pi\rho R^4/3$

35. 27π 37. $\pi/2$ 39. $\pi/2$ 41. $-81\pi/2$

43. 3π 45. 3π 47. 81π 49. -20π 51. 6π

53. $\frac{8\kappa\pi}{r}k$, where k is the constant of proportionality

59. $I_x = I_y = 2625k\pi/4$, $I_z = 875k\pi/2$, where k is the constant of proportionality

61. $I_x = I_y = \frac{425\sqrt{5}-163}{840}\pi\rho$, $I_z = \frac{25\sqrt{5}+1}{60}\pi\rho$

Section 15.7

1. Both integrals are equal to 4π .

3. Both integrals are equal to 2π .

5. 4π ; the answers are equal.

7. 2π 9. 20 11. π 13. 16π 15. 2π

17. $8/15$ 19. 35π 21. 4π

Section 15.8

1. Both integrals are equal to $4\pi/3$.

3. Both integrals are equal to $3a^4/2$.

5. $64\pi/3$ 7. $380\pi/3$ 9. 6π 11. $3\pi/4$

13. π 15. 15π 17. $121\pi/5$

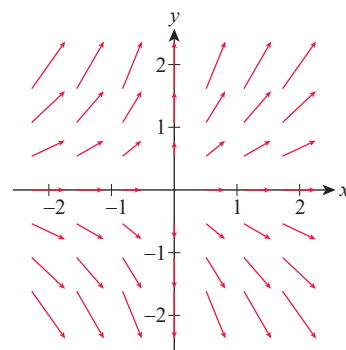
19. Both integrals are equal to 16π .

21. $16(e^3 + 26)$ 23. $V = abc$ 25. $V = \frac{1}{3}\pi R^2 h$

Chapter 15 Review

1. C 3. B

5.



7. C 9. D 11. $\langle 4xy + z^3, 2x^2, 3xz^2 \rangle$

13. Conservative 15. Conservative

17. $f(x, y) = 4x^3 - 2x^2y + y^3$

19. $32/3$ 21. $7\sqrt{19}$

23. $\left(\frac{17-3\sqrt{21}}{20}, \frac{23-3\sqrt{21}}{4}, 2\right)$

25. Center of mass:

$$\left(\frac{3}{2 + \sqrt{1 + \pi^2}}, \frac{2\sqrt{1 + \pi^2} - \pi}{2\pi + \pi\sqrt{1 + \pi^2}}, \frac{\sqrt{1 + \pi^2} - 2\sqrt{2}}{2(2 + \sqrt{1 + \pi^2})} \right)$$

27. $1344/5$ 29. 20 31. $\frac{12(3 + \pi)}{4\pi\epsilon_0} \cdot 10^{-7}$ volts

33. e 35. $110 - \ln 13$ 39. $1 + 4\sqrt{e}$

45. $\operatorname{div} \mathbf{F} = x^2 - x + \frac{1}{y^2 + 1};$

$$\operatorname{curl} \mathbf{F} = \left(2xy + \frac{2xy}{(y^2 + 1)^2} - y \right) \mathbf{k}$$

47. $\operatorname{div} \mathbf{F} = 2yz + x + \sin(yz);$

$$\operatorname{curl} \mathbf{F} = (x - y^2)\mathbf{i} + (xy \cos(yz) - y - z)\mathbf{j} - xz \cos(yz)\mathbf{k}$$

49. Both integrals are equal to $-1/3$.

51. 0 53. 4 55. $(6\pi + 32)/3$ 57. 14.75

59. B 61. A 63. C

65. $\mathbf{r}(s, t) = \left\langle \frac{\cos t}{s^2}, s, \frac{\sin t}{s^2} \right\rangle, 1 \leq s \leq 4, 0 \leq t \leq 2\pi$

67. The grid curves are ellipses if s is held constant and lines through the origin if t is held constant. The graph is an infinite elliptic double cone.

69. $\mathbf{r}(s, t) = \langle s \cos t, \sqrt{2}s \sin t, 2s^2 \rangle, 0 \leq t \leq 2\pi, 0 < s < \infty$

71. $z = 3x + 9y - 9$

73. $27\pi/2$ 75. $2(9 - \sqrt{3})\pi$ 77. $\pi/6$

79. $(2/\pi, 2/\pi, 3/4)$ 81. 12π 83. 28

87. Both integrals are equal to -12π .

89. $\pi/2$ 91. -72π 93. 12π 95. 80π

97. Both integrals are equal to π .

99. $405\pi/2$ 101. 12π 103. $93\pi/5$ 105. True

107. True (If the domain is simply connected.)

109. False; the work is only guaranteed to be zero if the path is closed. See Exercise 3 of Section 15.6.

111. True 113. Mass: $8\pi\rho$; center of mass: $(0, 0, 1)$

115. Mass: $\frac{(289\sqrt{17} - 41)k\pi}{60};$

center of mass: $\left(0, 0, \frac{4913\sqrt{17} - 1177}{7(289\sqrt{17} - 41)} \right)$

Chapter 15 Conceptual Project

7. If we choose $C_2 = C_3 = 0$, one possible answer is

$$\mathbf{P}(x, y, z) = \left\langle 0, \frac{x^3 y}{3}, x^2 y^2 z \right\rangle.$$

Chapter 15 Application Project

1. $\nabla \cdot \mathbf{F} = 3$

3. $\iiint_D \nabla \cdot \mathbf{F} dV = 3V$ and $\left| \iint_S \mathbf{F} \cdot \mathbf{n} d\sigma \right| \leq \iint_S |\mathbf{F} \cdot \mathbf{n}| d\sigma \leq rA,$
so $3V \leq rA$, or $V \leq \left(\frac{r}{3}\right)A$.

5. $A(k) = \frac{k}{2} \sin \frac{2\pi}{k}$ and $L(k) = 2k \sin \left(\frac{\pi}{k}\right)$, so
 $\frac{A(k)}{L(k)} = \frac{1}{2} \cos \left(\frac{\pi}{k}\right) < \frac{1}{2}$ for all k and the ratio
approaches $\frac{1}{2}$ as $k \rightarrow \infty$.