

Chapter 6 Application Project:

3 Calculations to Blast Off!

In this project, we will examine the motion of a rocket after launch. As you know, a rocket is propelled by the force caused by the exiting hot gases that result from rapidly burning fuel. This force is called the *thrust* of the rocket. The difficulty of analyzing rocket propulsion arises from the fact that the mass of the fuel, which comprises a large portion of the rocket's mass, decreases rapidly during flight due to the high rate of fuel burn. This rapid decrease in mass occurs up until the moment when all fuel is used up (a moment known as the *burnout point*). This means that the net force acting on the rocket is also a nonconstant function of time but, as you will discover, integration helps us overcome the challenge of rapidly changing mass. Throughout the project, we will ignore air resistance and assume that the acceleration caused by gravity $g \approx 9.81 \text{ m/s}^2$ is constant (in the case of rockets that fly to high altitudes, this is not necessarily the case).

Under the above assumptions, we use Newton's Second Law of Motion to obtain the following equation:

$$F_n(t) = F_t(t) - m(t)g = m(t)a(t), \quad (1)$$

where F_t denotes the propelling force or thrust. This force arises from the fact that the mass of the burnt fuel is rapidly leaving the rocket-fuel system, thus giving it upward momentum.

Although we omit the details of deriving it here, we will also use the fact that the thrust can be obtained as the product of v_f , the relative speed at which the burnt fuel is exiting the rocket, and the rate of decrease in mass:

$$F_t(t) = v_f \cdot \frac{dm(t)}{dt}.$$

1. Suppose a rocket of mass eighteen metric tons is fired vertically upward. Of its total mass, fuel accounts for twelve metric tons. The hot gases resulting from burned fuel are leaving the rocket at a relative speed of 2500 meters per second and at a rate of 150 kilograms per second. Calculate the thrust F_t propelling the rocket upward.
2. Use Equation (1) to calculate the net force F_n acting on the rocket
 - a. at the moment of blastoff;
 - b. just before all fuel burns away;
 - c. after all fuel has burnt away.

3. Use Equation (1) to find the rocket's velocity function $v(t)$, as follows. From Equation (1), we obtain

$$m(t)a(t) = m(t) \cdot \frac{dv(t)}{dt} = v_f \cdot \frac{dm(t)}{dt} - m(t)g;$$

then we use differential notation to arrive at

$$mdv = v_f dm - mgdt.$$

Use this equation to solve for dv , and then obtain $v(t)$ by integration:

$$v(t) = \int_0^t dv.$$

4. Use your answer to Question 3 to find the terminal velocity of the rocket after all fuel has been used up. (**Hint:** Be careful. The velocity of the exiting fuel is negative.)