Chapter 5 Conceptual Project: Looking For a Sine

The topic of this project is the so-called *sine integral function*, which is important for its applications, most notably in electrical engineering and signal processing.

1. Consider the following piecewise-defined function.

$$f(t)=\begin{cases}\dfrac{\sin t}{t} & \text{if } t>0\\ 1 & \text{if } t=0\end{cases}$$

Prove that for any $x \geq 0$, $f(t)$ is integrable on $[0,x]$.

2. The **sine integral function** is defined as follows.

$$\mathrm{Si}(x)=\int_0^x f(t)\,dt, \text{ for } x \geq 0$$

Prove that $\mathrm{Si}(x)$ is continuous.

3. Find the derivative $\dfrac{d}{dx}\mathrm{Si}(x)$.

4. Without graphing first, write a short paragraph on why you would expect the graph of $\mathrm{Si}(x)$ to be oscillating. Explain why its amplitude is expected to decrease as $x \to \infty$.

5. Find the x-values where the relative maxima and minima of $\mathrm{Si}(x)$ occur.

6. Extend the definition of $\mathrm{Si}(x)$ to negative x-values and prove that for any $a > 0$, $\int_{-a}^{a} \mathrm{Si}(x)\,dx = 0$.

7. Use a graphing utility to plot the graph of $\mathrm{Si}(x)$ on the interval $[-8\pi, 8\pi]$.

8. Use a graphing utility to approximate the range of $y = \mathrm{Si}(x)$ to four decimal places.