Chapter 5 Application Project: Too Math Too Furious

When we talk about acceleration of cars (for example, when discussing times necessary to accelerate from zero to sixty miles per hour), we often assume their acceleration is constant. This makes it easy to perform speed and distance calculations (see Question 1 below). However, in real life, constant acceleration over long periods of time may not be realistic. Air resistance, an engine's torque delivery, changing road conditions, and potential wheel spin can all lead to variable acceleration. Air drag alone, which is proportional to the square of velocity, has a strong effect on acceleration (see Question 7 of the Chapter 3 Application Project). For example, at the very high speeds experienced by race cars, air resistance is strong enough that simply stepping off the accelerator creates a sense of hitting the brakes with full force! It would therefore be realistic to expect acceleration to decrease at higher speeds. In this project, we will illustrate the power of integration by considering motion problems where the accelerating vehicles have time-dependent (nonconstant) acceleration. We will start with an ultrafast Porsche model.

1. The 2021 Porsche 911 Turbo S reaches 150 mph from a standstill in 13.6 seconds. In the analysis that follows, we will initially use the (unrealistic) simplifying assumption that acceleration is constant throughout the 0–150 mph test run.

a. Find the presumably constant acceleration a (in ft/s^2).

b. Use antidifferentiation (as in Example 4 of Section 4.7) to find the accelerating car's velocity $v = v(t)$ as a function of time (in ft/s).

c. Find an integral formula for the velocity function in part b. (**Hint**: Use the Fundamental Theorem of Calculus, Part I.)

d. Find the distance (in feet) covered by the car during the acceleration run.

2. The table below shows the actual acceleration times of the 2021 Porsche 911 Turbo S from zero to various speeds up to 150 mph. (Units are in miles per hour and seconds. As a side note, the car is actually capable of a top speed of 205 mph!)

2021 Porsche 911 Turbo S Acceleration Times

Increase in Speed (mph)	0–30	0–40	0–50	0–60	0–70	0–80	0–90	0–100	0–150
Time (s)	0.9	1.4	1.8	2.3	3.0	3.7	4.5	5.6	13.6

Source: *Motor Trend*

Table 1

a. Use the data from Table 1 to explain why the acceleration $a = a(t)$ is actually a nonconstant function of time, rendering our simplifying assumption in Question 1 unrealistic.

b. What features would you anticipate for the graph of $a(t)$ to possess? Describe these features, mentioning the first derivative and concavity. (You may want to plot a few points using data from the table.)

Notice that Table 1 gives us values of the Porsche's velocity function $v = v(t)$ at various points on the time axis during the acceleration run. We will use these values to approximate the total distance covered during the run. (Note that this is the same as the displacement of the car from its starting position, since there is no change of direction during this type of test run.) To start off, notice from Table 1 that the Porsche reaches 30 mph in the first 0.9 seconds. A crude approximation of the distance covered while doing so can be obtained by taking the average of the speeds at the two endpoints of this time interval, at $t = 0$ and $t = 0.9$ (where the speeds are 0 mph and 30 mph, respectively), and assuming that the speed is constant and equal to this average value throughout the entire time interval.

3. **a.** Use the technique described above to approximate the displacement during the first 0.9 seconds of the run, and then on the second time interval, from 0.9 seconds to 1.4 seconds. Add up the results to obtain an estimate for the total displacement during the first 1.4 seconds of the run. Express your answer in feet.

b. Continue the process from part a. over all consecutive time intervals from Table 1 and add up the results to obtain an estimate for the total displacement (in feet) during the entire 0–150 mph acceleration run. What is the name of the sum you just formed?

Notice that if we had more data in Table 1, we could work with shorter time intervals in order to arrive at more accurate estimates for displacement. Better yet, if we had a formula for $v = v(t)$, we could use a definite integral to calculate the actual displacement, much like we did in Example 1 of Section 5.2. You can use this observation to answer Question 4. Then, in the subsequent problems, we will generalize our analysis.

4. Use your work on Question 3 to give a definite integral interpretation of the total distance d traveled by the Porsche during its acceleration run. Find a formula in terms of $v(t)$.

5. Suppose an object is accelerating along a straight line from $t = t_0$ to $t = t_1$ and its acceleration is given by $a = a(t)$, while its velocity is $v = v(t)$.

a. Use the definite integral to give a formula for the total displacement of the object in terms of $v(t)$. (**Hint**: Generalize your answer to Question 4.)

b. Given $v(t)$, find an integral formula for the displacement function $d(t)$ on the interval $[t_0, t_1]$. (**Hint**: Generalize your answer to part a.)

c. Use $a(t)$ and integration to arrive at an integral formula for the velocity function $v(t)$ on the interval $[t_0, t_1]$. (**Hint**: Generalize your answer to Question 1c.)

d. Explain the validity of the formula you have given in part c. above. Use Riemann sums in your argument.

6. An experimental race car starts at a standstill and accelerates in a straight line. Suppose its acceleration can be described by the function $a(t) = (31 - 2t)\ \text{ft/s}^2$.

a. What is its velocity (in mph) five seconds later?

b. How far is it from the start at that instant?

c. Use technology to find the vehicle's quarter-mile time. (This means the time needed for it to run a quarter mile from its starting point.)