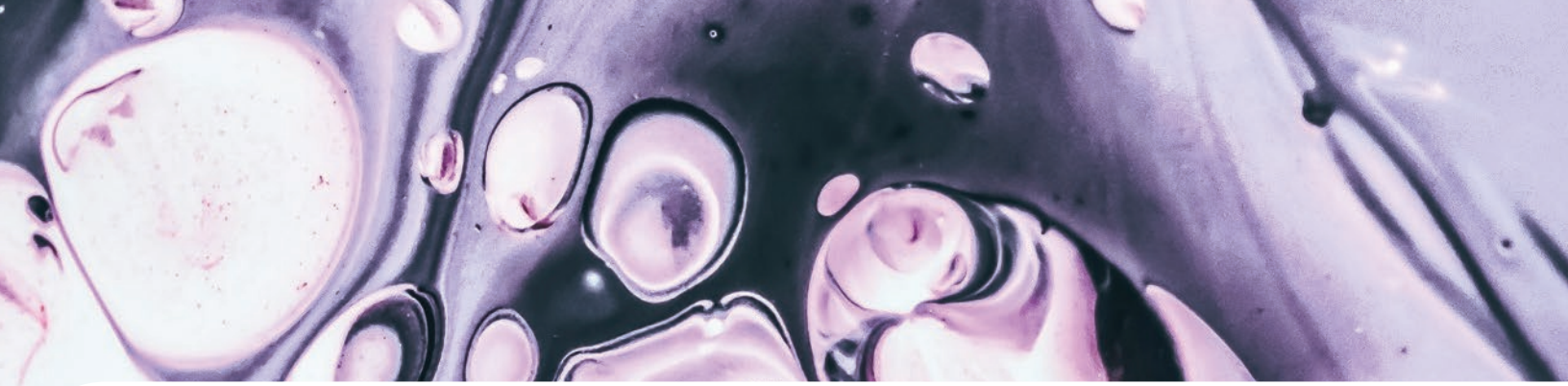

Chapter 4 Application Project: Cutting Corners with Cappuccinos

Suppose the management of a coffee shop chain wants to minimize the daily costs associated with delivery and storage for its franchises. In this project, we will find a formula for accomplishing that, given certain simplifying assumptions.

We will assume that the *holding costs* for a franchise are directly proportional to storage time as well as the total amount of unsold (whole bean or ground) coffee they have on hand. (Holding costs in general are the costs associated with storing unsold inventory. Some of the contributors to these expenses are the facts that the capital already invested in the inventory cannot earn interest elsewhere, as well as the costs of storage space, utilities, insurance, labor, damaged or spoiled items, etc.) We will assume that each delivery comes with a flat charge, regardless of the amount of coffee delivered. In addition, daily consumer demand is assumed to be constant, meaning the total amount of coffee purchased at the franchise is a linear function of time. Our notation will be the following.

f = delivery fee in dollars
a = delivery amount in pounds
h = holding costs in dollars per pound per day
c = customer demand in pounds per day
t = time in days
x = number of days between two successive deliveries

1. Suppose a delivery in the amount of a_0 pounds of coffee arrives at a franchise. In a coordinate system where the horizontal axis represents time t measured in days and the vertical axis stands for the amount A of coffee measured in pounds, sketch the graph of $A(t)$, the amount of coffee at the store as a function of time, from the point of delivery until the store runs out of supplies. What type of function is it? (Suppose delivery happens at $t = 0$. Be careful. Neither time, nor the coffee amount stored, can ever be negative.)

2. Using a_0 and c as *parameters* (unspecified, but fixed values), answer the following questions.

a. Find the equation of the graph you sketched in Question 1.

b. Use your answer from part a. to find a formula for t_0, the number of days it takes for the franchise to run out of supplies. This length of time is called *days sales of inventory*, or DSI in the business world.

3. Suppose that during the first day after delivery, the franchise's coffee supply decreases from the initial a_0 pounds to a_1 pounds. Explain why it is reasonable to calculate their holding cost for the day as follows.

$$H_1 = \frac{a_0 + a_1}{2} h$$

(In other words, by multiplying the *average daily inventory* by the holding cost per pound per day. **Hint:** Refer to Question 1.)

4. Now consider the first two days of the DSI cycle. Assuming that there are still a_2 pounds of coffee left at the end of the second day, use your answer to Question 3 to show that the total holding costs the franchise incurs during the first two days of the cycle can be obtained as follows.

$$H_2 = \frac{a_0 + a_2}{2}(2h) = (a_0 + a_2)h$$

(In other words, we can again take the *average inventory* for the first two days and multiply by twice the daily holding cost per pound of coffee.)

5. Generalizing the result from Question 4, answer the following questions.

a. Find a formula for the total holding cost H_{t_0} for a full DSI cycle of t_0 days. (**Hint:** Use the fact that the amount stored at the endpoint of the cycle is zero pounds, and again work with the average inventory.)

b. Find a formula for the total cost C_{t_0} incurred by the franchise from delivery and storage over the cycle in part a.

The remainder of this project is devoted to finding a formula for the length of the DSI cycle that minimizes the storage-and-delivery expenses for a franchise. Customer demand, the delivery fee, and the daily holding costs per pound will be assumed to be constant, but we will treat the delivery amount and the length of the DSI cycle as variables (denoted by *a* and *x*, respectively). As you might have discovered already from answering Questions 1–4, the delivery amount will determine the length of time before the next shipment becomes necessary; in other words, the two variables are strongly related.

6. Letting *a* denote the amount of coffee (in pounds) delivered to the franchise and *x* stand for the time it takes (in days) for the store to run out of supplies, answer the following.

a. Find an equation relating the variables *a* and *x*. (**Hint:** Refer to Question 2b.)

b. Use your answers from Questions 5b and 6a to express the total cost *C* as a function of DSI cycle length; that is, find a formula for the function below.

$$C = C(x)$$

c. Find a formula for the function $D(x)$, the average daily cost stemming from delivery and storage expenses. (**Hint:** There are *x* days in a DSI cycle.)

7. Find the (positive) critical point of the daily cost function $D(x)$ of Question 6c, and use the Second Derivative Test to obtain a formula for the length of the DSI cycle that minimizes the daily delivery and storage-related expenses for the franchise.

8. Sketch a possible graph for the daily cost function $D(x)$. Briefly discuss the main features of the graph.