

Chapter 2 Conceptual Project: Before Unlimited Calls

Some years ago, it was common for long-distance phone companies to charge their customers in one-minute increments. In other words, the company charges a flat fee for the first minute of a call and another fee for each additional minute or any fraction thereof (see Exercise 82 in Section 2.5). In this project, we will explore in detail a function that gives the cost of a telephone call under the above conditions.

1. Suppose a long-distance call costs 75 cents for the first minute plus 50 cents for each additional minute or any fraction thereof. In a coordinate system where the horizontal axis represents time t and the vertical axis price p , draw the graph of the function $p = C(t)$ that gives the cost (in dollars) of a telephone call lasting t minutes, $0 < t \leq 5$.
2. Does $\lim_{t \rightarrow 1.5} C(t)$ exist? If so, find its value.
3. Does $\lim_{t \rightarrow 3} C(t)$ exist? Explain.
4. Write a short paragraph on the continuity of this function. Classify all discontinuities; mention one-sided limits and left or right continuity where applicable.
5. In layman's terms, interpret $\lim_{t \rightarrow 2.5} C(t)$.
6. In layman's terms, interpret $\lim_{t \rightarrow 3^-} C(t)$.
7. In layman's terms, interpret $\lim_{t \rightarrow 3^+} C(t)$.
8. If possible, find $C'(3.5)$.
9. If possible, find $C'(4)$.
10. Find and graph another real-life function whose behavior is similar to that of $C(t)$. Label the axes appropriately and provide a brief description of your function.