

**Step 3:** Subtract the margin of error from and add the margin of error to the point estimate.

Finally, add and subtract  $E$  from  $\bar{d}$  to obtain the endpoints of the interval.

$$\begin{aligned}\text{Lower endpoint: } & -10.461538 - 6.983759 \\ & \approx -17.4\end{aligned}$$

$$\begin{aligned}\text{Upper endpoint: } & -10.461538 + 6.983759 \\ & \approx -3.5\end{aligned}$$

Hence we find the 99% confidence interval to be  $(-17.4, -3.5)$ .

**TI-83/84 Plus:** Begin by entering the data in the lists L1 and L2. Let L1 be the “before” levels and L2 be the “after” levels. Since we need the differences between the pairs of data, we use L3 to calculate those differences for us. To do so, highlight L3 and enter the formula to subtract the “before” levels from the “after” levels ( $L2-L1$ ) by using the 2ND button to select the lists. The screenshot in the margin shows how the data and the paired differences will appear in the calculator.

Now that we have the paired differences in L3, we can create a one-sample  $t$ -interval using those paired differences as our raw data. Press STAT, scroll to TESTS, and choose option TInterval. We want to calculate the confidence interval from the data, so choose the Data option. Our data are in List 3, so enter L3 by pressing 2ND and then 3. The frequency of the data (Freq) is the default value, which is 1. Also, we want a 99% confidence interval, so enter 0.99 for C-Level. Highlight Calculate and press ENTER. The calculator returns a confidence interval of  $(-17.4, -3.5)$  as shown in the margin.

L1	L2	L3	3
238	235	-3	
240	241	1	
220	219	-1	
246	235	-11	
202	198	-4	
222	208	-14	
210	202	-8	

L3(1) = -3

TInterval  
 $(-17.44, -3.479)$   
 $\bar{x} = -10.46153846$   
 $Sx = 8.242323546$   
 $n = 13$

Note that both endpoints of the confidence interval are negative, which indicates that the mean cholesterol level decreased significantly. We are 99% confident that, after taking the new cholesterol-lowering drug for four weeks, the mean decrease in the total cholesterol levels for the population from which the participants were sampled is between 3.5 and 17.4 mg/dL.

## 9.3 Section Exercises

**Note:** For all exercises in this section, you may assume that the requirements mentioned in this section are met; namely, the samples are dependent samples of paired data, and the population distributions of the paired differences are approximately normal.

### Means and Standard Deviations of Paired Differences

Calculate  $\bar{d}$  and  $s_d$  for each set of paired data.

1.

Sample A	22	21	19	20	17	18	20	19	22
Sample B	24	23	23	19	20	21	23	19	23

2.

Sample 1	2	1	1	2	1	1	2	1	2
Sample 2	4	3	2	1	2	2	2	1	3

3.

Diastolic Blood Pressure Readings for Patients at Consecutive Check-ups									
Check-up 1	78	79	91	83	79	92	45	68	
Check-up 2	67	85	90	84	86	96	51	66	

4.

Number of Minutes Taken to Complete a 5K Race for 9 Runners									
Fall 5K	18	23	24	19	17	22	18	16	20
Spring 5K	24	23	23	19	20	21	23	19	23

### Confidence Intervals for Means of Paired Differences for Two Populations ( $\sigma$ Unknown, Dependent Samples)

*Formula practice; construct a confidence interval for the mean of the paired differences for the two populations using the given information.*

5.  $n = 41$ ,  $\bar{d} = 2.230$ ,  $\alpha = 0.01$ ,  $s_d = 0.567$
6.  $n = 30$ ,  $\bar{d} = 12.0$ ,  $\alpha = 0.05$ ,  $s_d = 2.7$
7.  $n = 5$ ,  $\bar{d} = 1.14$ ,  $\alpha = 0.10$ ,  $s_d = 1.30$
8.  $n = 25$ ,  $\bar{d} = 3.40$ ,  $\alpha = 0.05$ ,  $s_d = 1.08$

*Construct and interpret each specified confidence interval.*

9. To determine if a new cold medicine works better than the traditional cold medicine, 16 people who had a cold volunteered for a study. The volunteers were matched based on age to create eight pairs. A double-blind study was constructed where one of the volunteers in the pair was given the new cold medicine and the other member of the pair was given the traditional medicine. The duration of the cold (in days) was measured for each person and the results are shown in the following table. Construct and interpret a 99% confidence interval for the true mean difference between the durations of a cold for those taking the traditional medication and those taking the new medication.

Duration of Cold (in Days)								
New Medicine	4	5	3	6	3	4	5	7
Traditional Medicine	6	5	4	8	4	5	7	7

10. To determine if his teaching method increases students' learning, a professor administers a pretest to his class at the beginning of the semester and then a posttest at the end of the semester. The results from 15 randomly chosen students are given below. Construct and interpret a 95% confidence interval for the true mean difference between the scores to determine if the teaching method increases students' knowledge of the course material.

Test Scores	
Pretest	Posttest
60	80
61	87
65	91
71	97
68	89
67	86
65	85
62	83
63	89
68	93
69	94
70	99
65	92
62	91
57	83

11. A personal trainer believes that walking every day can produce the same health benefits as jogging. Ten volunteers are paired based on significant characteristics; half of the group is asked to walk every day and half of the group is asked to jog every day. The amounts of weight lost (in pounds) over a 30-day period are recorded in the following table. Construct and interpret a 95% confidence interval for the true mean difference between the amount of weight lost by jogging and the amount of weight lost by walking.

Weight Loss (in Pounds)					
Walking	8	9	10	7	9
Jogging	10	12	14	9	12

12. Researchers have developed a method to improve memory. To test their method, 12 participants are asked to memorize a list of words and the number of words remembered correctly is recorded. The participants are then taught the method to improve their memory and are asked to memorize another list of words and the number of words remembered correctly is again recorded. The results are shown in the following table. Construct and interpret a 98% confidence interval to estimate the true mean increase in the number of words that people can memorize after learning the memory method.

Number of Words Memorized	
Before	After
8	13
5	16
6	12
7	16
6	15
3	13
8	17
10	20
12	19
6	17
8	18
5	13

13. A pharmaceutical company is running tests to see how well its new drug lowers cholesterol. Ten adults volunteer to participate in the study. The total cholesterol level of each participant (in mg/dL) is recorded once at the start of the study and then again after three months of taking the drug. The results are given in the following table. Construct and interpret a 99% confidence interval for the true mean difference between the cholesterol levels for people who take the new drug.

Total Cholesterol Levels (in mg/dL)	
Initial Level	Level after Three Months
210	201
200	195
215	208
194	197
206	200
221	203
203	190
189	188
208	210
211	210

14. An infomercial claims that its new cooking device will dramatically reduce the time you spend preparing meals. Wondering if the claim is true, you set out to determine how much time the new cooking device will really save someone on average. Eight people who have purchased the item agree to participate in your study, and they each estimate the time they spent cooking dinner before they bought the new device and then after they began using it. Use the table of results to create and interpret a 90% confidence interval for the true mean change in the amount of time spent cooking dinner by using the infomercial's item.

Time Spent Preparing Dinner (in Minutes)	
Without Device	With Device
50	45
60	50
45	30
30	30
45	60
50	40
20	15
25	30

15. Philip wants to take a speed-reading course, but his wife thinks that it is a waste of time. To convince her that the course will really change the way that he reads, Philip decides to conduct an informal study. He polls seven people, asking them to tell the number of pages they were able to read in an hour before and then after they took the course. The results he obtained are found in the following table. Construct and interpret a 95% confidence interval for the true mean increase in reading speeds for people who have taken the speed-reading course.

Number of Pages Read in One Hour							
Before Course	35	50	45	50	60	70	65
After Course	50	60	80	70	85	100	90

16. A home improvement show gives tips on ways to improve your house before it is listed for sale. The show claims that their tips will help your house sell faster. To test the show's claim, a researcher found five pairs of houses that were similar in condition, area, and asking price. The homeowners from one house in each pair were asked to follow the tips given in the show before putting their houses on the market. (No major renovations were allowed.) The researchers then kept tabs on the subsequent length of time that it took for each house to sell. Use the following results to find and interpret a 90% confidence interval for the true mean difference between the numbers of weeks required to sell a house for homeowners who follow the show's tips and those who do not follow the tips.

Number of Weeks to Sell a House					
Without Tips	9	10.5	3	5.5	14.5
With Tips	7	6	5	1	4

17. During the fall semester of biology, the instructor teaching the class became ill and had to have another instructor stand in for him. After the students had taken a test under both instructors, the chairman of the biology department wanted to know if there was a significant difference in the performance of students under the different instructors. The results from each test are given in the table below. Construct and interpret a 90% confidence interval for the true mean difference between students' test scores under the two instructors.

Students' Test Scores												
Instructor A	58	82	78	91	83	77	45	87	94	92	68	77
Instructor B	52	60	83	90	87	70	48	81	90	90	71	70

18. At the beginning of an elementary physical education class, students are asked to do as many sit-ups as possible in a one-minute period. After practicing the proper technique for doing sit-ups, the students are once again timed to see if they increased the number of sit-ups they can do in one minute. Use the table of results to create and interpret a 95% confidence interval for the true mean change in the number of sit-ups that students were able to complete in one minute after the training.

Number of Sit-Ups								
<b>Before</b>	10	20	15	30	31	40	20	25
<b>After</b>	28	30	27	42	50	45	19	36

19. A learn-to-type software program claims that it can improve your typing skills. To test the claim and possibly help yourself out, you and three of your friends decide to try the program and see what happens. Use the table below to construct and interpret an 80% confidence interval for the true mean change in the typing speeds for people who have completed the typing program.

Typing Speeds (in Words per Minute)	
Before	After
42	54
50	52
37	41
22	30

20. After students were not doing so well in her math class, Ms. Comeaux decided to try a different approach and use verbal positive reinforcement at least once every hour. Use the following results to find and interpret a 99% confidence interval for the true mean change in the students' test scores after Ms. Comeaux started using positive reinforcement.

Students' Test Scores	
Without Reinforcement	With Reinforcement
66	72
57	55
61	80
72	79
38	49
47	56