

$$\begin{aligned}\hat{p} &= \frac{x}{n} \\ &= \frac{84}{383} \\ &\approx 0.219321\end{aligned}$$

- c. To construct the confidence interval, we need to calculate the margin of error. Substituting the appropriate values into the formula, we have the following.

$$\begin{aligned}E &= z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ &= 1.44 \sqrt{\frac{0.219321(1-0.219321)}{383}} \\ &\approx 0.030447\end{aligned}$$

Notice that the margin of error we calculated is very close to the value of  $E$  that we were willing to accept in part a. Since we used the minimum sample size calculated in part a., this is to be expected.

Therefore, subtracting the margin of error from the sample proportion and then adding the margin of error to the sample proportion gives us the following endpoints for the confidence interval.

$$\begin{aligned}\text{Lower endpoint: } \hat{p} - E &= 0.219321 - 0.030447 \\ &\approx 0.189\end{aligned}$$

$$\begin{aligned}\text{Upper endpoint: } \hat{p} + E &= 0.219321 + 0.030447 \\ &\approx 0.250\end{aligned}$$

Thus, the 85% confidence interval for the population proportion ranges from 0.189 to 0.250. The confidence interval can be written mathematically using either inequality symbols or interval notation, as follows.

$$\begin{aligned}0.189 &< p < 0.250 \\ &\text{or} \\ (0.189, &0.250)\end{aligned}$$

In other words, the state education commission can be 85% confident that the proportion of tenth-grade students in the state who read at or below the eighth-grade level is between 18.9% and 25.0%.

## 8.4 Section Exercises

**Determine if the conditions  $n\hat{p} \geq 10$  and  $n(1-\hat{p}) \geq 10$  are met for the following scenarios.**

- $n = 80, \hat{p} = 0.43$
- $n = 110, \hat{p} = 0.95$
- A hospital administrator wishes to estimate the proportion of his medical staff that follows the appropriate handwashing protocol. Out of 35 staff members randomly chosen, 29 were found to follow the appropriate handwashing protocol.

4. A market analyst wants to predict the proportion of mortgages in the San Francisco Bay area that are distressed, meaning that borrowers are more than 3 months behind on payments. Out of 350 mortgages selected at random, 17 are found to be distressed.

*Note:* For the following exercises in this section, you may assume that the necessary requirements are met; namely, all samples are simple random samples, the conditions for a binomial distribution are met, and the sample size is large enough to ensure that  $n\hat{p} \geq 10$  and  $n(1 - \hat{p}) \geq 10$ .

### Point Estimates and Confidence Intervals for Population Proportions

*Find each specified point estimate or confidence interval.*

5. Out of 50 randomly selected students who were surveyed, 41 felt that dorm renovation should be the administration's top priority. What is the best point estimate for the proportion of all students who feel that dorm renovation should be the administration's top priority?
6. In a random sample of 130 students, only 7 had been placed in the wrong math class. What is the best point estimate for the population proportion of all students who have been placed in the wrong math class?
7. A survey revealed that 48 out of 112 randomly selected homeowners in one neighborhood pay for lawn service every month. What is the best point estimate for the population proportion of all homeowners in that neighborhood who pay for lawn service every month?
8. Out of 210 randomly selected registered voters who were polled, 107 declared that they intended to vote for the incumbent in the upcoming election. What is the best point estimate for the proportion of all registered voters who intend to vote for the incumbent?
9. Out of 50 randomly selected students who were surveyed, 41 felt that dorm renovation should be the administration's top priority. Using a margin of error of 8.9%, give the interval estimate for the proportion of all students who feel that dorm renovation should be the administration's top priority.
10. Out of 210 randomly selected registered voters who were polled, 107 declared that they intended to vote for the incumbent in the upcoming election. Using a margin of error of 4.4%, give the interval estimate for the proportion of all registered voters who intend to vote for the incumbent.

### Confidence Intervals for Population Proportions

*Construct and interpret each specified confidence interval.*

11. Thirteen out of 147 randomly selected faculty members who were surveyed at a community college know sign language. Construct and interpret a 90% confidence interval for the proportion of all faculty members at the community college who know sign language.
12. A random sample of 200 computer chips is obtained from one factory and 4% are found to be defective. Construct and interpret a 95% confidence interval for the proportion of all computer chips from that factory that are defective.
13. Out of 140 randomly selected kindergartners who were surveyed, 32 said that pancakes are their favorite breakfast food. Construct and interpret a 90% confidence interval for the percentage of all kindergartners who say pancakes are their favorite breakfast food.
14. Out of 54 randomly selected patients of a local hospital who were surveyed, 49 reported that they were satisfied with the care they received. Construct and interpret a 95% confidence interval for the percentage of all patients satisfied with their care at that hospital.
15. A survey of 145 randomly selected students at one college showed that only 87 checked their campus e-mail account on a regular basis. Construct and interpret a 90% confidence interval for the percentage of students at that college who do not check their e-mail account on a regular basis.

16. A random sample of 200 computer chips is obtained from one factory and 4% are found to be defective. Construct and interpret a 99% confidence interval for the percentage of all computer chips from that factory that are *not* defective.
17. Out of 190 randomly selected adults in the United States who were surveyed, 71 exercise on a regular basis. Construct and interpret a 95% confidence interval for the proportion of all adults in the United States who exercise on a regular basis.
18. A survey of 47 randomly selected members of a health club reported that 68.1% prefer walking or running on a treadmill to any other type of aerobic exercise. Construct and interpret a 95% confidence interval for the proportion of health club members who prefer walking or running on a treadmill.
19. Suppose that a national wireless phone company is interested in determining how many of its customers prefer paperless billing. In a survey of 730 customers, 323 said they prefer paperless billing. Construct and interpret a 95% confidence interval for the percentage of all customers who prefer paperless billing.
20. Of 542 mortgages made to borrowers with credit scores between 620 and 679, 41 defaulted on their loans within 5 years. Create a 90% confidence interval for the percentage of all borrowers with credit scores between 620 and 679 who default on their mortgages.

### Minimum Sample Sizes for Estimating Population Proportions

#### *Calculate the minimum sample size needed to construct a confidence interval with the desired characteristics.*

21. Suppose you wish to determine the proportion of college students in your state who receive some form of financial aid. You want to be 98% confident of your results and have a maximum error of 5%. Calculate the minimum sample size that you must have to meet these requirements, given that the financial aid office at a local institution estimates the percentage to be 78%.
22. Suppose that you wish to determine the proportion of college students in the United States who work at least 20 hours per week while enrolled as full-time students. You want to be 95% confident in your results and have a maximum error of 3%. Calculate the minimum number of college students that you must survey to meet these requirements, given that a previous study estimated that 68% of all college students in the United States work at least 20 hours per week while enrolled as a full-time student.
23. A market researcher wishes to determine the proportion of American consumers who shop online. The results must be accurate at the 90% level of confidence with a maximum error of 2%. Calculate the minimum sample size needed to meet these requirements, given that a previous study estimated that 59% of American consumers shop online.
24. Suppose a researcher needs to verify the proportion of patients who will have mild side effects when taking a recently approved drug. The researcher must be 99% sure of her results with a maximum error of 1%. Calculate the minimum sample size the researcher needs to meet her requirements, given that the studies used to get the drug approved reported that only 12% of patients experienced mild side effects.

#### *Respond thoughtfully to the following exercises.*

25. The Deseret News in Salt Lake City, Utah reported in April 2012 that the Zions Bank Consumer Attitude Index for Utah increased to 85.3, which was 5.4 points higher than the month prior. It stated that the index was “based on a representative sample of 500 Utah households . . . and has a confidence interval of plus or minus 4.38 percent at a 95 percent confidence level.” Based on the information given in the article, is it possible that the index reported in April actually did not increase over the previous month’s index? Why or why not?

**Source:** “Utah Consumers Remain Confident, New Report States.” *Deseret News*. 24 Apr. 2012. <http://www.deseretnews.com/article/765571132/Utah-consumers-remain-confident-new-report-states.html> (20 May 2012).

26. A Bloomberg Politics National Poll surveyed 982 Americans in November 2016 regarding the upcoming Presidential election. Of this group, 46% favored Hillary Clinton and 43% favored Donald Trump, with a margin of error of 3.5%. Donald Trump went on to win the election a few weeks later. Based on this information, is it true to say that the Bloomberg poll predicted the election wrong? Explain your answer.

**Source:** Bloomberg Politics Poll. Selzer & Company. 7 Nov. 2016. <https://assets.bwbx.io/documents/users/iqjWHBFdfxIU/rkIC>

27. Solve the formula for the margin of error of a confidence interval for a proportion,

$E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ , for  $n$ , to show that it is equivalent to the formula for the minimum sample size for estimating a population proportion,  $n = \hat{p}(1-\hat{p}) \left( \frac{z_{\alpha/2}}{E} \right)^2$ . (Recall that a value of  $\hat{p}$  from a previous study is typically used as an estimate for  $p$  in the formula for the minimum sample size.)