8.1 Section Exercises

Note: *For all exercises in this section, you may assume that the requirements mentioned in this section are met; namely, the population standard deviation is known, all samples are simple random samples, and either the sample size is at least 30 or the population distribution is approximately normal.*

Point Estimates and Confidence Intervals for Population Means

Find each specified point estimate or confidence interval.

1. A survey of 42 randomly selected teachers finds that they spend a mean of $18 per week on lunch. What is the best point estimate for the mean amount of money spent per week on lunch for all teachers?
2. The mean number of pets per student for a random sample of 47 students at Brown Elementary is 2.5 pets. What is the best point estimate for the mean number of pets per student for all students at Brown Elementary?
3. A survey of teachers reports that a point estimate for the mean amount of money spent each week on lunch is $18.00. If the margin of error for a 95% confidence interval for the mean amount of money spent each week on lunch by all teachers is $1.70, construct a 95% confidence interval for the mean amount of money spent each week on lunch for all teachers.
4. The mean batting average for a random sample of 35 professional baseball players is .283. If the margin of error for the population mean with a 99% level of confidence is .051, construct a 99% confidence interval for the mean batting average for professional baseball players.

Margins of Error of Confidence Intervals for Population Means (σ Known)

Calculate the margin of error of a confidence interval for the population mean at the given level of confidence.

5. $n = 56, \quad \sigma = 3.14, \quad c = 0.90$
6. $n = 81, \quad \sigma = 2.45, \quad c = 0.95$
7. $n = 93, \quad \sigma = 1.25, \quad c = 0.95$
8. $n = 134, \quad \sigma = 0.27, \quad c = 0.99$

Confidence Intervals for Population Means (σ Known)

Construct a confidence interval for the population mean at the given level of confidence.

9. $n = 89, \quad \sigma = 2.01, \quad c = 0.95, \quad \bar{x} = 45.00$
10. $n = 64, \quad \sigma = 8.01, \quad c = 0.90, \quad \bar{x} = 90.40$
11. $n = 607, \quad \sigma = 1.92, \quad c = 0.99, \quad \bar{x} = 18.45$
12. $n = 1123, \quad \sigma = 7.31, \quad c = 0.95, \quad \bar{x} = 87.12$

Minimum Sample Sizes for Estimating Population Means

Calculate the minimum sample size needed to construct a confidence interval with the desired characteristics. The value of the population standard deviation is an estimate based on a previous reliable study.

13. $E = 0.5, \quad \sigma = 5.25, \quad c = 0.95$
14. $E = 2, \quad \sigma = 12.10, \quad c = 0.90$
15. $E = 1.5, \quad \sigma = 4.75, \quad c = 0.99$
16. $E = 3, \quad \sigma = 15.03, \quad c = 0.95$

Confidence Intervals for Population Means (σ Known)

Construct and interpret each specified confidence interval.

17. A professor wants to estimate how many hours per week her students study. A simple random sample of 78 students had a mean of 15.0 hours of studying per week. Construct and interpret a 90% confidence interval for the mean number of hours a student studies per week. Assume that the population standard deviation is known to be 2.3 hours per week.

18. A faculty advocacy group is concerned about the amount of time teachers spend each week doing schoolwork at home. A simple random sample of 56 teachers had a mean of 8.0 hours per week working at home after school. Construct and interpret a 95% confidence interval for the mean number of hours per week a teacher spends working at home. Assume that the population standard deviation is 1.5 hours per week.

19. A writer for a computer magazine is working on an article about computer usage in American households. A simple random sample of 120 American households has a mean computer usage time of 19.2 hours per week. Construct and interpret a 95% confidence interval for the mean computer usage time per week for all American households. Assume that the population standard deviation is 3.3 hours per week.

20. A survey of 85 randomly selected homeowners finds that they spend a mean of \$67 per month on home maintenance. Construct and interpret a 99% confidence interval for the mean amount of money spent per month on home maintenance by all homeowners. Assume that the population standard deviation is \$14 per month.

21. A survey of 97 randomly selected homeowners found that the mean amount spent on lawn service was \$720 per year. Construct and interpret a 98% confidence interval for the mean amount of money spent on lawn service per household each year. Assume that the population standard deviation is \$123 per year.

22. A survey of a simple random sample of 140 dieters revealed that the numbers of times they "cheated" on their diets had a mean of 7.0 times per week. Construct and interpret a 99% confidence interval for the mean number of times dieters "cheat" on their diets each week. Assume that the population standard deviation is 1.5 times per week.

23. A physical therapist is investigating the mean recovery time after ACL surgery for patients involved in a new therapy regimen. For the purpose of the study, a successful recovery was defined to be the ability to walk without crutches. For 38 randomly selected patients, the mean recovery time after ACL surgery was found to be 22.6 days. Assume that the population standard deviation is 3.7 days. Find and interpret a 99% confidence interval for the mean recovery time for all ACL surgery patients undergoing the same new therapy.

24. The manufacturers of Caudill automotive oil wish to estimate the mean number of miles that motorists drive between oil changes. A random sample of 54 motorists has a mean of 5900 miles driven between oil changes. Assume that the population standard deviation is 1350 miles. Construct and interpret a 95% confidence interval for the mean number of miles driven between oil changes for all motorists.

Minimum Sample Sizes for Estimating Population Means

Calculate the minimum sample size needed to construct a confidence interval with the desired characteristics. The value of the population standard deviation is an estimate based on a previous reliable study.

25. The upper management at a bank would like to estimate the mean number of credit cards held by Millennials. They would like to create a 98% confidence interval with a maximum error of 1 card. Assuming a standard deviation of 3.25 cards, what is the minimum number of Millennials they must include in their sample?

26. A social worker is concerned about the number of prescriptions her elderly clients have. She would like to create a 99% confidence interval for the mean number of prescriptions per client with a maximum error of 2 prescriptions. Assuming a standard deviation of 5.2 prescriptions, what is the minimum number of clients she must sample?

27. Suppose you are interested in determining the mean number of hours students spend working out each week. You want a 95% level of confidence and a maximum error of 0.5 hours. Assuming the standard deviation is 2.5 hours, what is the minimum number of students you must include in your sample?

28. For a psychology experiment, Emma is assigned the task of finding the mean number of hours students sleep per night. Her results must be at the 99% level of confidence with a maximum error of 0.25 hours. Assuming the standard deviation is 1.4 hours, how many students must Emma survey?

Respond thoughtfully to the following exercises.

29. Given a 99% confidence interval for a population mean of $(1.01, 1.97)$, is it possible to determine the original point estimate for the interval? Explain your answer.

30. Two individual researchers report a confidence interval for the mean number of unreported domestic violence incidents per month in one small county over the past year. One gives a 95% confidence interval of $(4.93, 6.14)$ while the other gives a 98% confidence interval of $(3.72, 5.01)$. Is it possible for both of the intervals to contain the true mean number of unreported domestic violence incidents? Explain your answer.

31. Last quarter, your sales team reported a mean amount of revenue of \$131,540 per salesperson. Based on a survey of a portion of the sales team and last quarter's data, you estimate that this quarter, the mean revenue will be between \$129,915 and \$154,798 per salesperson, with a 95% level of confidence. Would you report to your supervisor that this quarter's sales will be up from last quarter? Why or why not?

32. The formula for the margin of error for a confidence interval for a population mean (when the population standard deviation is known) is given as $E = \left(z_{\alpha/2}\right)\left(\sigma_{\bar{x}}\right)$ or $E = \left(z_{\alpha/2}\right)\left(\dfrac{\sigma}{\sqrt{n}}\right)$. Explain how the Central Limit Theorem can be used to rewrite the first formula as the second one.