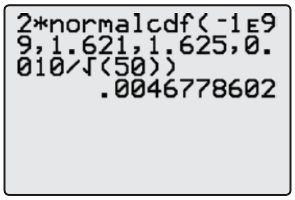
$$z = \frac{\overline{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$
$$= \frac{0.004}{\left(\frac{0.010}{\sqrt{50}}\right)}$$
$$\approx 2.83$$

Thus, the two z-scores are $z_1 = -2.83$ and $z_2 = 2.83$. Because we want to find the total area in the two tails, using the symmetry property of the normal distribution, we can find the area to the left of $z_1 = -2.83$ and double it. Using the tables we find that the area to the left of $z_1 = -2.83$ is 0.0023, so the total area in the two tails is $(0.0023)(2) = 0.0046$.

TI-83/84 Plus: In order to find the total area located in two symmetric tails of the distribution, we will use the calculator to double the area found in the left tail of the curve. To do this, go to the `DISTR` menu and compute `2*normalcdf(lower bound, upper bound, μ, σ)` using the following values.



lower bound: −1E99
upper bound: 1.621
$\mu = 1.625$
$$\sigma = \frac{0.010}{\sqrt{50}}$$

As shown in the margin screenshot, the calculator gives the more accurate value of approximately 0.0047 for the probability when this method is used.

Hence, the probability would be reported as 0.0046 if using tables or 0.0047 if using the calculator. This means that the probability of the sample batch not meeting the requirements is very small, approximately 0.5%.

7.2 Section Exercises

Calculating Standard Scores for Sample Means

Calculate the standard score (z-score) of the given sample means. Round your answer to two decimal places.

1. $\mu = 36$ and $\sigma = 39$; $n = 73$; $\overline{x} = 38$
2. $\mu = 11.0$ and $\sigma = 10.2$; $n = 100$; $\overline{x} = 10.0$
3. $\mu = 1.10$ and $\sigma = 1.10$; $n = 40$; $\overline{x} = 0.55$
4. $\mu = 0.49$ and $\sigma = 0.60$; $n = 1500$; $\overline{x} = 0.52$
5. $\mu = 114{,}023$ and $\sigma = 30{,}000$; $n = 100$; $\overline{x} = 110{,}045$
6. $\mu = 514.2$ and $\sigma = 30.2$; $n = 1000$; $\overline{x} = 513.0$

Probability

Find each specified probability.

7. Suppose that the walking step lengths of adults are normally distributed with a mean of 2.4 feet and a standard deviation of 0.3 feet. A sample of 34 step lengths is taken.

   a. Find the probability that an individual adult's step length is less than 2.1 feet.

b. Find the probability that the mean of the sample taken is less than 2.1 feet.

c. Find the probability that the mean of the sample taken is more than 2.5 feet.

d. Find the probability that the sample mean differs from the population mean by more than 0.06 feet.

8. The average number of miles between oil changes for vehicles serviced in one large facility is 6520 with a standard deviation of 1100 miles. A random sample of 50 vehicles serviced at this facility is taken.

a. Calculate the probability that the sample mean is more than 6800.

b. Calculate the probability that the sample mean is less than 6700.

c. Calculate the probability that the sample mean is between 6200 and 6400.

9. Intelligence quotient (IQ) scores are often reported to be normally distributed with $\mu = 100.0$ and $\sigma = 15.0$.

a. What is the probability of a person chosen at random having an IQ score of less than 95?

b. If a random sample of 50 people is taken, what is the probability that their mean IQ score will be less than 95?

c. If a random sample of 50 people is taken, what is the probability that their mean IQ score will be more than 95?

d. If a random sample of 50 people is taken, what is the probability that their mean IQ score will be more than 105?

e. If a random sample of 50 people is taken, what is the probability that their mean IQ score will differ from the population mean by more than 5?

10. Suppose that the diameters of oak trees are normally distributed with a mean of 4.000 feet and a standard deviation of 0.375 feet.

a. What is the probability of walking down the street and finding an oak tree with a diameter of more than 5 feet?

b. What is the probability of sampling a set of 87 oak trees and finding their mean to be more than 4.1 feet in diameter?

c. What is the probability of sampling a set of 87 oak trees and finding their mean to be less than 3.92 feet in diameter?

d. What is the probability of sampling a set of 87 oak trees and finding their mean to differ from the population mean by less than 0.1 feet in diameter?

11. A glass tube maker claims that his tubes have lengths that are normally distributed with a mean of 9.00 cm and a variance of 0.25.

a. What is the probability that a quality control regulator will pull a tube off the assembly line that has a length between 8.6 and 9 cm?

b. What is the probability that a random sample of 40 tubes will have a mean of less than 8.8 cm?

c. What is the probability that a random sample of 35 tubes will have a mean of more than 9.2 cm?

d. What is the probability that a random sample of 75 tubes will have a mean that differs from the population mean by more than 0.1 cm?

12. A pencil manufacturer claims that its pencils have lengths that are normally distributed with a mean of 6.0 inches and a variance of 0.2. What is the probability that a randomly chosen pencil will have a length of more than 6.4 inches?

13. A vineyard claims that the mean berry count per grape cluster is 43 with a standard deviation of 3.6 berries. What is the probability that, in next year's crop, the mean berry count for a sample of 50 clusters will be between 42 and 44?

14. A book publisher claims that its mean book length is 250 pages with a standard deviation of 70 pages. As a reviewer, you get paid per book, and not per page, to read over a manuscript. What is the probability that for a sample of 45 randomly selected books, the mean length of a book is less than 230 pages?

15. Airlines predict that the mean number of "no shows" per 100 seats on each flight is 10.0 with a standard deviation of 3.4. What is the probability that a random sample of 45 flights has a mean of more than 12 "no shows" per 100 seats?

16. In a Scrabble tournament, the scores were normally distributed and the mean score was 420.2 points with a standard deviation of 105.0 points. What is the probability that the score of a randomly selected competitor differs from the mean score by less than 50 points?

17. A tea bag manufacturer needs to place 2 g of tea in each bag. If the machinery places a mean of 2.6 g of tea in each bag with a standard deviation of 0.3 g, what is the probability that a randomly chosen bag will have between 2 and 2.8 g of tea, assuming that the amounts of tea per bag are normally distributed?

18. A pollster claims that the mean amount spent on Christmas gifts by an American family is \$927 with a standard deviation of \$200. What is the probability that the mean amount spent on Christmas gifts for a sample of 500 families differs from the population mean by less than \$15?

19. A medical journal reports the mean fetal heart rate to be 140 beats per minute (bpm) with a standard deviation of 12 bpm. What is the probability that a fetal heart rate differs from the mean by more than 25 bpm, assuming that fetal heart rates are normally distributed?

20. A local sports magazine reports the mean length of a baseball game to be 175.9 minutes with a standard deviation of 27.0 minutes. For a random sample of 30 games, what is the probability that the mean game length is at most 170 minutes?

21. Teenagers reportedly spend a mean of 17.5 hours per week playing computer and video games. For a random sample of 110 teenagers, what is the probability that the mean amount of time spent playing games is more than 18 hours per week? Let $\sigma = 3.0$.

22. The mean wait time for a drive-through chain is 193.2 seconds with a standard deviation of 29.5 seconds. What is the probability that for a random sample of 45 wait times, the mean is between 185.7 and 206.5 seconds?

23. The mean amount spent per order at one fast food restaurant is \$8.43 with a standard deviation of \$1.52. What is the probability that for a random sample of 75 orders from this week's customers, the mean amount spent will be between \$8 and \$9?

24. The mean hourly rate for babysitters in one town is \$7.05 with a standard deviation of \$0.55. What is the probability that a babysitter chosen at random will charge an hourly rate that differs from the mean by less than \$1.00, assuming that the hourly rates for babysitters in this town are normally distributed?

25. The mean sale price for a piece of art from members of a large artists' guild is \$545 with a standard deviation of \$76. If, at one show, 45 pieces are for sale, what is the probability that the mean sale price for the show will be higher than \$575?

26. The mean cost for plumbing repairs in one area is \$208 with a standard deviation of \$64. If 31 homeowners in that area are surveyed, what is the probability that the mean cost for their plumbing repairs will be over \$200?

27. One pediatric clinic sees a mean of 42.0 patients per day with a standard deviation of 3.4 patients per day. What is the probability that the mean number of patients per day for March (31 days) will be between 41 and 44?

28. At a large university, the mean amount spent by students for cellular phone service is \$38.90 per month with a standard deviation of \$3.64 per month. Consider a group of 44 randomly chosen university students. What is the probability that the mean amount of their monthly cell phone bills differs from the mean for the university by more than \$1?

29. A large bakery sells a mean of 11.0 dozen cookies per day with a standard deviation of 1.1 dozen cookies per day. Consider the mean number of cookies sold per day in July (31 days) of one year. What is the probability that the mean differs from the population mean by less than 0.5 dozen?

30. According to data released by the Jackson City Chamber of Commerce, the weekly wages of office workers have a mean of \$723 and a standard deviation of \$151. If 57 office workers are chosen at random from Jackson City, what is the probability that the mean weekly wage of these workers will be greater than \$750?