What do these results tell us? Comparing the standard deviations, we see that not only does Plan B have a higher expected value, but its profits vary slightly less than those of Plan A. We may conclude that Plan B carries a slightly lower amount of risk than Plan A.

5.1 Section Exercises

Properties of a Probability Distribution

For each table, determine whether it could represent a valid discrete probability distribution. If not, explain why.

1.

x	$P(X = x)$
1	0.2
2	0.6
3	0.05
4	0.15
5	0.0

2.

x	$P(X = x)$
–22	0.4
53	1.05
–15	0.05

3.

x	$P(X = x)$
0.5	0.4
2.5	0.7
4.5	–0.3
6.5	0.2

4.

x	$P(X = x)$
–15	0.4
–20	0.3
–25	0.4

Discrete Probability Distributions

Create the probability distribution for each random variable described.

5. The number of tails showing when flipping four coins.

6. The number of even numbers showing when a pair of standard six-sided dice are rolled.

7. The difference between the two numbers showing when a pair of standard six-sided dice are rolled (largest value – smallest value).

8. The number of heads showing in five tosses of a coin.

Mean and Standard Deviation for Discrete Probability Distributions

For each discrete probability distribution, find the mean and the standard deviation.

9.

x	$P(X = x)$
15	0.6
22	0.4

10.

x	$P(X = x)$
–55	0.45
30	0.55

11.

x	$P(X = x)$
14	0.3
21.5	0.4
–2	0.3

12.

x	$P(X = x)$
–$1.50	0.3
$0.00	0.5
$2.75	0.1
$5.00	0.1

Expected Values

Determine the expected values for each scenario.

13. Scott likes to trade stocks online. On a good day, he averages a $2200.00 gain. On a bad day, he averages a $1600.00 loss. Suppose that he has good days 25% of the time, bad days 35% of the time, and the rest of the time he breaks even.

a. What is the expected value for one day of Scott's online trading?

b. If Scott trades online every weekday for three weeks, how much money should he expect to gain or lose?

c. What is the variance for one day of Scott's online trading?

14. Mike's older brother, Jack, bets him that he can't roll two dice and get doubles three times in a row. If Mike does it, Jack will give him $100.00. Otherwise, Mike has to give Jack $5.00.

a. What is the expected value of Mike's bet?

b. What is the expected value of Jack's bet?

c. If Mike and Jack make the same bet 30 times, how much can Mike expect to win or lose?

15. An insurance company offers Mississippi adults between the ages of 25 and 34 a $100,000 life insurance policy for $18 a month. They use the fact that Mississippi has a yearly death rate of 172.8 per 100,000 residents aged 25–34 years.

a. Find the expected value per customer for the insurance company at the end of one year for the policy described.

b. If the insurance company has 10,000 customers with these life insurance policies in Mississippi, what is its profit at the end of the year?

16. Suppose the same insurance company as in the previous question insures adults ages 25 to 34 in California for the same amount of money per month, but offers a $175,000 policy for that amount of money. The reason for the difference in the payout is that the death rate in California for that age group is 81.6 per 100,000 residents.

a. Find the expected value per customer for the insurance company at the end of one year for the policy described in California.

b. If the insurance company has 10,000 customers with these life insurance policies in California, what is its profit at the end of the year?

c. Which state is more profitable for the insurance company (as compared to Mississippi in the previous problem)?

17. A carnival game consists of choosing eight winning numbers, then randomly choosing one ball out of 50 balls, numbered 1 - 50. If the numbered ball you pick is one of your winning numbers, you win a $5 prize.

a. What is the expected value of the game if it costs $2 to play?

b. How much can you expect to win or lose if you play the game 10 times?

18. A church in town is raffling off $50.00. You can buy one ticket for $1.00, three tickets for $2.50, or five tickets for $4.00. Assume that the church sells 100 tickets.

a. Find the expected value for each of the three ticket options.

b. Should you buy one, three, or five tickets in order to maximize your expected winnings?

19. A department store is running a promotion one Saturday by giving out coupons for $10 of free merchandise. Based on data collected in the past, only one-fourth of customers who shop on that Saturday use the coupon but do not purchase any other merchandise. However, one-third of customers purchase $40 in merchandise and then use the coupon. Another one-third of customers use the coupon after ringing up a total of $75 in merchandise. The remainder of customers who come in the store do not take advantage of the promotion at all.

a. Find the expected value of the promotion per customer for the department store.

b. If the store has 720 customers on the promotional Saturday, what is its expected revenue for the day?

20. A car dealership is offering an interesting incentive in order to get people to come and test drive its new sports cars. Everyone who agrees to a test drive gets to choose a key. There are 75 car keys in the bag, and 4 of them unlock a sports car. For the customers who choose a winning key, the dealership agrees to knock $1000 off of the price if they buy a new car. (Assume that each key is returned after being drawn.)

a. From the perspective of the car dealership, what is the expected value of the incentive for one customer who chooses a key and buys a new car?

b. If 90 customers come in and choose a key and all of them buy new cars, how much can the dealership expect to give up in sales?

21. a. In the following probability distribution for the cost of textbooks in a fall semester for various liberal arts majors, each probability represents the chance that the total cost of a student's books will be the given amount. Find the expected value for the cost of books for the semester.

Cost of Textbooks for Liberal Arts Majors

x	$P(X = x)$
$262	0.19
$410	0.21
$590	0.17
$653	0.43

b. In the following probability distribution for the cost of textbooks in a fall semester for various business majors, each probability represents the chance that the total cost of a student's books will be the given amount. Find the expected value for the cost of books for the semester.

Cost of Textbooks for Business Majors

x	$P(X = x)$
$378	0.35
$389	0.14
$392	0.25
$401	0.26

c. Which of the groups of majors would you guess is more likely to feel that their textbooks are priced fairly? Explain your answer. (**Hint:** Use the standard deviations to help you make your *informed* decision.)

22. The managing director of a traveling carnival needs to add a new game to the carnival lineup. Given below are the probability distributions for his top two choices. The values of the random variable are the amounts the carnival would either gain (positive values) or have to pay out (negative values). Which game would you advise the director to choose? Why?

Carnival Game 1

x	$P(X = x)$
\$1.00	0.8
–\$5.00	0.135
–\$10.00	0.06
–\$100.00	0.005

Carnival Game 2

x	$P(X = x)$
\$1.00	0.65
–\$1.00	0.25
–\$2.00	0.1