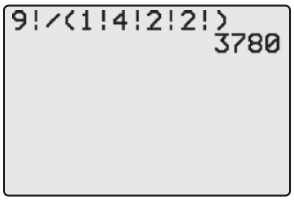
Since there are a total of 9 letters in TENNESSEE, substituting these values into the special permutation formula gives the following.

By Hand:

$$\frac{9!}{1!4!2!2!} = \frac{9\cdot \overset{4}{\cancel{8}}\cdot 7\cdot \overset{3}{\cancel{6}}\cdot 5\cdot \cancel{4\cdot 3\cdot 2\cdot 1}}{(1)\cancel{(4\cdot 3\cdot 2\cdot 1)}\cancel{(2\cdot 1)}\cancel{(2\cdot 1)}}$$
$$= 9\cdot 4\cdot 7\cdot 3\cdot 5$$
$$= 3780$$



TI-83/84 Plus: Using the calculator we obtain the same result shown in the screenshot. Remember that you need parentheses around the calculation in the denominator.

Thus, there are 3780 ways to arrange the letters in the word TENNESSEE.

4.4 Section Exercises

Factorials

Evaluate each factorial expression.

1. $6!$ **2.** $8!$ **3.** $\frac{6!}{4!}$ **4.** $\frac{8!}{5!}$

5. $\frac{6!}{4!2!}$ **6.** $\frac{8!}{5!3!}$ **7.** $\frac{6!}{4!(6-4)!}$ **8.** $\frac{8!}{5!(8-5)!}$

9. $0!$ **10.** $\frac{7!}{0!}$

Combinations and Permutations

Evaluate each combination or permutation expression.

11. ${}_5C_2$ **12.** ${}_8C_5$ **13.** ${}_4C_4$ **14.** ${}_5C_1$

15. ${}_{12}P_1$ **16.** ${}_7\mathrm{P}_4$ **17.** ${}_5\mathrm{P}_3$ **18.** ${}_7\mathrm{P}_6$

19. ${}_8P_1$ **20.** ${}_3P_3$ **21.** $\frac{{}_3C_2}{{}_3P_1}$ **22.** $\frac{{}_{10}P_2}{{}_{10}C_2}$

23. ${}_6C_4 + {}_6C_3 + {}_6C_2 + {}_6C_1$ **24.** ${}_5P_4 + {}_5P_3 + {}_5P_2 + {}_5P_1$

Simplify the formula for each expression.

25. ${}_nC_n$ **26.** ${}_nC_1$ **27.** ${}_nP_1$ **28.** ${}_nP_n$

29. ${}_nP_{n-1}$ **30.** ${}_nC_{n-1}$

Use a combination or permutation expression to determine the total number of outcomes.

31. Farmer John has nine prize-winning cows. How many ways can he choose three of his cows to show at the state fair?

32. When a large group visits the Library of Congress, 3 members of the group are randomly chosen as "Library Ambassadors" for the day. How many ways can the ambassadors be chosen from a group of 38 visitors?

33. There are 12 members in the local garden club. In how many ways can a president and secretary be chosen? (Assume that no member can hold both positions at the same time.)

34. A teacher must choose parts for the upcoming Thanksgiving play from her class of 17 students. How many ways can she choose the parts of pilgrim, Native American, and turkey?

35. A teacher must choose parts for the upcoming Thanksgiving play from her class of 18 students. She needs a group of four students to serve as program attendants before the start of the play. In how many ways can this group be chosen?

36. There are eight people hosting a party. Three people are needed to stay and clean up after the party is over. How many ways can the clean-up crew be chosen?

37. There are eight people hosting a party. One person must set up the catering, another must bring flowers, and someone else needs to bring drinks. In how many ways can these tasks be assigned?

38. In assigning seats for a classroom, how many ways can a teacher place 8 students in the front row from her roll of 35?

39. In how many ways can a graduate student fulfill the master's degree requirements in mathematics if 10 classes are needed from a choice of 15 classes?

40. In how many ways can 1st, 2nd, and 3rd place prizes be awarded in a local science fair if there are 25 participants?

41. In how many ways can a task force of 4 people be chosen from a group of 12 employees?

42. If 3 people need to serve as chaperones on a school trip, in how many ways can they be chosen from the parents of the 20 students? (Assume that each child has two parents available.)

43. The Seago family is planning their vacation. Each of the five family members is allowed to nominate three places they would like to visit. If they want to visit four different places during the trip, in how many ways can they plan their road trip, assuming that no family members choose the same place?

44. In how many ways can the letters in the word STATISTICS be arranged?

45. In how many ways can the letters in the word PROBABILITY be arranged?

46. Karran was born on 11/21/1992. He would like to make an eight-digit code from all of the digits in his birth date. How many different eight-digit codes could he create?

47. Employees at a local factory need a unique seven-digit code to access the building. The manager wants to make each person's code from the factory's phone number, 555-9313.

a. If there are 509 employees who need codes, will the manager have enough unique codes using only the digits in the phone number?

b. Would there be enough ten-digit codes if he used the area code, 514, as well?

48. Which of the following words would produce the greatest number of different five-letter arrangements?

a. BEAST

b. ORDER

c. TESTS

d. GOING

Counting Techniques and Probability

Use counting techniques to compute each classical probability.

49. At a carnival entrance, tickets are assigned five-digit numbers using the digits 0–9. If one ticket number is chosen randomly for a prize, what is the probability that every number on the ticket is even? (Assume that all possible ticket numbers are eligible to be chosen.)

50. Suppose that your boss must choose three employees in your office to attend a conference in Jamaica. Because all 20 of you want to go, he decides that the only fair way is to draw names out of a hat. What is the probability that you, Suzanne, and Alex are chosen?

51. Rhonda and Laura are planning to watch two movies over the weekend from Laura's collection of 24 DVDs. Rhonda has two favorites among the collection. What is the probability that the girls would randomly choose those two movies to watch?

52. Bill is planting tulip bulbs in the front yard. There are two white bulbs and two red bulbs mixed together in a bucket. What is the probability that Bill plants the four bulbs in a row so that they are alternating in color?

53. Nye is playing Scrabble. What is the probability that she chooses the tiles with the letters of her name, in order, when she draws three tiles from the bag? (Assume that when she begins there is one tile of each letter in the alphabet in the bag.)

54. Every 6 months, university email requires that a new 5-digit password be set up. No digits are allowed to be repeated and it must be different from your last two passwords. If you let your computer randomly choose a 5-digit code for you with no repeating digits, what is the probability that it will choose one of the last 2 passwords you've had?