Solution

Since we are not told in the problem whether the distribution of the data is bell-shaped, we cannot apply the Empirical Rule here. However, we can apply Chebyshev's Theorem to find a minimum estimate. To do so, we need to know how many standard deviations \$1989 and \$5229 are from the mean. By subtracting, we can find how far each of these figures lie from the mean. Then, dividing by the standard deviation, we can convert these differences into numbers of standard deviations. Here are the calculations.

Rent = \$1989

$$\begin{aligned}\text{Distance from Mean} &= x - \mu \\ &= \$1989 - 3609 \\ &= -\$1620\end{aligned}$$

$$\begin{aligned}\text{Dev. from Mean} &= \frac{\text{Distance}}{\text{Standard Deviation}} \\ &= \frac{-\$1620}{\$540} \\ &= -3\end{aligned}$$

Rent = \$5229

$$\begin{aligned}\text{Distance from Mean} &= x - \mu \\ &= \$5229 - \$3609 \\ &= \$1620\end{aligned}$$

$$\begin{aligned}\text{Dev. from Mean} &= \frac{\text{Distance}}{\text{Standard Deviation}} \\ &= \frac{\$1620}{\$540} \\ &= 3\end{aligned}$$

Thus these rent prices lie three standard deviations above and below the mean. Chebyshev's Theorem can then be applied for $K = 3$. Using the calculation previously shown in the box with the theorem, we can say that at least 88.9% of the rent prices lie within this range.

3.2 Section Exercises

Range, Population Standard Deviation, and Population Variance

Calculate the range, population standard deviation, and population variance of the given data set.

1. 3, 5, 7, 8
2. 8, 12, 10, 12, 4, 13, 5, 17, 10
3. 5, 5, 5, 5, 5, 5, 5
4. –5, 6, 3, –5, –7, –7, 8, –25
5. 17, 19, 21, 16, 20, 19, 17, 13
6. 1.6, 2.2, 1.1, 3.0, 4.9, 2.8, 5.7, 4.5, 6.0

Range, Sample Standard Deviation, and Sample Variance

Calculate the range, sample standard deviation, and sample variance of the given data set.

7. 2, 2, 3, 4, 7
8. 15, 12, 13, 14, 15, 17, 18, 11, 12, 15
9. 4, 4, 4, 4, 4, 4, 4
10. –1, 2, 3, –2, –4, 4, 5, –35, 2
11. 2.0, 1.7, 1.9, 1.5, 1.6, 1.9, 1.7, 3.0
12. 8.1, 9.5, 10.7, 12.3, 4.4, 3.9, 5.1, 7.3, 10.1

Sample Standard Deviation

Calculate the sample standard deviation for the given data set.

13. The following data represent weights of yorkshire terriers (in pounds).

6.8	9.1	8.7	7.5	8.2
5.4	6.5	8.5	7.3	6.6
5.9	7.3	9.3	7.5	7.8

14. The following data represent average numbers of Tweets per day posted on Twitter for 16 high school students.

0.8	42.2	20.6	2.8
36.7	18.6	23.3	11.5
3.7	14.9	9.4	1.5
14.9	31.1	23.5	9.5

15. The following data represent high temperatures for cities in the Southeast (in degrees Fahrenheit).

High Temperatures for Cities in the Southeast

Stem	Leaves
7	
7	7 9
8	2 3 4
8	5 5 7 8 8 9 9
9	0 0 1 1 2 2 3
9	5

Key: 7 | 7 = 77 °F

16. The following data represent ages of 20 American entrepreneurs (in years).

Ages of American Entrepreneurs

Stem	Leaves
2	2 3
2	5 8 9 9
3	0 1 1 2 2 3 4
3	5 6 9
4	1 3
4	7
5	3
5	

Key: 2 | 2 = 22 years old

Standard Deviation and Variance

Decide if each statement is true or false. Explain why.

17. If the standard deviation of a data set is zero, then all entries in the data must equal zero.

18. The population variance and sample variance are the same value for the same set of data.

19. It is possible to have a standard deviation of –3 for some data set.

20. It is possible to have a standard deviation of 435,000 for some data set.

Coefficient of Variation

Calculate the coefficient of variation, CV, for each data set and then answer the question.

21. The data in set A represent numbers of hours worked in one week for a sample of employees of a fast food restaurant. The data in set B represent numbers of minutes spent waiting for food for a sample of customers at the same restaurant. Which of the two data sets has the *larger* spread relative to its own mean?

A: 31, 33, 35, 39, 31, 32, 30, 40, 13, 41, 38, 32, 37, 33

B: 2.3, 3.5, 4.3, 2.1, 4.8, 3.9, 2.0, 3.3, 4.0, 1.6, 2.2

22. The data in set A represent prices (with tax included) of cookies sold at a sample of bakeries. The data in set B represent number of cookies sold in one weekend for the same sample of bakeries. Which of the two data sets has the *larger* spread relative to its own mean?

A: $1.23, $1.55, $1.01, $1.89, $2.35, $2.56, $2.71, $1.75, $2.01, $1.59

B: 119, 145, 97, 121, 118, 98, 102, 114, 118, 99

23. The data in set A represent numbers of orders received by an online retailer for a random sample of months from the past two years. The data in set B represent package weights (in pounds) for a random sample of customer orders from the same online retailer in the past two years. Which of the two data sets has the *smaller* spread relative to its own mean?

A: 21,568 20,888 20,037 21,932 22,000 21,123 21,567 22,298

B: 4.5, 4.3, 6.5, 3.3, 4.7, 3.67, 4.01, 3.89, 4.4, 2.99, 4.88, 3.77

24. The data in set A represent numbers of graduates from a high school for a random sample of years since the school opened. The data in set B represent the numbers of graduates who started college during the year after graduating from the same high school for the same sample of years. Which of the two data sets has the *smaller* spread relative to its own mean?

A: 328, 444, 283, 289, 345, 327, 298, 277, 419, 402, 399, 418, 401

B: 78, 73, 92, 89, 74, 88, 91, 71, 70, 89, 81, 83, 84

Empirical Rule

Use the Empirical Rule to answer the questions.

25. Suppose that private school tuitions in one region of the country have a bell-shaped distribution with a mean of $25,400 and a standard deviation of $1300. Approximately what percentage of tuitions are between $24,100 and $26,700?

26. Suppose that private school tuitions in one region of the country have a bell-shaped distribution with a mean of $25,400 and a standard deviation of $1300. Approximately what percentage of tuitions are between $22,800 and $28,000?

27. Suppose that electric bills for the month of May in one city have a bell-shaped distribution with a mean of $119 and a standard deviation of $22. Approximately what percentage of electric bills are greater than $97?

28. Suppose that electric bills for the month of May in one city have a bell-shaped distribution with a mean of $119 and a standard deviation of $22. Approximately what percentage of electric bills are less than $163?

29. Suppose it is known that verbal SAT scores have a bell-shaped distribution with a mean of 500 and a standard deviation of 100. Approximately what percentage of verbal SAT scores are no more than 600?

30. Suppose it is known that verbal SAT scores have a bell-shaped distribution with a mean of 500 and a standard deviation of 100. Approximately what percentage of verbal SAT scores are at least 300?

Chebyshev's Theorem

Use Chebyshev's Theorem to answer the questions.

31. Suppose that household electric bills for the months of May through August in a city in Florida have a mean of \$230 and a standard deviation of \$58. What is the minimum percentage of electric bills between \$56 and \$404?

32. Suppose that salaries for associate mathematics professors at one university have a mean of \$64,900 and a standard deviation of \$9400. What is the minimum percentage of associate professors with salaries between \$46,100 and \$83,700?

33. Car insurance premiums in one region have a quarterly mean of \$246 and a standard deviation of \$31. What is the minimum percentage of car insurance premiums between \$184 and \$308?

34. The average weight of cows auctioned at a large livestock event is 1614 with a standard deviation of 59 pounds. What is the minimum percentage of cows that weigh between 1437 and 1791 pounds?

Standard Deviation and Variance of Grouped Data

Estimate the sample standard deviation or variance of the data in each frequency distribution using the given formula.

35. Let's extend the concept of standard deviation to include the standard deviation for a frequency distribution. The following frequency distribution gives the final grades for students in a statistics class.

Final Grades

Grade	Frequency
66–72	4
73–79	7
80–86	12
87–93	8
94–100	5

Since we do not know the exact value of each final grade, we will estimate that each value in a class is equal to the midpoint of that class. Use the following formula to estimate the sample standard deviation of the data in the frequency distribution, if you calculate this value by hand. To calculate this estimate using a TI-83/84 Plus calculator, use the same directions given in Section 3.1 for calculating a weighted mean, entering the midpoints in L1 and the frequencies in L2.

$$s = \sqrt{\frac{n\left[\sum\left(f_i \cdot x_i^{\,2}\right)\right] - \left[\sum\left(f_i \cdot x_i\right)\right]^2}{n(n-1)}},$$

where n = sample size,

f_i = frequency of class i, and

x_i = midpoint of class i.

36. Use the formula given in Exercise 35 to estimate the sample standard deviation of the gas prices in the following frequency distribution.

Gas Prices

Price in Dollars per Gallon	Frequency
3.55–3.59	1
3.60–3.64	3
3.65–3.69	5
3.70–3.74	6
3.75–3.79	2
3.80–3.84	1

37. What is the approximate sample variance of the final grades in the frequency distribution given in Exercise 35?

38. What is the approximate sample variance of the gas prices in the frequency distribution given in Exercise 36?