11.2 Section Exercises

Test Statistics and Degrees of Freedom for Hypothesis Tests for Two Population Means (σ Unknown)

Calculate the test statistic and determine the number of degrees of freedom for a hypothesis test for two population means using the given information. Assume that both population distributions are approximately normal.

1. $\bar{x}_1 = 93.0, \quad s_1 = 10.4, \quad n_1 = 21, \quad \bar{x}_2 = 89.2, \quad s_2 = 9.5, \quad n_2 = 18, \quad H_0: \mu_1 - \mu_2 = 0$
Assume that the population variances are not the same.

2. $\bar{x}_1 = 3.4, \quad s_1 = 0.3, \quad n_1 = 5, \quad \bar{x}_2 = 3.7, \quad s_2 = 0.5, \quad n_2 = 7, \quad H_0: \mu_1 - \mu_2 \leq 0$
Assume that the population variances are not the same.

3. $\bar{x}_1 = 33.5, \quad s_1 = 2.1, \quad n_1 = 14, \quad \bar{x}_2 = 31.1, \quad s_2 = 2.8, \quad n_2 = 11, \quad H_a: \mu_1 - \mu_2 > 0$
Assume that the population variances are equal.

4. $\bar{x}_1 = 24.1, \quad s_1 = 1.3, \quad n_1 = 19, \quad \bar{x}_2 = 23.0, \quad s_2 = 1.1, \quad n_2 = 22, \quad H_0: \mu_1 - \mu_2 = 0$
Assume that the population variances are equal.

Note: *The* `2-SampTTest` *was used to obtain the p-value answers for all exercises.*

Hypothesis Tests for Two Population Means (σ Unknown)

Perform each hypothesis test using the method of your choice or the one assigned by your instructor. For each exercise, complete the following steps. Assume that both population distributions are approximately normal in each scenario.

a. State the null and alternative hypotheses.
b. Determine which distribution to use for the test statistic, and state the level of significance.
c. Calculate the test statistic.
d. Draw a conclusion and interpret the decision.

5. A manufacturer fills soda bottles. Periodically the company tests to see if there is a difference between the mean amounts of soda put in bottles of regular cola and diet cola. A random sample of 14 bottles of regular cola has a mean of 501.6 mL of soda with a standard deviation of 3.9 mL. A random sample of 16 bottles of diet cola has a mean of 498.9 mL of soda with a standard deviation of 5.3 mL. Test the claim that there is a difference between the mean fill levels for the two types of soda using a 0.01 level of significance. Assume that the population variances are not equal since different machines are used to fill bottles of regular cola and diet cola.

6. A professor is concerned that the two sections of college algebra that he teaches are not performing at the same level. To test his claim, he looks at the mean exam score for a random sample of students from each of his classes. In Class 1, the mean exam score for 12 students is 78.7 with a standard deviation of 6.5. In Class 2, the mean exam score for 15 students is 81.1 with a standard deviation of 7.4. Test the professor's claim at the 0.05 level of significance. Assume that the population variances are equal.

7. While shopping for a cookout, Ian notices that a particular brand of charcoal briquettes claims to burn longer because the briquettes are 60% thicker than the competitor's briquettes. Feeling tired of being taken for a shopper who just believes what the manufacturer wants him to believe, Ian decides to test the manufacturer's claim that the charcoal briquettes from Brand A are thicker than those from Brand B. He buys a bag of each kind of charcoal, randomly selects a few briquettes from each brand, and measures the thicknesses of the briquettes in his samples. His findings are given in the following table. Test the manufacturer's claim that the charcoal briquettes from Brand A are thicker than those from Brand B at the 0.01 level of significance. Assume that the population variances are different.

Thickness of Charcoal Briquettes (in cm)

	Brand A	Brand B
Sample Size	8	6
Mean Thickness	3.21	2.13
Standard Deviation	0.50	0.85

8. A new small business wants to know if its current radio advertising is effective. The owners decide to look at the mean number of customers who make a purchase in the store on days immediately following days when the radio ads are played as compared to the mean for those days following days when no radio advertisements are played. They found that for 11 days following no advertisements, the mean was 17.8 purchasing customers with a standard deviation of 3.5 customers. On 6 days following advertising, the mean was 22.8 purchasing customers with a standard deviation of 2.8 customers. Test the claim, at the 0.01 level, that the mean number of customers who make a purchase in the store is lower for days following no advertising compared to days following advertising. Assume that the population variances are equal.

9. Gary has discovered a new painting tool to help him in his work. If he can prove to himself that the painting tool reduces the amount of time it takes to paint a room, he has decided to invest in a tool for each of his helpers as well. From records of recent painting jobs that he completed before he got the new tool, Gary collected data for a random sample of 6 medium-sized rooms. He determined that the mean amount of time that it took him to paint each room was 4.2 hours with a standard deviation of 0.5 hours. For a random sample of 4 medium-sized rooms that he painted using the new tool, he found that it took him a mean of 3.9 hours to paint each room with a standard deviation of 0.7 hours. At the 0.10 level, can Gary conclude that his mean time for painting a medium-sized room without using the tool was greater than his mean time when using the tool? Assume that the population variances are equal.

10. A supermarket chain is convinced that customers spend more money at the grocery store when the store plays music with a slow tempo over the loud speaker rather than music with a faster tempo. They select 2 stores to test their theory. The first store plays music with a slow tempo for the noon hour and randomly chooses 4 of the receipts with a mean of $122.56 and a standard deviation of $13.12. The second store plays music with a fast tempo for the noon hour. Five randomly chosen receipts from that time have a mean of $108.31 with a standard deviation of $17.06. Assume that the population variances are different. At the 0.10 level, can the supermarket chain say that music with a slower tempo makes customers buy more groceries?

11. While running some quality control tests, a manager at a factory that makes potato chips noticed a difference in the mean bag weights for chips coming from two different production lines. To see if the bags in Line A did actually have a mean weight lower than those in Line B, he randomly selected some of the bags from each line. His results are given in the following table. If he assumes that the population variances for the two lines are different, can he conclude at the 0.10 level that the mean weight of bags from Line A is lower than the mean weight of bags from Line B?

Weights of Bags of Chips (in g)

	Line A	Line B
Sample Size	20	15
Mean Weight	309.63	311.87
Standard Deviation	15.91	13.21

12. A physician wants to test the claim that the average adult height of premature baby boys is different from that of full-term baby boys. To do this, he finds a random sample of 18 men who were born prematurely and calculates that their mean height is 68.1 inches with a standard deviation of 2.3 inches. He also finds a random sample of 20 men who were carried full term and calculates their mean height to be 68.9 inches with a standard deviation of 2.0 inches. Assume that the population standard deviations are unequal, and test the claim at the 0.05 level of significance.

13. A pharmaceutical company needs to know if its new cholesterol drug, Praxor, is effective at lowering cholesterol levels. It believes that people who take Praxor will average a greater decrease in cholesterol level than people taking a placebo. After the experiment is complete, the researchers find that the 25 participants in the treatment group lowered their cholesterol levels by a mean of 23.5 points with a standard deviation of 5.8 points. The 25 participants in the control group lowered their cholesterol levels by a mean of 18.9 points with a standard deviation of 4.1 points. Assume that the population variances are not equal, and test the company's claim at the 0.01 level.

14. A speech pathology professor believes from experience that, on average, boys begin talking at a later age than girls. To test her theory, she gathers information from the parents of random samples of 12 boys and 14 girls. The boys began talking at a mean of 1.33 years of age with a standard deviation of 0.15 years. The girls began talking at a mean of 1.23 years of age with a standard deviation of 0.12 years. Assume that the population standard deviations are equal, and test the professor's claim at the 0.05 level of significance.

15. Insurance Company A claims that its customers pay less for car insurance, on average, than customers of its competitor, Company B. You wonder if this is true, so you decide to compare the average monthly costs of similar insurance policies from the two companies. For a random sample of 9 people who buy insurance from Company A, the mean cost is \$152 per month with a standard deviation of \$17. For 11 randomly selected customers of Company B, you find that they pay a mean of \$155 per month with a standard deviation of \$14. Assume that the population variances are equal, and test Company A's claim at the 0.01 level of significance.

16. A women's group believes that women are offered lower starting salaries than men applying for similar jobs. To test this claim, the group sends 10 women and 10 men to interviews for similar positions at various companies. The women were offered a mean starting salary of \$29,500 with a standard deviation of \$1100. The men were offered a mean starting salary of \$30,500 with a standard deviation of \$950. Assume that the population standard deviations are different, and test the group's claim at the 0.05 level of significance.

17. A psychologist wants to test the claim that the mean lengths of time spent to complete a piece of art are different for trauma patients and non-trauma patients. He randomly chooses 12 trauma patients and 11 non-trauma patients and asks them to estimate the amount of time it takes for them to complete a piece of artwork. According to the survey, the trauma patients spent a mean of 4.6 hours to complete the artwork with a standard deviation of 1.3 hours. The non-trauma patients spent a mean of 5.4 hours to complete a piece of artwork with a standard deviation of 1.1 hours. Assume that the population variances are equal, and test the claim at the 0.05 level of significance.

18. The results of a state-wide 9th grade literature state assessment for random samples of students in two neighboring counties are shown in the table.

Grappling County	Hammond County
$\bar{x}_1 = 517$	$\bar{x}_2 = 495$
$s_1 = 39.7$	$s_2 = 24.8$
$n_1 = 134$	$n_2 = 40$

Can you conclude that there is a difference in the mean literature test scores for the students of the two counties? Use $\alpha = 0.01$. Assume the populations are normally distributed and the population variances are not equal.