Step 4: Draw a conclusion and interpret the decision.

Method 1: Rejection Regions

The number of degrees of freedom for the chi-square distribution for this test is $df = 5 - 1 = 4$, and $\alpha = 0.05$. Using the table, we find that the critical value is $\chi^2_{0.05} = 9.488$. Comparing the test statistic to the critical value, we have $2.950 < 9.488$, so $\chi^2 < \chi^2_{0.050}$, and thus we must fail to reject the null hypothesis.

Method 2: *p*-Values

We see that $\chi^2 \approx 2.950$ and the *p*-value ≈ 0.5662 from the output screen for the TI-83/84 Plus calculator. This *p*-value can be compared to the level of significance, $\alpha = 0.05$, to draw a conclusion. Remember that if *p*-value $\leq \alpha$, then the conclusion is to reject the null hypothesis. In this case, *p*-value $> \alpha$, so the conclusion is to fail to reject the null hypothesis.

Interpretation: The evidence does not support the claim that the proportions of kids playing youth sports have changed. The educational campaign does not appear to have made any difference based on this evidence.

10.6 Section Exercises

Null and Alternative Hypotheses for Chi-Square Tests for Goodness of Fit

State the null and alternative hypotheses in words for each scenario.

1. A pharmacist believes that more people get their prescriptions filled on Fridays than on any other day of the week.
2. A game warden believes that the numbers of ducks, geese, and swans in the national park are not equal.
3. The hairstylists at a salon think that they are twice as busy on Thursdays and Fridays than the rest of the week.
4. A music producer believes that Southerners prefer country music over either pop music or R&B.

Test Statistics for Chi-Square Tests for Goodness of Fit

Calculate the test statistic, χ^2, for a chi-square test for goodness of fit using the given information.

5. The following data represent numbers of drive-through customers at a fast-food restaurant on a weekday.

Drive-Through Customers

	8:00–9:00 a.m.	12:00–1:00 p.m.	3:00–4:00 p.m.	5:00–6:00 p.m.
Observed Values, O_i	35	54	11	21
Expected Values, E_i	$\frac{121}{4}$	$\frac{121}{4}$	$\frac{121}{4}$	$\frac{121}{4}$

6. The following data represent numbers of four different candy bars sold at a concession stand in one day.

Candy Bar Sales

	A	B	C	D
Observed Values, O_i	11	12	5	15
Expected Values, E_i	$\frac{43}{4}$	$\frac{43}{4}$	$\frac{43}{4}$	$\frac{43}{4}$

7. The following data represent job applicants' scoring levels on a company's entrance tests.

Applicants' Scoring Levels			
	Level 1	Level 2	Level 3
Observed Values, O_i	29	201	169
Expected Values, E_i	20.5	218.8	159.7

8. The following data represent numbers of hamburgers sold at three competing restaurants between 12 p.m. and 2 p.m. one weekday.

Hamburger Sales			
	Handy Andy	Philip's Grocery	Proud Larry
Observed Values, O_i	37	26	24
Expected Values, E_i	29	29	29

9. The following data represent numbers of kids' jeans sold in various sizes at a department store in one day.

Kids' Jeans Sales				
	Size 6	Size 8	Size 10	Size 12
Observed Values, O_i	4	7	5	5
Expected Values, E_i	5.25	5.25	5.25	5.25

10. The following data represent numbers of children at the local library on a Saturday.

Children at the Library				
	9:00–10:00 a.m.	10:00–11:00 a.m.	1:00–2:00 p.m.	3:00–4:00 p.m.
Observed Values, O_i	6	10	9	8
Expected Values, E_i	8.25	8.25	8.25	8.25

Critical Values for Chi-Square Tests for Goodness of Fit

Find the critical value of χ^2 for a chi-square test for goodness of fit using the given information.

11. $\alpha = 0.10, \quad k = 47$

12. $\alpha = 0.05, \quad k = 22$

13. $\alpha = 0.005, \quad k = 13$

14. $\alpha = 0.01, \quad k = 9$

Conclusions of Chi-Square Tests for Goodness of Fit

State the critical value of χ^2 and determine the appropriate conclusion for a chi-square test for goodness of fit using the given information.

15. $\alpha = 0.005, \quad k = 15, \quad \chi^2 = 29.674$

16. $\alpha = 0.10, \quad k = 8, \quad \chi^2 = 13.219$

17. $\alpha = 0.05, \quad k = 29, \quad \chi^2 = 43.222$

18. $\alpha = 0.025, \quad k = 17, \quad \chi^2 = 27.008$

Hypothesis Tests for Goodness of Fit

Perform each test for goodness of fit. For each exercise, complete the following steps.

a. State the null and alternative hypotheses.
b. Determine which distribution to use for the test statistic, and state the level of significance.
c. Find the expected value for each possible outcome, and calculate the test statistic.
d. Draw a conclusion and interpret the decision.

19. A school principal claims that the number of students who are tardy to school does not vary from month to month. A survey over the school year produced the following results. Using a 0.05 level of significance, test a teacher's claim that the number of tardy students does vary by the month.

Tardy Students

	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
Number	7	18	16	5	8	12	15	18	11	15

20. A service station owner believes that equal numbers of customers prefer to buy gasoline on every day of the week. A manager at the service station disagrees with the owner and claims that the number of customers who prefer to buy gasoline on each day of the week varies. Test the manager's claim using $\alpha = 0.10$. The owner surveyed 739 customers over a period of time to record each customer's preferred day of the week. Here's what he found.

Preferred Day to Buy Gasoline

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Number	103	103	126	103	111	96	97

21. At the emergency room at one hospital, the nurses are convinced that the number of patients that they see during the midnight shift is affected by the phases of the moon. The doctors think this is an old wives' tale. The nurses decide to test their theory at the 0.10 level of significance by recording the number of patients they see during the midnight shift for each moon phase over the course of one lunar cycle. The results are summarized in the table below.

ER Patients During the Midnight Shift

	New Moon	1st Quarter	Full Moon	3rd Quarter
Number of Patients	85	66	97	68

22. The management of the local zoo wants to know if all of their animal exhibits are equally popular. If there is significant evidence that some of the exhibits are not being visited frequently enough, then changes may need to take place within the zoo. A tally of visitors is taken for each of the following animals throughout the course of a week, and the results are contained in the following table. At $\alpha = 0.05$, determine whether there is sufficient evidence to conclude that some exhibits are less popular than others.

Animal Exhibits at the Zoo

	Elephants	Lions/Tigers	Giraffes	Zebras	Monkeys	Birds	Reptiles
Number of Visitors	157	154	168	162	185	129	133

23. The manager of the city pool has scheduled extra lifeguards to be on staff for Saturdays. However, he suspects that Fridays may be more popular than the other weekdays as well. If so, he will hire extra lifeguards for Fridays, too. To test his theory that the daily number of swimmers varies on weekdays, he records the number of swimmers each day for the first week of summer. Test the manager's theory at the 0.01 level of significance.

Swimmers at the City Pool

	Monday	Tuesday	Wednesday	Thursday	Friday
Number	46	47	43	53	54

24. A manufacturer of children's vitamins claims that its vitamins are mixed so that each batch has exactly the following percentages of each color: 20% green, 40% yellow, 10% red, and 30% orange. To test the claim that these percentages are incorrect, 100 bottles of vitamins were pulled and the colors of the vitamins were tallied. The results are listed in the following table. At $\alpha = 0.05$, determine whether there is sufficient evidence to conclude that the percentages stated by the vitamin manufacturer are incorrect.

Children's Vitamins

	Green	Yellow	Red	Orange
Number	1149	1948	552	1401