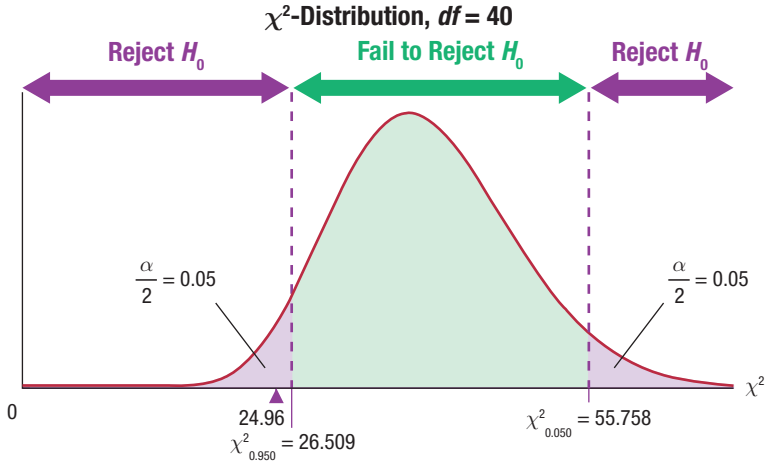

Since $\chi^2 = 24.96$, which is in the rejection region, we reject the null hypothesis.

Interpretation: Thus, there is sufficient evidence at the 0.10 level of significance to support the farmer's claim that the variance of the heights of the two-year-old trees is not 16.

10.5 Section Exercises

Rejection Regions for Hypothesis Tests for Population Variances and Population Standard Deviations

State the critical value(s) of the test statistic, and determine the rejection region for a hypothesis test for the population variance or population standard deviation using the given information. Assume that the population distribution is approximately normal.

1. $\alpha = 0.05, \quad df = 26, \quad$ left-tailed test

2. $\alpha = 0.10, \quad df = 15, \quad$ left-tailed test

3. $\alpha = 0.05, \quad n = 18, \quad H_a: \sigma > 0.04$

4. $c = 0.90, \quad n = 31, \quad H_a: \sigma^2 > 0.65$

5. $c = 0.95, \quad n = 25, \quad H_a: \sigma^2 \neq 0.80$

6. $c = 0.99, \quad n = 30, \quad H_0: \sigma \neq 0.009$

Hypothesis Tests for Population Variances and Population Standard Deviations

Perform each hypothesis test. For each exercise, complete the following steps. Assume that each population is normally distributed.

a. State the null and alternative hypotheses.
b. Determine which distribution to use for the test statistic, and state the level of significance.
c. Calculate the test statistic.
d. Draw a conclusion and interpret the decision.

7. A dairy supplier fills hundreds of cartons with milk each day. They are contracted to fill each carton with exactly one gallon of milk. Because of the moving parts on the machine that fills the cartons, the amount of milk dispensed begins to vary slightly over time. When this happens, the machine must be serviced to realign it correctly. When testing the accuracy of the machine, the amount of milk dispensed into each carton sampled is measured in milliliters (mL). A variance of 4 is acceptable. Servicing must occur when the variance in the amounts of milk in the cartons is more than 4 with a level of significance of 0.01. Twenty-five cartons are randomly chosen to be tested and the amounts of milk in the cartons are found to have a standard deviation of 2.5 mL. Perform a hypothesis test to determine if the machine needs servicing.

8. A potato chip manufacturer produces bags of potato chips that are supposed to have a net weight of 326 grams. Because the chips vary in size, it is difficult to fill the bags to the exact weight desired. However, the bags pass inspection so long as the standard deviation of their weights is 3 grams. A quality control inspector wished to test the claim that one batch of bags has a standard deviation of more than 3 grams, and thus does not pass inspection. If a sample of 25 bags of potato chips is taken and the standard deviation is found to be 3.4 grams, does this evidence, at the 0.05 level of significance, support the claim that the bags should fail inspection?

9. At an archery range, instructors can determine if an archer is consistently missing the target because of the sight or because of the archer's ability. If a sight is off, the variance of the distances between the shots and the center of the shot pattern will be small (even if the shots are not in the center of the target). A student claims that it is the sight that is off, not his aim, and wants the instructor to confirm his claim. If a skilled archer shoots an arrow at a target multiple times, the distances between the shots and the center of the shot pattern, measured in centimeters (cm), will have a variance of 0.33. After the student shoots 20 shots at the target, the instructor calculates that the distances between his shots and the center of the shot pattern, measured in cm, have a variance of 0.17. Does this evidence support the student's claim that the sight is off? Use a 0.10 level of significance.

10. To ensure that there is not a large disparity in the quality of education at various schools in a school district, the school board wants to make sure that the variance of the mean standardized-test scores for all students at each school in the district is less than 0.05. To test this claim, they looked at the mean student scores for the standardized test from a random sample of 18 schools in the district. The results from the survey found that the overall mean was a score of 192.560 with a standard deviation of 0.162. With $\alpha = 0.10$, perform a hypothesis test to determine if the variance is less than 0.05.

11. The manufacturer of a popular antibiotic must ensure that each 5-mL dose contains 250 milligrams (mg) of the active ingredient. It is also essential that the variance of the amounts of active ingredient per dose be less than 0.1. For testing purposes, a random sample of 100 doses is taken, and the standard deviation of the amounts of active ingredient per dose in the sample is found to be 0.3 mg. Does the evidence support the claim that the variance is within the necessary bounds, at the 0.01 level of significance?

12. Claudia's bakery business has taken off. Her goods are so popular that she has decided to invest in kitchen equipment that will do some of the work for her. One machine, in particular, prepares bread dough, and it is set to measure 8 grams of yeast per loaf. Because baking requires precise measurements, she wants to test the new machine to make sure that the variance in the amounts of yeast per loaf is less than 0.30, at the 0.05 level of significance. A sample of the amounts of yeast added to the dough for 8 loaves has a variance of 0.25. Does this evidence support the claim that the new machine produces dough with a variance in the amounts of yeast per loaf of less than 0.30?

13. The temperatures in chicken incubators on a chicken farm, measured in degrees Fahrenheit (°F), are generally believed to have a variance of 0.50. The manager of the chicken farm claims that the variance has changed. A random test of 25 incubators finds that their temperatures have a standard deviation of 0.55 °F. At the 0.05 level of significance, does this evidence support the manager's claim that the variance is not 0.50?

14. A grocery store needs the refrigeration section to have its coolers stay at the same temperature on a daily basis with little variance to help ensure quality. Daily temperatures are measured in degrees Fahrenheit (°F), and the manager of the store assumes that the variance in the daily temperatures is 3.8. The assistant manager claims that the variance is not 3.8 and decides to test the claim using a hypothesis test. For a random sample of 30 days, the assistant manager finds that the standard deviation in the daily temperatures for one cooler is 2.9 °F. At the 0.01 level of significance, does this evidence support the claim that the variance in the daily temperatures for that cooler is not 3.8?

15. A health club needs to ensure that the temperature in its heated pool stays constant throughout the winter months. Otherwise, it needs to invest in a new heater for the pool. It is assumed that the daily water temperatures, measured in degrees Fahrenheit (°F), have a variance of 2.25, which is considered to be within normal limits for a properly operating heater. The pool manager needs to determine if the variance is still 2.25, so a hypothesis test is performed to test the claim that the variance in the temperature is not 2.25. After testing the pool water for a random sample of 15 winter days, the pool manager finds a mean daily temperature of 78.60 °F with a variance of 3.81. At the 0.10 level of significance, does this evidence support the manager's claim that the variance in the water temperature is no longer 2.25?

16. Many properties of the fabric used in different textiles, such as sheets and clothing, are influenced by whether the diameter of yarn used in the creation of the textile is consistent. Therefore, various methods can be used to measure the diameter of yarn at specified intervals, such as every 2 mm, to determine the consistency of the diameter. These measurements are normally distributed. Suppose that one textile manufacturer will not use any yarn in which the standard deviation of the diameters is greater than 0.02 mm. To ensure that the yarn is usable, the diameter of a length of yarn is measured at 100 random intervals and the standard deviation of those measurements is found to be 0.021402 mm. Does this evidence provide support for the manufacturer to use this batch of yarn? Use a 0.05 level of significance.

Source: Ibrahim, S. et.al. "Characterization of Yarn Diameter Measured on Different." 4 July 2012. http://textileconference.rmutp.ac.th/wp-content/uploads/2012/10/005-Characterization-of-Yarn-Diameter-Measured-on-different.pdf (27 Mar. 2019).