**Example 10.1.8**

**Determining the Type of Error**

A study regarding the effects of television viewing on children reports that children watch a mean of 4.0 hours of television per night. Kiko believes the mean number of hours that children in her neighborhood watch television per night is not 4.0. She performs a hypothesis test and rejects the null hypothesis. Assume that in reality, children in her neighborhood do watch a mean of 4.0 hours of television per night. Did she make an error? If so, what type?

**Solution**

Begin by writing the null and alternative hypotheses. Kiko wishes to gather data in support of her belief that the mean is not 4.0 hours per night. Therefore, her research hypothesis is written mathematically as $H_a: \mu \neq 4.0$. Thus, the null and alternative hypotheses are written as follows.

$$H_0: \mu = 4.0$$
$$H_a: \mu \neq 4.0$$

The decision was to reject the null hypothesis, when in reality, $\mu = 4.0$, so Kiko rejected a true null hypothesis. This is a Type I error.

**Memory Booster**

We would usually have no basis for determining if a Type I error or a Type II error has been made because population parameters are usually unknown.

Now that you have had practice distinguishing between Type I and Type II errors, it is important to note that in almost every case, we would not know the "reality" of a situation because population parameters are usually unknown. Thus, we would usually have no basis for determining if a Type I error or a Type II error has been made. Knowing what types of errors exist and how they are related to each other, as well as being able to distinguish between them, allows us to interpret decisions in the context of possible realities.

In conclusion, in this section we discussed the nuances of each step of a hypothesis test as well as the types of errors that might be made when stating the conclusion. In the next several sections, we will discuss the details of determining the appropriate probability distribution to use as well as the test statistic to use for a given population parameter and the various methods for determining the values of that test statistic. In these sections, we will look at complete examples of hypothesis tests from start to finish for several different population parameters. These complete examples will put all the general discussion from this section into specific focus.

# 10.1 Section Exercises

## Fundamentals of Hypothesis Testing

***Decide if each statement is true or false. Explain why.***

1. When we reject a null hypothesis, we have proven the alternative hypothesis to be true.
2. There are only two possible conclusions in a hypothesis test: reject or fail to reject the null hypothesis.
3. A Type I error is made when a true null hypothesis is rejected.
4. A Type II error is made when we fail to reject a true null hypothesis.
5. The level of significance is the probability of making a Type II error.
6. The probability of making a Type I error is inversely related to the probability of making a Type II error.

## Null and Alternative Hypotheses

***State the null and alternative hypotheses for each scenario.***

**7.** Based on past sales, a shoe manufacturer considers the mean size of women's shoes to be 7.5. The manufacturer would like to test if this is still the case.

**8.** Austin's company manufactures test tubes. He needs his tubes to be exactly 4.0 mm in diameter. If they are too narrow or too wide, he must recalibrate his machine. Austin randomly measured 150 tubes off the production line to perform a hypothesis test.

**9.** A sports analyst is testing his claim that the mean weight of this year's NFL linemen is heavier than in past years. Suppose that over the past five years, the mean weight of NFL linemen was 320.0 pounds.

**10.** A nationwide study shows that children watch a mean of 3.0 hours of television per day. A mother in Louisiana believes that this is an underestimate for children in her area. She conducts a local survey to test her belief.

**11.** After reading a headline claiming that half of Americans think space travel will become routine during the next 50 years of space exploration, a science teacher had students survey friends and relatives nationwide to research whether the true proportion is less than half.

**12.** Madison, an ambulance driver in a small town, believes a typical 24-hour shift receives fewer than 5 emergency calls on average. To test this belief, the number of calls each shift receives is recorded for 10 randomly selected shifts.

**13.** While continuing to keep abreast of local trends in education, a school administrator read a journal article that reported only 42% of high school students study on a regular basis. The administrator claims that this percentage is too low for the district.

**14.** The city council of Oxford is thinking of building a road around the city. The council wants to know if the majority of the residents are in favor of the new road before pursuing the issue further.

**15.** As a general guideline, tap water should contain 3 ppm (parts per million) chlorine. A local council wants to test the chlorine levels on 15 randomly selected days over the next two months to see if the guidelines are being exceeded.

**16.** Seawater is believed to have a mean fluoride concentration of 1.3 mg/L (milligrams per liter). A marine biologist is concerned that the level of fluoride is too high in a particular area and is killing the ocean life.

**Source:** "Fluoride in Drinking-Water: Background Document for Development of WHO *Guidelines for Drinking-Water Quality*." World Health Organization. 2004. http://www.who.int/water_sanitation_health/dwq/chemicals/fluoride.pdf (9 Nov. 2011).

## Interpreting Conclusions of Hypothesis Tests

***Answer each question.***

**17.** A television network has believed for many years that 40% of its viewers are below the age of 22. For marketing purposes, a potential advertiser wishes to test the claim that the percentage is actually less than 40%. After performing the test at the 0.05 level of significance, the advertiser decides to reject the null hypothesis. What does this conclusion lead us to believe about the potential advertiser's claim?

**18.** A radio station has always believed that the mean age of its listeners is 26. Because their ratings are slipping, the executives need to know if the mean age has changed, so that they can alter their programs accordingly. After information is collected from 329 listeners and a hypothesis test is completed, the radio station executives decide that they should reject the null hypothesis with a 95% level of confidence. Based on this conclusion, should the radio station look into changing its programming?

**19.** A pharmaceutical company has publicized that approximately 4% of people who take a particular drug experience significant side effects. A researcher is concerned that the percentage is more than 4%, and she decides to test her claim with a hypothesis test. Based on the sample she collects, she decides to fail to reject the null hypothesis at the 0.01 level of significance. What does this conclusion tell us about the researcher's claim?

**20.** A cellular phone company promotes the claim that its customers spend a mean of \$60 per month on cell phone service. One skeptical college student is convinced that the mean is higher than \$60 per month. He surveys a random sample of the company's customers and performs a hypothesis test to test his claim. In the end, he decides to fail to reject the null hypothesis with $\alpha = 0.01$. What does this conclusion tell us about the college student's claim?

## Types of Errors in Hypothesis Testing

***For each scenario, determine the type of error that was made, if any. (Hint: Begin by determining the null and alternative hypotheses.)***

**21.** A software company advertises that its software can improve students' grades by 15%. One student conducts a hypothesis test to see if the percentage increase is less than 15%. The conclusion of the hypothesis test is to reject the null hypothesis. If the true percentage increase in grades for all students using the software is 13%, was an error committed? If so, what type?

**22.** A prominent travel agency claims that the mean cost of a four-day theme park vacation is \$1500 per person. A researcher believes that the mean cost is much higher, and decided to test the theory with a hypothesis test. According to the sample obtained, the decision is to fail to reject the null hypothesis. If, in reality, the mean cost of a four-day theme park vacation is \$1500 was an error made with the hypothesis test? If so, what type?

**23.** The mathematics department at one university reports that the failure rate for College Algebra is 35% in any given semester. A group of students believes the rate is higher, and they decided to use a hypothesis test to see if they are correct. Based on the sample they obtained, they decided to reject the null hypothesis. If, in reality, the failure rate is 35%, did the students make an error? If so, what type?

**24.** Dreamfilms Studios boasts that its summer features are so good that half of all tickets sold one summer are for its films. A competing studio claims that less than half of all tickets sold are for Dreamfilms' movies. After the hypothesis test is completed, the conclusion is to fail to reject the null hypothesis. If the true percentage of tickets sold for Dreamfilms Studios' movies that summer was only 48%, was an error committed? If so, what type?

**25.** An ice cream company must keep the temperature inside its delivery trucks at 30 degrees Fahrenheit. If the temperature is too warm, the ice cream will melt. The company also does not want to allow the temperature to be too low, because it does not want to waste the cost and energy it takes to keep the trucks cool. Periodically the company runs a hypothesis test to ensure that the temperature is as claimed. Suppose that after one hypothesis test, the company fails to reject the null hypothesis. If, in reality, the mean temperature of its trucks is 29 degrees Fahrenheit, was an error committed? If so, what type?

**26.** A survey of undergraduate college students found that 52% of students think the FAFSA is only for federal financial aid. A local college wants to make sure that their students understand that this is not true. After presenting numerous programs aimed at helping students understand the reasons for filling out the FAFSA forms for financial aid, the college believes that the percentage of students who believe this myth is now lower at their college. A hypothesis test of their claim results in failing to reject the null hypothesis. If, in reality, 52% of students at this college believe that the FAFSA is only for federal financial aid, was an error made? If so, what type?