

## Looking Ahead

In the following example, problem solving is used to determine the best way for the business owner to meet their objective.

### Example Preview

Determine whether an observational or experimental study is appropriate to address the following statement.

A car wash operator wants to study whether customers are satisfied with the service they receive.

### Solution

Since the car wash operator is merely observing data that already exists, the question would be answered through an observational study.

## 1.R.1 Exercises

### Concept Check

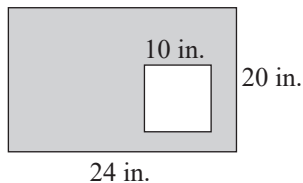
**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Averages are found by performing addition and then division.
2. The sum of 312 and 4 is 1248.
3. The word “quotient” indicates multiplication.
4. After reading a problem carefully, the next step might be to make a diagram or draw a figure.

## Applications

Solve.

5. **Nutrition Facts:** Steven is calculating how many calories are in his lunch. He has a hamburger that has 354 calories, a medium fry that has 365 calories, and a chocolate milkshake that has 384 calories. How many total calories is his meal?
6. **Purchases:** For a class in statistics, Anthony bought a new graphing calculator for \$95, special graphing paper for \$8, a USB flash drive for \$10, a textbook for \$105, and a workbook for \$37. How much did he spend for this class?
7. **Area:** A square that is 10 inches on a side is placed inside a rectangle that has a width of 20 inches and a length of 24 inches. What is the area of the region inside the rectangle that surrounds the square? (Find the area of the shaded region in the figure.)



8. **Average Purchases:** The Lee family spent the following amounts for groceries: \$338 in June; \$307 in July; \$318 in August. What was the average amount they spent for groceries in these three months?

## Writing & Thinking

9. Make up three word problems that include keywords to indicate operations such as addition, subtraction, multiplication, and division. Underline the keywords.

10. Give an example where you might use average (other than in a class).

## 1.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

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1. Two hundred thousand, four hundred six and twelve hundredths can be written as 200,406.12.
2. 92.586 is greater than 92.6.
3. On a number line, any number to the left of another number is larger than that other number.
4. When a decimal number is rounded, all numbers to the right of the place of accuracy become zeros in the final answer.

### Practice

5. Write  $2\frac{57}{100}$  in decimal notation.
6. Write 20.7 in words.
7. Write six and twenty-eight thousandths in decimal notation.

8. Arrange 0.2, 0.26, and 0.17 in order from smallest to largest. Then, graph the data on a number line.

Fill in the blanks to correctly complete each statement.

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9. Round 3.00652 to the nearest ten-thousandth.
- The digit in the ten-thousandths position is \_\_\_\_.
  - The next digit to the right is \_\_\_\_.
  - Since \_\_\_\_ is less than 5, leave \_\_\_\_ as it is and replace \_\_\_\_ with 0.
  - So 3.00652 rounds to \_\_\_\_\_ to the nearest ten-thousandth.

## Applications

In each exercise, write the decimal numbers that are not whole numbers in words.

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10. **Unicycles:** The tallest unicycle ever ridden was 114.8 feet tall, and was ridden by Sam Abrahams (with a safety wire suspended from an overhead crane) for a distance of 28 feet in Pontiac, Michigan, on January 29, 2004.<sup>1</sup>
11. **Water Weight:** One quart of water weighs approximately 2.0825 pounds.
12. **Average Height:** A basketball coach wants to know the average height of his starting lineup. To study this the coach could do an observational study, which involves observing data that already exists, or he could conduct an experiment to gather the data. Should he conduct an observational study or an experiment to determine the average height of his starting lineup?

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<sup>1</sup> Source: <http://semcycle.biz/record>

13. **Average Lifespan:** The average lifespan of Americans in 1990 was 75.37 years. If a *parameter* is a numerical description of a population characteristic and a *statistic* is a numerical description of a sample characteristic, does the underlined value describe a parameter or a statistic?
14. **Rounding:** If you were asked to round the underlined value in the previous question to the nearest tenth, what would your answer be? To the nearest whole number?

## Writing & Thinking

15. Discuss situations where you think it is particularly appropriate (or necessary) to write numbers in English word form.
16. With **a.** and **b.** as examples, explain in your own words how you can tell quickly when one decimal number is larger (or smaller) than another decimal number.
- The decimal number 2.765274 is larger than the decimal number 2.763895.
  - The decimal number 17.345678 is larger than the decimal number 17.345578.

## Looking Ahead

To calculate even the simplest of statistics, it is necessary to be able to work with decimal numbers in calculations and to understand the order in which operations should be performed in those calculations.

### Example Preview

Use the following body temperature data recorded on a patient taking fever-reducing medication to determine the patient's average body temperature for the week. The temperatures are in degrees Fahrenheit.

Day	Body Temperature (in °F)
Monday	103.1
Tuesday	102.6
Wednesday	101.0
Thursday	99.5
Friday	99.7
Saturday	99.2
Sunday	98.9

### Solution

To find the average of the temperatures you must first add the temperature data values together. After adding the temperatures together, you then divide by 7 to get the average body temperature.

$$\frac{103.1 + 102.6 + 101.0 + 99.5 + 99.7 + 99.2 + 98.9}{7} = \frac{704}{7} \approx 100.6^{\circ}F$$

## 1.R.3 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. An estimate of the sum  $71.369 + 49.1$  is 120.

2. One way to estimate the product of decimal numbers is to round the numbers to the rightmost nonzero digit before performing the multiplication.
  
3. An estimate of the quotient  $16.469 \div 3.87$  would be 4.
  
4. Experience and understanding are needed to decide whether or not a particular answer is reasonably close to an estimate.
  
5. According to the rules for order of operations, addition and subtraction should be performed before multiplication and division.

## Practice

Estimate each answer, then find the actual answer. Round to the nearest hundredth, if necessary.

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6. 
$$\begin{array}{r} 29.03 \\ + 3.79 \\ \hline \end{array}$$

7. 
$$\begin{array}{r} 51.21 \\ - 25.13 \\ \hline \end{array}$$

8.  $(6.3)(1.6)$

9.  $3.1 \overline{)6.36}$

Simplify.

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10.  $8.6 \div 2.15 + 3.6 \cdot 20.3$

## Applications

Solve.

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11. **Shipping:** Jim is packing three sculptures in a box for shipping. The weights of the sculptures are 5.63 pounds, 12.4 pounds, and 3 pounds. The shipping materials weigh 17.4 pounds.

a. Estimate the total weight.

b. Find the actual weight.

12. **Bicycling:** Peter Sagan rode 125.09 miles in 5.35 hours.

a. Estimate how fast he was riding per hour.

b. What was his average speed per hour (to the nearest hundredth)?

13. **Average Stock Price Change:** Compute the average daily change in stock price over the course of 5 days if the following changes were recorded.

Day	Daily Net Change (Closing Price – Opening Price)
Monday	+25.88
Tuesday	-12.65
Wednesday	+10.31
Thursday	+15.47
Friday	-30.49

## Writing & Thinking

14. Suppose you are only interested in an approximate answer for a product. Would there be any difference in the products produced by the following two procedures?
- First multiply the two numbers as they are and then round the product to the desired place of accuracy.
  - First round each number to the desired place of accuracy and then multiply the rounded numbers.

Explain why you think these two procedures would produce the same result or different results.

## Looking Ahead

Survey results are often converted from raw data to a percent.

### Example Preview

Sandra asked 20 of her friends for their favorite flavor of ice cream. Convert her survey results below to percents.

#### Solution

Sandra surveyed 20 people, so we divide the actual survey result in each category by 20 and then convert to a decimal and then to a percent.

Favorite Flavors of Ice Cream

Ice Cream Flavor	Survey Result	Survey Result as a Fraction	Survey Result as a Decimal	Survey Result as a Percent
Vanilla	2	$\frac{2}{20}$	0.1	10%
Chocolate	6	$\frac{6}{20}$	0.3	30%
Cookies and Cream	4	$\frac{4}{20}$	0.2	20%
Rocky Road	3	$\frac{3}{20}$	0.15	15%
Other	5	$\frac{5}{20}$	0.25	25%

## 1.R.4 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a decimal number is less than 1, then the equivalent percent will be less than 100%.
2. It is not possible to have a percent greater than 100%.
3. A decimal number that is between 0.01 and 0.10 is between 10% and 100%.
4. To change from a percent to a decimal, simply omit the percent sign.

**Practice**Change each fraction to a percent.

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5.  $\frac{20}{100}$

6.  $\frac{125}{100}$

Change each decimal number to a percent.

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7. 0.02

8. 2.3

Change each percent to a decimal number.

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9. 7%

10. 179%

**Applications**Solve.

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11. **Interest:** A savings account is offering an interest rate of 0.04 for the first year after opening the account. Change 0.04 to a percent.

12. **Sales Tax:** Suppose that sales tax is figured at 7.25%. Change 7.25% to a decimal.
13. **Artificial Intelligence Survey:** According to a survey conducted from Sept. 15 to Oct. 10, 2017 by Gallup and Northeastern University, 73% of Americans surveyed say artificial intelligence will eliminate more jobs than it creates. Results were based on a mail survey of 3,297 Americans over the age of 18.<sup>1</sup>
- What is the population being studied?
  - What is the sample?
  - What are the sample statistics?
  - Convert the sample statistics to decimal numbers.

## Writing & Thinking

14. Describe the relationship between percent and the number 100.
15. Describe a situation where more than 100% is possible. Describe a situation where it is impossible to have more than 100%.

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<sup>1</sup> Source: Gallup, Inc. and Northeastern University. "Optimism and Anxiety." January 2018. <https://www.northeastern.edu/gallup/pdf/OptimismAnxietyNortheasternGallup.pdf> (23 July 2019).

**Solution**

To find a relative frequency, we will need to divide the frequency of the class,  $f$ , we are looking at by the sample size,  $n$ .

To begin with, the sample size in this case is

$$n = 4 + 3 + 9 + 5 + 6 = 27 .$$

Therefore, the relative frequency of the second class will be

$$\frac{f}{n} = \frac{3}{27} = \frac{1}{9} .$$

**2.R.1 Exercises****Concept Check**

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. In  $\frac{11}{13}$ , the denominator is 11.
2.  $\frac{0}{6} = 0$
3.  $\frac{17}{0}$  is undefined.

**Practice**

For the figure, a) write the fraction for the number of days remaining in June (not crossed out) and b) write the fraction for the number of days that have been crossed out for June.

4.

June						
S	M	T	W	T	F	S
<del>1</del>	<del>2</del>	<del>3</del>	<del>4</del>	<del>5</del>	<del>6</del>	<del>7</del>
<del>8</del>	<del>9</del>	<del>10</del>	<del>11</del>	<del>12</del>	<del>13</del>	<del>14</del>
<del>15</del>	<del>16</del>	<del>17</del>	<del>18</del>	<del>19</del>	<del>20</del>	<del>21</del>
<del>22</del>	<del>23</del>	24	25	26	27	28
29	30					

5. Graph  $\frac{3}{5}$  on a number line

Write the remaining amount as **a.** a mixed number and **b.** an improper fraction.

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6. Isabella brought 2 boxes of doughnuts to a meeting. The figure shows the remaining amount of doughnuts.



7. Graph  $3\frac{1}{4}$  on a number line.
8. Change  $1\frac{3}{5}$  to an improper fraction.
9. Change  $\frac{4}{3}$  to a mixed number.

## Applications

Solve.

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10. **Grades:** In a class of 35 students, 6 students received As on a mathematics exam. What fraction of students received an A? What fraction of students did not receive an A?

11. **Nutrition:** A certain brand of plain bagels has 146 calories per bagel. 115 calories come from the carbohydrates in the bagel. What fraction of the calories is from carbohydrates?
12. **Frequency Distribution:** Given the following frequency distribution of grades, compute the relative frequency for each class grade as a simplified fraction. The first class grade has been done for you.

Class Grade	Frequency	Relative Frequency
A	6	$\frac{6}{33} = \frac{2}{11}$
B	14	
C	9	
D	3	
F	1	
Total	$n = 33$	

### Writing & Thinking

13. In your own words, list the parts of a fraction and briefly describe the purpose of each part.

14. Show and explain, using diagrams and words, why  $2\frac{3}{5} = \frac{13}{5}$ .

## Looking Ahead

Relative frequencies are often given as decimal numbers, converted from fractions.

### Example Preview

Calculate the relative frequency for each class in the frequency distribution.

#### Solution

We first find the sample size by summing the class frequencies.

$$\begin{aligned} n &= \sum f_i \\ &= 2 + 5 + 4 + 5 + 4 \\ &= 20 \end{aligned}$$

Then divide each class frequency by 20.

Class	Frequency	Relative Frequency, $f_i$	
\$1500 – \$1599	2	$\frac{2}{20} = \frac{1}{10} = 0.1 = 10\%$	Each fraction is converted to a decimal number.
\$1600 – \$1699	5	$\frac{5}{20} = \frac{1}{4} = 0.25 = 25\%$	
\$1700 – \$1799	4	$\frac{4}{20} = \frac{1}{5} = 0.2 = 20\%$	
\$1800 – \$1899	5	$\frac{5}{20} = \frac{1}{4} = 0.25 = 25\%$	
\$1900 – \$1999	4	$\frac{4}{20} = \frac{1}{5} = 0.2 = 20\%$	

## 2.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- When a terminating decimal number is changed to a fraction, the denominator will be the power of 10 that names the rightmost digit of the decimal number.

2. When a terminating decimal number is changed to a fraction, the numerator can be determined by using the whole number that is formed by all the digits of the decimal number.
  
3. Fractions can always be converted to decimal form without losing accuracy.
  
4. In decimal form,  $\frac{1}{3}$  is repeating and nonterminating.

## Practice

Change each decimal number to a fraction or mixed number in lowest terms.

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5. 0.18

6. 2.75

Change each fraction to a decimal number rounded to the nearest hundredth.

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7.  $\frac{20}{3}$

8.  $\frac{40}{9}$

Simplify the expression by first writing all of the numbers in decimal form. Round to the nearest hundredth, if necessary.

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9.  $\frac{1}{4} + 0.25 + \frac{1}{5}$

10. Arrange the following data in order from smallest to largest.  $0.76, \frac{3}{4}, \frac{7}{10}$

## Applications

Solve.

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11. **Rectangles:** A rectangle measures 6.4 inches in length, and has a width that measures  $\frac{2}{5}$  of the length. Find the perimeter of the rectangle.
12. **Groceries:** A loaf of bread weighs 21.6 ounces. Mauricio cut off a third of the loaf to save for later and then cut the remaining portion into 16 equal slices. What was the weight of each slice of the 16 slices he cut?
13. **Statistical Survey:** A student mailed a survey to 132 people and  $\frac{1}{4}$  of those surveyed did not respond. How many respondents were there?
14. **Advertising:** An ad states that 9 out of 10 dentists would recommend Freshbright toothpaste. Marge wants to know how many dentists she can expect to recommend Freshbright if she surveys 150 dentists. To answer Marge's question compute  $\frac{9}{10}$  of the total number of dentists surveyed, 150.

**Writing & Thinking**

15. Describe the process used to change a terminating decimal number to a fraction.

16. List 2 different ways to solve this problem:  $\frac{1}{2} + 3.67 - \frac{1}{8}$ . State which method you prefer and why.

## 2.R.3 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

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1. The rules for order of operations are the same for fractions, mixed numbers, and whole numbers.
2. According to the rules for order of operations, multiplication occurs before division.
3. The mean is found by adding all the numbers in the set and then dividing by the quantity of numbers in the set.

### Practice

For each pair of fractions, determine which fraction is larger and by how much it is larger.

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4.  $\frac{2}{3}, \frac{3}{4}$

5.  $\frac{4}{5}, \frac{17}{20}$

6. Arrange  $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$  in order from smallest to largest. Then find the difference between the largest and smallest fractions.

Simplify.

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7.  $\frac{1}{2} \div \frac{7}{8} + \frac{1}{7} \cdot \frac{2}{3}$

8.  $\frac{1}{2} \div \frac{2}{3} + \left(\frac{1}{3}\right)^2$

9. Find the average of the numbers  $\frac{5}{6}$ ,  $\frac{1}{15}$ , and  $\frac{17}{30}$ .

## Applications

Solve.

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10. **Painting:** Two painters paint  $76\frac{1}{2}$  feet of fencing in one day. The first painter contributes  $2\frac{1}{2}$  hours and the second works for  $5\frac{3}{5}$  hours. How many feet of fencing are painted in each hour? (**Hint:** Add the number of hours then divide the length of fencing by this sum.)
11. **Books:** An art book has 40 two-sided pages of pictures, each of which is  $\frac{1}{32}$  inch thick. Each two-sided page is protected by a  $\frac{1}{80}$  inch thick piece of paper. Each side of the book is bound by a  $\frac{1}{6}$  inch cover. What is the total thickness of the book?
12. **Statistical Survey:** Jade spoke to students on campus and found that  $\frac{3}{11}$  of the students surveyed are freshmen and  $\frac{3}{8}$  of the students surveyed are sophomores. Did Jade survey more freshmen or sophomores?

13. **Marketing:** A marketing team set up a booth in a grocery store and asked 150 shoppers to taste samples of potato chips and select their favorite. The Good Foods Company that hired the marketing team produced the chips in Sample A and Sample B. The chips in Sample C and Sample D are competitor products. The marketing team results are below. How many shoppers chose one of the Good Foods Company potato chips?

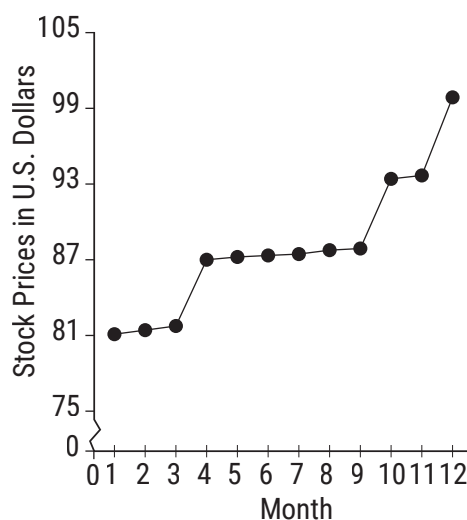
Sample	Shoppers Favorite
A	26%
B	22%
C	32%
D	20%

### Writing & Thinking

14. a. If two fractions are between 0 and 1, can their sum be more than 1? Explain.
- b. If two fractions are between 0 and 1, can their product be more than 1? Explain.

Month	Closing Price (1st Day of 12 consecutive months)
7	87.45
8	87.76
9	87.89
10	93.41
11	93.68
12	99.57

- Using the graph with the data plotted, determine which type of process the time series data represent: Stationary, Nonstationary and Increasing, or Nonstationary and Decreasing.



- What percent of the months have a closing price above \$90?

### Solution

- From the time series graph, we can see that there is strong evidence of an upward trend in stock prices. Therefore, the process is Nonstationary and Increasing.
- Months 10 through 12 have closing prices of \$93.41, \$93.68, and \$99.57 respectively, which are all greater than \$90. Therefore 3 months out of 12 have a closing stock price above \$90. As a percent this is

$$\frac{3}{12} \cdot 100 = 25\%$$

## 2.R.4 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- Fractions that have denominators other than 100 cannot be changed to a percent.

2. The fraction  $\frac{1}{5}$  is equivalent to  $\frac{1}{5}\%$ .

3. When changing from a percent to a mixed number, the fraction does not need to be reduced.

4.  $75\% = 0.75 = \frac{3}{4}$

## Practice

Change each fraction or mixed number to a percent. If necessary, round to the nearest tenth of a percent.

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5.  $\frac{3}{4}$

6.  $5\frac{3}{10}$

Change each percent to a fraction or mixed number and reduce, if possible.

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7. 120%

8. 12.5%

## Applications

Solve. If necessary, round to the nearest hundredth of a percent.

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9. **Exam Grades:** Out of a possible total of 240 points on an exam, David received 204 points. What percent of the exam did David get correct?
10. **College Degrees:** To receive a Bachelor of Science (BS) degree at Bluefield State College, the student must complete a total of 128 credit hours, of which 41 of these credits must be general education Core Skills courses. What percent of the total curriculum is dedicated to general education courses? <sup>1</sup>
11. **Coin Toss:** A coin was tossed 450 times and landed on heads 243 times. What percent of the tosses landed on heads?
12. **Sports Statistics:** Kamal is practicing his free throw shots to improve for the basketball team. On Monday he made 28 out of 35 of his shots. What percent of his shots did he make on Monday?

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1 Source: 2010–2011 Bluefield, WV State College Catalogue, p.76

## Writing & Thinking

13. Justify why mixed numbers are a larger percentage than proper fractions alone. (Consider the value of 100%.)

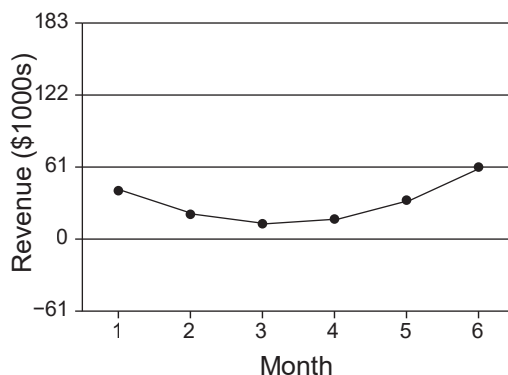
14. Describe the process to change a percent to a fraction or mixed number.

## Looking Ahead

The following example features a line graph, as we have reviewed in this section. Looking carefully at the elements of the graph and considering what data the graph is representing, we can consider the appropriateness of the graph.

### Example Preview

Is the  $y$ -scale appropriate for the following graph?



### Solution

The values on the  $y$ -axis represent data values that are positive and negative. Since the graph is representing revenue, which means a financial gain, negative values would not appear on the graph. Therefore, this graph has a  $y$ -scale which is too large for the data given.

## 2.R.5 Exercises

### Concept Check

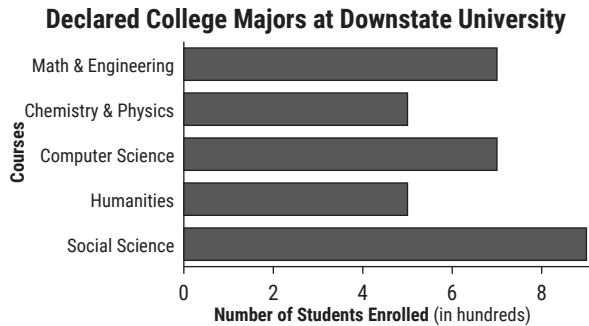
**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Graphs should always be clearly labeled, easy to read, and have appropriate titles.
2. Circle graphs show trends over a period of time.
3. The frequency is the number of data items in a class.
4. Numbers that are halfway between the upper limit of one class and the lower limit of the next class are the class boundaries.

## Applications

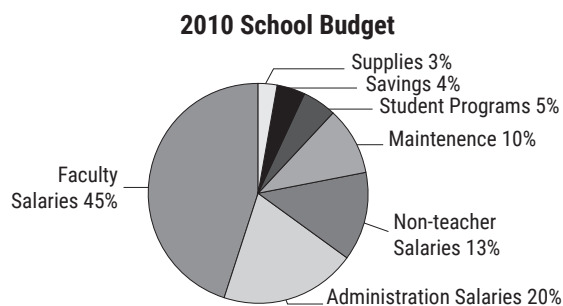
Answer the questions using the given graphs.

5. **Education:** The following bar graph shows the number of students in five fields of study at a university.



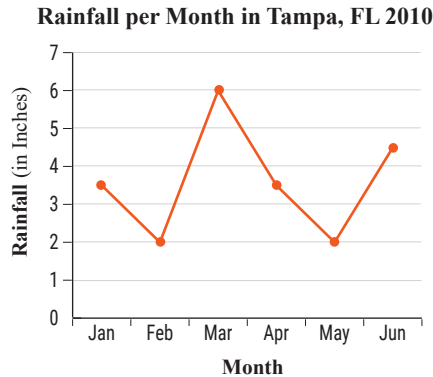
- Which field(s) of study has the largest number of declared majors?
- Which field(s) of study has the smallest number of declared majors?
- How many declared majors are indicated in the entire graph?
- What percent are computer science majors? Round your answer to the nearest tenth of a percent.

6. **Budgeting:** The following circle graph represents the various areas of spending for a school with a total budget of \$34,500,000.



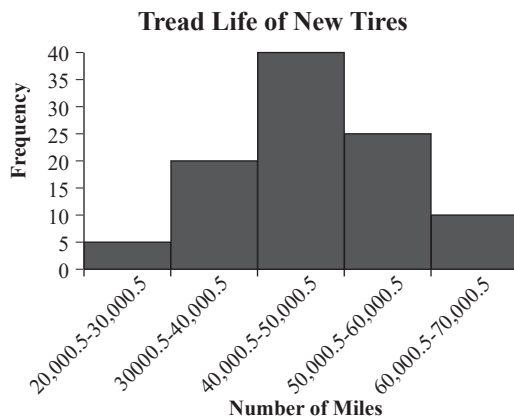
- What amount will be allocated to each category?
- What percent will be for expenditures other than salaries?
- How much will be spent on maintenance and supplies?
- How much more will be spent on teachers' salaries than on administration salaries?

7. **Weather:** The following line graph shows the total monthly rainfall in Tampa, Florida for the first 6 months of 2010.<sup>1</sup>



- a. Which months had the least rainfall?
- b. What was the most rainfall in a month?
- c. What month had the most rainfall?
- d. What was the mean rainfall over the six-month period (to the nearest hundredth)?

8. **Tires:** The following histogram summarizes the tread life for 100 types of new tires.



- a. How many classes are represented?
- b. What is the width of each class?
- c. Which class has the highest frequency?

<sup>1</sup> Source: weather.gov

## Writing & Thinking

9. State three properties or characteristics that should be true of all graphs so that they can communicate numerical data quickly and easily.

10. Compare and contrast a bar graph and a histogram.

## Looking Ahead

The following example highlights a portion of the process in considering what graphical display is best to represent a particular set of data.

### Example Preview

Could a pie chart be constructed from the following set of data?

Item	Percent of Income Spent on Item
Rent	19
Food	13
Car	19
Entertainment	15

### Solution

The total sum of the percents given in the second column is 66.

Since the percents sum to a value less than 100%, the parts listed in the table do not make up the total budget, so some items must be missing. This data could not be represented by a pie chart since the corresponding wedges would not form a complete circle.

## 2.R.6 Exercises

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. In creating a vertical bar graph, a bar's width should vary based on the number it represents.
2. The first step in constructing a vertical bar graph is to draw a vertical and a horizontal axis.
3. Bar graphs can have either vertical or horizontal bars.
4. Titles are unnecessary for circle graphs.

## Applications

For each set of data, construct the specified graph.

5. **Geography:** Construct a bar graph that represents the following data.

Island	Area in Square Miles (nearest ten thousand)
Greenland	840,000
New Guinea	310,000
Borneo	290,000
Madagascar	230,000
Baffin	200,000
Sumatra	180,000
Honshu	90,000
Great Britain	90,000

6. **Movies:** Construct a bar graph that represents the following data.

Motion Picture	Box Office(in millions of dollars)
Rogue One: A Star Wars Story	\$532.2
Finding Dory	\$486.3
Captain America: Civil War	\$408.1
The Secret Life of Pets	\$368.4
The Jungle Book (2016)	\$364.1
Deadpool	\$363.1
Zootopia	\$341.3
Batman v Superman: Dawn of Justice	\$330.4
Suicide Squad	\$325.1
Sing	\$270.3

Source: Information courtesy of Box Office Mojo. Used with permission. [www.boxofficemojo.com](http://www.boxofficemojo.com)

7. **Anatomy:** Construct a circle graph that represents the following data.

Percent of Population with  
Particular Blood Types

Type of Blood	Percent of Population
O positive (O <sup>+</sup> )	38%
O negative (O <sup>-</sup> )	7%
A positive (A <sup>+</sup> )	34%
A negative (A <sup>-</sup> )	6%
B positive (B <sup>+</sup> )	9%
B negative (B <sup>-</sup> )	2%
AB positive (AB <sup>+</sup> )	3%
AB negative (AB <sup>-</sup> )	1%

Source: AABB.org

8. **Energy:** Construct a circle graph that represents the following data.

Sources for World Electricity  
Generation in 2014

Source of Energy	Percent
Coal/Peat	40.8%
Natural Gas	21.6%
Hydro	16.4%
Nuclear	10.6%
Oil	4.3%
Other	6.3%

Source: Key World Energy Statistics, 2016.  
International Energy Agency.

## Writing & Thinking

9. While most graphs can be created through the use of a computer, give at least one benefit from constructing a graph yourself.

10. List three mistakes a person might make when constructing a graph.

## Absolute Value

The **absolute value** of a real number is \_\_\_\_\_ Note that the absolute value of a \_\_\_\_\_

**PROPERTIES**

## Looking Ahead

The following example involves a dot plot, which is a statistical graph representing data on a real number line.

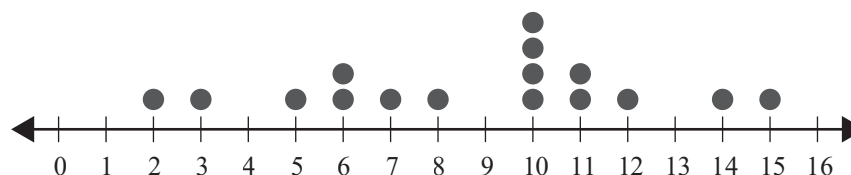
### Example Preview

The following data represent the number of oranges, in hundreds, which were sold by various grocery stores in the month of May. Construct a dot plot for the given data.

10	12	6	3	10	10	8	11
7	5	10	15	14	6	2	11

### Solution

Recall that a dot plot is a graphical display of data where a dot is used to represent each data value on a number line. If any data value repeats, that value will have more than one dot, corresponding to the number of times that data value appears. The given data are plotted on the following number line.



## 2.R.7 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. On a number line, smaller numbers are always to the left of larger numbers.
  
2. The absolute value of a negative number is a positive number.

3. All whole numbers are also integers.

4. Zero is a positive number.

Graph each set of real numbers on a real number line.

---

5.  $\{-3, -2, 0, 1\}$

6.  $\{-2, -1, -\frac{1}{3}, 2\}$

List the numbers in the set  $A = \{-7, -\sqrt{6}, -2, -\frac{5}{3}, -1.4, 0, \frac{3}{5}, \sqrt{5}, \sqrt{11}, 4, 5.9, 8\}$  that are described in each exercise.

---

7. Whole numbers

8. Rational numbers

Determine whether each statement is true or false. If a statement is false, rewrite it in a form that is a true statement.

(There may be more than one way to correct a statement.)

---

9.  $0 = -0$

10.  $|-8| \geq 4$

## Applications

Solve. Represent each quantity with a signed integer.

---

11. **Oceans:** The Alvin is a manned deep-ocean research submersible that has explored the wreck of the Titanic. The operating depth of the Alvin is 4500 meters below sea level.

12. **Oceans:** The Mariana trench is the deepest known location on the Earth's ocean floor. The deepest known part of the Mariana Trench is approximately 11 kilometers below sea level.

13. *Finance:* The closing price of a stock dropped 37 points in one day.

14. *Auto Mechanics:* Last winter the average air pressure in Ian's car tires decreased by 5 psi.

### Writing & Thinking

15. Explain, in your own words, how an expression such as  $-y$  might represent a positive number.

16. Compare and contrast absolute value with opposites.

## 3.R.1 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. The sum of a positive number and a negative number is always positive.
2. When adding two numbers with unlike signs, the result uses the sign of the number with the larger absolute value.
3. The sum of two positive numbers can equal zero.

### Practice

Add. Reduce any fractions to lowest terms.

---

4.  $8 + (-3)$

5.  $2 + (-8)$

6.  $-\frac{1}{6} + \frac{7}{15}$

7.  $3.2 + (-1.2) + (-2.5)$

Add. Be sure to find the absolute value first.

---

8.  $13 + |-5|$



12. **Find a Data Value Given the Mean:** A small plane has strict guidelines for the weight allowed for passenger luggage. Consequently, the six passengers are limited to a maximum average luggage weight of 35 pounds (lb.). The weights of the luggage for five of the passengers is as follows: 39 lb., 22 lb., 35 lb., 37 lb., and 27 lb. Determine the maximum possible weight allowance for the remaining bag.

### Writing & Thinking

13. Describe, in your own words, how the sum of the absolute values of two numbers might be 0. (Is this even possible?)
14. Describe in your own words the conditions under which the sum of two integers will be 0.

## Looking Ahead

In the following example, we will look at a statistical measure known as the range, which is calculated as the difference between the largest and smallest values in the data set.

### Example Preview

Calculate the range for the following data.

6, 12, 10, 11, 14, 12, 7, 12, 14, 17

#### Solution

To find the largest and smallest data value, we will order the values from the smallest to the largest.

6, 7, 10, 11, 12, 12, 12, 14, 14, 17

The largest value is 17 and the smallest value is 6, so the range of this data set is

Range = Largest – Smallest

$$= 17 - 6$$

$$= 11$$

You will apply subtraction of real numbers to find the range.

## 3.R.2 Exercises

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The sum of a number and its additive inverse is the number itself.
2. The additive inverse of negative seven is seven.
3. We can think of addition of numbers as accumulating numbers.
4. The expression “ $15 - 7$ ” can be thought of as “fifteen plus negative seven.”

## Practice

Find the additive inverse (opposite) of each real number.

---

5. 11

6.  $-3.4$

Subtract. Reduce fractions to lowest terms.

---

7.  $-8 - (-11)$

8.  $\frac{7}{15} - \frac{2}{15}$

Perform the indicated operation to find the net change in value.

---

9.  $-6 + (-4) - 5$

10.  $-11.3 + 5.3 - 7.9$

## Applications

Solve.

---

11. **Temperature:** At 2 p.m. the temperature was  $76^{\circ}\text{F}$ . At 8 p.m. the temperature was  $58^{\circ}\text{F}$ . What was the change in temperature?
12. **Real Estate:** A couple sold their house for  $\$135,000$ . They paid the realtor  $\$8100$ , and other expenses of the sale came to  $\$800$ . If they owed the bank  $\$87,000$  for the mortgage, what were their net proceeds from the sale?

13. **Construct a Data Set:** Construct a data set with 5 values that has a range of zero.
14. **Construct a Data Set:** Construct a data set with 7 values that has a range of zero and a mean of 2.5.

### Writing & Thinking

15. Explain, in your own words, how to find the difference between a positive and a negative number.
16. What is the additive inverse of 0? Why?

**Solution**

To calculate the weighted average balance for the three-month period, we will need to use the weighted mean

equation,  $\bar{x} = \frac{\sum(x_i \cdot w_i)}{\sum w_i}$ .

First, it is helpful to determine which numbers are data values and which are weights. In this case, the data values,  $x_i$ , are the average monthly balances. The weights,  $w_i$ , are then the number of days in each month: 31, 30, and 31. The weighted mean is then calculated as follows:

$$\begin{aligned} \bar{x} &= \frac{(31)(2251.33) + (30)(2490.51) + (31)(1478.27)}{31 + 30 + 31} && \text{Simplifying the numerator of the weighted mean formula requires the} \\ & && \text{multiplication of real numbers.} \\ &= \frac{190,332.90}{92} && \text{The final step in calculating the weighted mean is to divide the} \\ &\approx 2068.84 && \text{calculated numerator and denominator.} \end{aligned}$$

## 3.R.3 Exercises

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a negative number is divided by a positive number, the result will be a negative number.
2. The product of zero and a number is zero.
3. If two numbers have the same sign, both the product and the quotient of the two numbers will be negative.
4. The mean of a set of numbers is always positive.

### Practice

Multiply.

5.  $12 \cdot 4$
6.  $(-7)(-16)(0)$

Divide. Reduce fractions to lowest terms. Round answers with decimals to the nearest tenth.

---

7.  $\frac{-20}{-10}$

8.  $\frac{-5.6}{7}$

## Applications

Solve.

---

9. **Animals:** According to the US Fish and Wildlife Service, migratory birds are imported at a value of about \$19 each. Suppose that about 800,000 live birds are imported each year. What is the total value of these imported birds?

10. **Summary Statistics:** Find the mean and range of the following data set.

-10, 15, 16, -17, -34, -42

## Writing & Thinking

11. If you multiply an odd number of negative numbers together, do you think that the product will be positive or negative? Explain your reasoning.

12. Explain the conditions under which the quotient of two numbers is 0.

## Looking Ahead

The following example is calculating a statistical measure called the sample variance. To calculate this statistic, the difference between each value and the sample mean is raised to the second power.

### Example Preview

Calculate the value of the sample variance for the following data. Round your answer to one decimal place.

$$-12, -12, 2, 7, 7, 2$$

### Solution

There are  $n = 6$  data values, and the mean of the values is

$$\bar{x} = \frac{-12 + (-12) + 2 + 7 + 7 + 2}{6} = -1$$

In the following table, we have used the mean and data values to calculate the sum of squared deviations.

Data $x_i$	Deviations $(x_i - \bar{x})$	Squared Deviations $(x_i - \bar{x})^2$
-12	$-12 - (-1) = -11$	$(-11)^2 = 121$
-12	$-12 - (-1) = -11$	$(-11)^2 = 121$
2	$2 - (-1) = 3$	$3^2 = 9$
7	$7 - (-1) = 8$	$8^2 = 64$
7	$7 - (-1) = 8$	$8^2 = 64$
2	$2 - (-1) = 3$	$3^2 = 9$
Total		388

We can substitute the values we have found into the formula for the sample variance.

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1} = \frac{388}{5} = 77.6$$

## 3.R.4 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a constant does not have an exponent written, it is assumed that the exponent is 0.

2. If  $a$  is a nonzero real number and  $n$  is an integer, then  $a^{-n} = -a^n$ .
3. Since the product rule is stated for integer exponents, the rule is also valid for 0 and negative exponents.
4. When using the quotient rule, you should subtract the smaller exponent from the larger exponent.

## Practice

Simplify each expression. The final form of the expressions with variables should contain only positive exponents. Assume that all variables represent nonzero numbers.

---

5.  $y^3 \cdot y^8$

7.  $x^{-3} \cdot x^0 \cdot x^2$

6.  $\frac{y^7}{y^2}$

8.  $\frac{10^4 \cdot 10^{-3}}{10^{-2}}$

## Applications

Solve.

9. **Computers:** Rylee wants to move all her files to a new hard drive that has  $2^{12}$  GB of storage on it. She wants to designate the same amount of storage for each of  $2^4$  projects. How much storage should be assigned to each project? Write your answer as a power of two.

10. **Bacteria:** Trey is studying patterns in bacteria. For a positive test result in his experiment, bacteria must grow in population at a minimum rate of  $3^2$  in 24 hours. If the initial population of the bacteria is  $3^5$  and his final measurement after 24 hours is  $3^8$ , should he mark the test as positive or negative?

11. **Calculating Variance Given a Data Set:** The following data represent the average number of tweets per day posted on Twitter for 16 high school students. Calculate the sample variance of the data set.

0.8	42.2	20.6	2.8
36.7	18.6	23.3	11.5
3.7	14.9	9.4	1.5
14.9	31.1	23.5	9.5

12. **Calculating Variance Given a Stem-and-Leaf Plot:** The following data represent the ages of 20 American entrepreneurs (in years). Calculate the sample variance of the data set.

**Ages of American Entrepreneurs**

Stem	Leaves
2	2 3
2	5 8 9 9
3	0 1 1 2 2 3 4
3	5 6 9
4	1 3
4	7
5	3
5	

**Key: 2 | 2 = 22 years old**

## Looking Ahead

In the following example, we will be looking at a statistical measure known as the standard score or  $z$ -score. The standard score of a value tells you how that value relates to the mean of the data, specifically how many standard deviations it is away from the mean.

### Example Preview

A sample of 113 hotels is selected from a large urban area and the price for a night of lodging for a single room was determined for each. The mean rate is computed to be \$93 and the standard deviation is \$13.

What is the standard score for a hotel charging \$105 per night? Round your answer to two decimal places.

### Solution

The standard score is calculated using the formula  $z = \frac{x - \bar{x}}{s}$ .

where  $x$  is the price of a single room at the hotel,  $\bar{x}$  is the mean, and  $s$  is the standard deviation.

We know that  $x = 105$ ,  $\bar{x} = 93$ , and  $s = 13$ .

Substitute these values into the formula and compute  $z$ .

Therefore,

$$z = \frac{105 - 93}{13} = 0.92$$

Thus, the standard score for a hotel charging \$105 per night is 0.92.

## 3.R.5 Exercises

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A variable that does not appear to have an exponent has an exponent of 1.
2. In the term  $-9x$ , nine is being subtracted from  $x$ .
3. In the term “ $12a$ ,” 12 is the constant.
4. Like terms have the same coefficients.

## Practice

Identify the like terms in each list of terms.

---

5.  $-5, 3, 7x, 8, 9x, 3y$

Simplify each expression by combining like terms.

---

6.  $8x + 7x$

7.  $3x - 5x + 12x$

8.  $13x + 12x^2 + 15x - 35 - 41 - 2x^2$

Simplify each expression and then evaluate the expression for  $y = 3$  and  $a = -2$ .

---

9.  $5y + 4 - 2y$

10.  $\frac{3a + 5a}{-2} + 12a$

## Applications

Solve.

---

11. **Profit:** An apartment management company owns a property with 100 units. The company has determined that the profit made per month from the property can be calculated using the equation  $P = -10x^2 + 1500x - 6000$ , where  $x$  is the number of units rented per month. How much profit does the company make when 80 units are rented?

12. **Physics:** A ball is thrown upward from an initial height of 96 feet with an initial velocity of 16 feet per second. After  $t$  seconds, the height of the ball can be described by the expression  $-16t^2 + 16t + 96$ . What is the height of the ball after 3 seconds?
13. **Applying a Standard Score:** Carlita scored 32 on the ACT Mathematics Test and 730 on the mathematics section of the SAT. If the ACT Mathematics Test had a mean score of 21.0 with a standard deviation of 5.3, and the mathematics section of the SAT had a mean score of 516 with a standard deviation of 116, on which exam did Carlita earn a better math score with respect to her peers? (Calculate the  $z$ -score for each test to make a decision.) Round to two decimal places.

## Writing & Thinking

14. Discuss like and unlike terms and give an example of each.

15. Explain the difference between  $-13^2$  and  $(-13)^2$ .

## Looking Ahead

The following example illustrates a statistical application of the square root. The standard deviation of a set of data is the square root of the variance.

### Example Preview

Calculate the value of the sample standard deviation for the following data. Round your answer to one decimal place.

$$-12, -12, 2, 7, 7, 2$$

### Solution

The sample variance of this data set was calculated in the Looking Ahead section of section 3.R.4 and found to be 77.6. To find the sample standard deviation we simply have to find the square root of this value.

$$s = \sqrt{s^2} = \sqrt{77.6} \approx 8.8 \quad \text{Evaluate the radical expression.}$$

## 3.R.6 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a number is squared and the principal square root of the result is found, that square root is always equal to the original number.
  
2. There is no real number that can be a square root of a negative number.
  
3. The index is the number underneath the radical sign.
  
4. The cube root of  $-27$  is a real number.

## Practice

Simplify the following square roots and cube roots.

---

5.  $\sqrt{49}$

6.  $\sqrt{289}$

7.  $\sqrt[3]{1000}$

8.  $\sqrt[3]{\frac{27}{64}}$

9.  $\sqrt{0.04}$

## Applications

Solve.

---

10. **Area:** The area of a square tile is 16 square inches.
- How long are the sides of the square tile?
  - How many tiles would be needed for a four-foot-long and four-inch-high backsplash in a newly designed bathroom?
11. **Volume:** The volume of a child's building block is 64 cubic centimeters.
- Assuming the building block is a perfect cube, find the length of each side of the block.
  - If a child stacks 5 blocks directly on top of each other, find the height of the structure that is created.

12. **Calculating Measures of Dispersion:** Use the following sample data and find the range, sample variance, and sample standard deviation.

2.0, 1.7, 1.9, 1.5, 1.6, 1.9, 1.7, 3.0

### Writing & Thinking

13. Discuss, in your own words, why the square root of a negative number is not a real number.

14. Discuss, in your own words, why the cube root of a negative number is a negative number.

**Solution**

Use the multiplication rule for dependent events since the coins are chosen without replacement.

The total number of coins in this jar is  $18 + 16 + 18 + 22 = 74$ . Furthermore, there are a total of 16 nickels and 18 dimes.

Let  $E$  be the event “choosing a nickel first,” and let  $F$  be the event “choosing a dime.”

We will then have  $P(E) = \frac{16}{74}$  and  $P(F|E) = \frac{18}{73}$ .

$$\begin{aligned} P(E \text{ and } F) &= P(E) \cdot P(F|E) \\ &= \frac{16}{74} \cdot \frac{18}{73} \\ &= \frac{288}{5402} && \text{Multiply the fractions.} \\ &= \frac{144}{2701} \approx 0.053 && \text{Reduce to lowest terms.} \end{aligned}$$

## 4.R.1 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- To find  $\frac{1}{2}$  of  $\frac{2}{9}$  requires multiplication.
- $\frac{3}{4} \cdot \frac{9}{10} = \frac{27}{40}$
- The statement  $\frac{1}{3} \cdot \frac{2}{5} = \frac{2}{5} \cdot \frac{1}{3}$  is an example of the associative property of multiplication.
- The number 1 is always a factor of the numerator and the denominator.

## Practice

Multiply and reduce to lowest terms. (**Hint:** Factor before multiplying.)

---

5.  $\frac{1}{3} \cdot \frac{3}{4}$

7.  $\frac{5}{16} \cdot \frac{16}{15}$

6.  $\frac{2}{3} \cdot \frac{4}{3}$

8.  $\frac{9}{10} \cdot \frac{35}{40} \cdot \frac{25}{15}$

## Applications

Solve.

---

9. **Recipes:** A recipe calls for  $\frac{3}{4}$  cups of flour. How much flour should be used if only half of the recipe is to be made?
10. **Demographics:** A study showed that  $\frac{3}{5}$  of the students in an elementary school were left-handed. If the school had an enrollment of 600 students, how many were left-handed?
11. **Conditional Probability:** There are 15 different colored crayons in a new box. What is the probability that the orange and then the green will be chosen at random, without replacement?
12. **Probability of Independent Events:** On awards day at the end of the year, Jasmine has an 85% chance of winning the top award in English and a 4 out of 5 chance of winning an award for athletics. What's the probability that Jasmine wins both awards?

13. **Medicine:** Insulin pens used to administer a patient's insulin at hospitals have a malfunction rate of 9%. This means that out of a box of 200 pens, 18 are defective in some way and must be thrown away. Find the probability of randomly selecting 3 defective insulin pens in a row from a brand new box of 200 pens, if a defective pen is immediately discarded.

## Writing & Thinking

14. If two fractions are between 0 and 1, can their product be more than 1? Explain.
15. Explain the process of multiplying two fractions. Give an example of a product that cannot be reduced.

## Looking Ahead

In the following example, we will use the Rule for Conditional Probability. This rule will require us to divide probabilities, which can be represented by fractions in many instances.

### Example Preview

A high school fundraiser sold 500 tickets for a raffle. 372 tickets were sold to parents of current students and 238 tickets were sold to parents of current students who are also graduates of the high school themselves. What is the probability that a randomly chosen ticket belongs to someone who is a graduate of the high school, given that the ticket holder is a parent of a current student?

### Solution

The question asks for  $P(\text{graduate} \mid \text{parent of current student})$ . Thus, we need to find  $P(\text{graduate and parent of current student})$  as well as  $P(\text{parent of current student})$ .

$$P(\text{graduate and parent of current student}) = \frac{238}{500}$$

$$P(\text{parent of current student}) = \frac{372}{500}$$

We can now calculate the conditional probability as follows.

$$\begin{aligned} P(\text{graduate} \mid \text{parent of current student}) &= \frac{P(\text{graduate and parent of current student})}{P(\text{parent of current student})} \\ &= \frac{238}{500} \div \frac{372}{500} && \text{The probabilities, expressed as fractions, are divided.} \\ &= \frac{238}{500} \cdot \frac{500}{372} \\ &= \frac{238}{372} = \frac{119}{186} \approx 0.6398 \end{aligned}$$

## 4.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The reciprocal of 1 is undefined.

2. The product of a nonzero number and its reciprocal is undefined.

3. The reciprocal of 12 is  $\frac{12}{1}$ .

4. The result of  $\frac{1}{3} \div \frac{1}{6}$  is 2.

## Practice

Divide and reduce to lowest terms.

---

5.  $\frac{2}{3} \div \frac{3}{4}$

7.  $\frac{5}{6} \div 0$

6.  $0 \div \frac{5}{6}$

8.  $\frac{14}{15} \div \frac{21}{25}$

## Applications

Solve.

---

9. **Geology:** The floor of the Atlantic Ocean is spreading apart at an average rate of  $\frac{3}{50}$  of a meter per year. How long will it take for the ocean floor to spread 12 meters?

- 10. Airplane Capacity:** An airplane is carrying 180 passengers. This is  $\frac{9}{10}$  of the capacity of the airplane.
- Is the capacity of the airplane more or less than 180?
  - If you were to multiply 180 times  $\frac{9}{10}$ , would the product be more or less than 180?
  - What is the capacity of the airplane?
- 11. Rolling a Die:** A six-sided die is rolled. What is the probability of rolling a two, assuming that you rolled an even number?
- 12. Probability:** Mrs. Harvey's algebra class has 42 students, classified by academic year and method of instruction as follows.

Mrs. Harvey's Algebra Class		
	In-Class Instruction	Online Instruction
Freshman	9	13
Sophomore	4	5
Junior	4	2
Senior	2	3

Mrs. Harvey randomly chooses one student's homework to grade first.

- What is the probability that she selects an in-class student, given that she chooses from only the sophomores?

- b. What is the probability that she selects a junior, given that she chooses an online student?
- c. What is the probability that she selects an online student, given that she chooses a junior?

## Writing & Thinking

13. Explain why the number 0 has no reciprocal.
14. Is division a commutative operation? Explain briefly and give three examples using fractions to help justify your answer.

## 4.R.3 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. The LCM of 15 and 25 is 50.
2. The first five multiples of 9 are 9, 18, 27, 36, and 45.
3. The first five multiples of 4 are 4, 8, 12, 20, and 24.
4. When given larger numbers, the most efficient way to find the LCM is to use the prime factorization method.

### Practice

Find the LCM of each set of numbers.

---

5. 6, 10
6. 3, 4, 8
7. For 14, 35, and 49, **a.** find the LCM and **b.** state how many times each number divides into the LCM.

For each equation, find the missing numerator that will make the fractions equivalent.

---

8.  $\frac{5}{8} = \frac{?}{24}$

9.  $\frac{5}{12} = \frac{?}{108}$

## Applications

Solve.

---

- 10. Security:** Three security guards meet at the front gate for coffee before they walk around inspecting buildings at a manufacturing plant. The guards take 15, 20, and 30 minutes, respectively, for the inspection trip.
- If they start at the same time, in how many minutes will they meet again at the front gate for coffee?
  - How many trips will each guard have made?
- 11. Fruit:** A fruit production company has three packaging facilities, each of which uses different-sized boxes as follows: 24 pieces/box, 36 pieces/box, and 45 pieces/box.
- Assuming that the truck provides the same quantity of uniformly-sized pieces of fruit to all three packaging facilities, what is the minimum number of pieces of fruit that will be delivered so that no fruit will be left over?
  - How many boxes will each facility package?

## Writing & Thinking

12. Explain, in your own words, why each number in a set divides evenly into the LCM of that set of numbers.

13. Explain why simply multiplying two numbers together will not necessarily find the LCM of those numbers. Give an example of when it would find the LCM and an example when it would not.

## 4.R.4 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

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1. The final step in adding fractions is to reduce, if possible.
2. The process for finding the LCD is the same as the process for finding the LCM.
3. LCD represents the Least Common Digit.
4. When subtracting fractions, simply subtract the numerators and the denominators.
5. Subtraction of fractions requires that the fractions have the same denominators.

### Practice

Add and reduce to lowest terms.

---

6.  $\frac{3}{25} + \frac{12}{25}$

7.  $\frac{2}{7} + \frac{4}{21} + \frac{1}{3}$

Subtract and reduce to lowest terms.

---

8.  $\frac{7}{8} - \frac{5}{8}$

10.  $2 - \frac{9}{16}$

9.  $\frac{9}{14} - \frac{2}{21}$



15. **Probability:** Ana is looking forward to starting high school. Of the 192 homeroom teachers at the school,  $\frac{1}{4}$  are new this year,  $\frac{1}{8}$  are bilingual, and  $\frac{1}{12}$  are both new and bilingual. If class rosters are generated at random, what is the probability that Ana's teacher is either new or bilingual?

## Writing & Thinking

16. Explain how finding the LCM relates to the LCD.
17. Give an example of a situation where you might add or subtract fractions (other than in class).



6. Find the union and intersection of  $E = \{1, 2, 4, 8, 16, 32\}$  and  $F = \{1, 4, 16, 64\}$ .

## Applications

Solve.

---

7. **Rolling a Die:** Consider rolling a fair 6-sided die. The set of all possible outcomes is  $S = \{1, 2, 3, 4, 5, 6\}$ . Since there are 6 outcomes for rolling the die and each outcome has an equal chance of occurring, the probability of rolling a specific outcome is  $\frac{1}{6}$ . Let  $E$  be the set of odd outcomes and let  $F$  be the set of outcomes greater than three.  $E = \{1, 3, 5\}$  and  $F = \{4, 5, 6\}$ .

a. What is the probability of rolling an outcome in  $(E \cup F)$ ?

b. What is the probability of rolling an outcome in  $(E \cap F)$ ?

8. **Biology:** In a biology lab, vials considered defective must be thrown away. The following chart shows the breakdown of vials in a testing center by distributor.

Lab Vials by Distributor

Lab Vials	BioVial	L.B.C.
Usable	1762	2001
Defective	3	2

- a. If a random vial is chosen from the biology lab, what is the probability that the vial is usable?
- b. Find the probability that two lab partners both randomly choose a defective vial, one after another, if all the vials are stored together.

## Looking Ahead

In this example, the order of operations with real numbers is applied when calculating an expected value.

### Example Preview

Find the expected value,  $E(X)$ , for the following data.

$x$	$P(X = x)$	$x \cdot P(X = x)$
-7	0.2	-1.4
-6	0.1	-0.6
-5	0.2	-1.0
-4	0.1	-0.4
-3	0.4	-1.2

### Solution

The expected value of a random variable  $X$  is the mean of the random variable  $X$ . It is denoted by  $\mu$  or  $E(X)$  and is calculated as follows.

$$\begin{aligned}
 E(X) = \mu &= \sum [x_i \cdot P(X = x_i)] \\
 &= (-7)(0.2) + (-6)(0.1) + (-5)(0.2) + (-4)(0.1) + (-3)(0.4) && \text{Multiply} \\
 &= -1.4 + (-0.6) + (-1.0) + (-0.4) + (-1.2) && \text{Add} \\
 &= -4.6
 \end{aligned}$$

## 5.R.1 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- If there are no grouping symbols, multiplication should always be performed before addition.
- When following the rules for order of operations, powers indicated by exponents should be evaluated last.
- The square root symbol is a grouping symbol.
- A well-known mnemonic device for remembering the rules for order of operations is SADMEP.

## Practice

Simplify.

---

5. a.  $24 \div 4 \cdot 6$

b.  $24 \cdot 4 \div 6$

7.  $3^2 \div (-9) \cdot (4 - 2^2) + 5(-2)$

8.  $14 \cdot 3 \div (-2) - 6(4)$

6.  $15 \div (-3) \cdot 3 - 10$

9.  $|16 - 20| + (-10)^2 + 5^2$

## Applications

Solve.

---

10. **Discounts:** The Matthews family, a family of 4, is planning a trip to New York City. During their visit, they want to see the Broadway play *Matilda*. The tickets cost \$102 each. The Matthews purchase the tickets online and the website charges a service fee of \$7.50 per ticket. The website is running a sale where the Matthews can get 10% off of their entire purchase.

a. Write an expression to describe how much of a discount the Matthews will receive on their purchase.

b. What is the final purchase price of the tickets?

11. **Banking:** Dennis overdrew his checking account and ended up with a balance of  $-\$42$ . The bank charged a \$35 overdraft fee and an additional \$5 fee for every day the account was overdrawn. Dennis left his account overdrawn for 3 days.

a. Write an expression to show the balance of Dennis's checking account after 3 days.

b. Simplify the expression in part a. to find the balance of Dennis's checking account after 3 days.

12. **Expected Value:** Find the expected value,  $E(X)$ , for the following data.

$x$	$P(X = x)$	$x \cdot P(X = x)$
-10	0.1	-1.0
-7	0.2	-1.4
-5	0.5	-2.5
0	0.1	0
5	0.1	0.5

### Writing & Thinking

13. Explain, in your own words, why the following expression cannot be evaluated.

$$(24 - 2^4) + 6(3 - 5) \div (3^2 - 9)$$

14. Consider any number between 0 and 1. If you square this number, will the result be larger or smaller than the original number? Is this always the case? Explain.

## 5.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. If only one endpoint is included in an interval, it is called a half-open interval.
2. When both sides of a linear inequality are multiplied by a negative constant, the sense of the inequality should stay the same.
3. To check the solution set of a linear inequality, every solution in the solution set must be checked in the original inequality.
4. The infinity symbol  $\infty$  does not represent a specific number.

### Practice

Graph each interval on a real number line and tell what type of interval it is.

---

5.  $x \leq -3$

6.  $-1.5 \leq x < 3.2$

Solve each inequality and graph the solution set. Write each solution set using interval notation.

---

7.  $x + 1 > 5$

8.  $-2x \geq 6$

9.  $4x - 7 \geq 9$

10.  $5x + 6 \geq 2x - 2$

## Applications

Solve.

---

11. **Test Scores:** A statistics student has grades of 82, 95, 93, and 78 on four hour-long exams. He must average 90 or higher to receive an *A* for the course. What scores can he receive on the final exam and earn an *A* if:
- The final is equivalent to a single hour-long exam (100 points maximum)?
  - The final is equivalent to two hourly exams (200 points maximum)?
12. **Postage:** Allison is going to the post office to buy 34¢ stamps and 3¢ adjustment stamps. Since the current postage rate is 49¢, she will need 5 times as many 3¢ adjustment stamps as 34¢ stamps. If she has \$12.25 to spend, what is the largest number of 34¢ stamps she can buy?

## Writing & Thinking

13. a. Write a list of three situations where inequalities might be used in daily life.

b. Illustrate these situations with algebraic inequalities and appropriate numbers.

## 5.R.3 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. The solution set of a compound inequality containing “and” is the union of the solution sets of the two inequalities.
  
  
  
  
  
  
  
  
  
  
2. The following set,  $\{x \mid x \text{ is a positive odd integer less than } 12\}$ , can be written in roster form as  $\{1, 3, 5, 7, 9, 11\}$ .

### Practice

3. Graph the set  $\{x \mid x > 3 \text{ or } x \leq 2\}$  on a real number line.

Solve each compound inequality and graph its solution set. Write each solution set using interval notation.

---

4.  $\{x \mid x + 3 > 2 \text{ and } x - 1 < 5\}$

5.  $\{x \mid x < -1 \text{ or } 2x + 1 \geq 3\}$

6.  $-4 < x + 5 < 6$

## Applications

Consider rolling two fair six-sided dice. The outcome is the sum of the digits on the top face of each die. The approximate probability of each of the possible outcomes are shown in the table below.

---

2	3	4	5	6	7	8	9	10	11	12
0.0278	0.0556	0.0833	0.1111	0.1389	0.1667	0.1389	0.1111	0.0833	0.0556	0.0278

- Find the probability that the outcome is less than 4.
- Find the probability that the outcome is greater than or equal to 6.
- Find the probability that the outcome is between 3 and 8 (both exclusive).
- Find the probability that the outcome is less than 6 or greater than 10.

## Looking Ahead

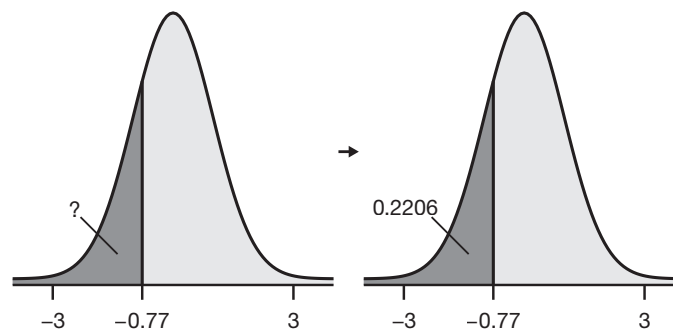
A thorough understanding of the concept of area with simple polygons will help you in determining the area under the standard normal curve, as is shown in the following example.

### Example Preview

Find the area under the standard normal curve to the left of  $z = -0.77$ .

#### Solution

A graphical representation of the problem is shown below. In order to find the desired area, which is labeled with a question mark, we will need to find the area under the standard normal curve from negative infinity to  $z = -0.77$ . Using technology or the standard normal distribution table we find this area to be 0.2206.



$$P(z < -0.77) = P(-\infty < z < -0.77) = 0.2206$$

## 6.R.1 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- The  $(b + c)$  in the trapezoid area formula represents the sum of the lengths of the base and the corners.
- The height of a triangle is the distance between the base and the vertex opposite the base.
- The area formula for a triangle is  $A = a + b + c$ .
- The area formula for a trapezoid is  $A = \frac{1}{2}h(b + c)$ .

**Practice**

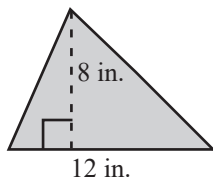
Calculate the area of each figure described.

5. A square with sides of length 9 ft.

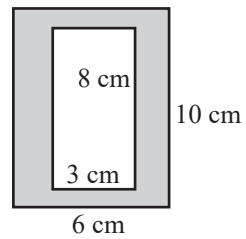
6. A parallelogram with height 2.3 ft and base 11.9 ft.

Calculate the shaded area of each figure.

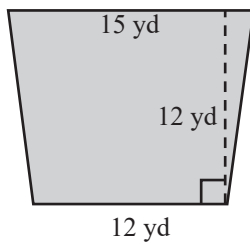
7.



9.



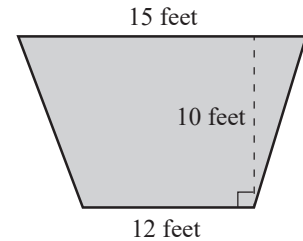
8.



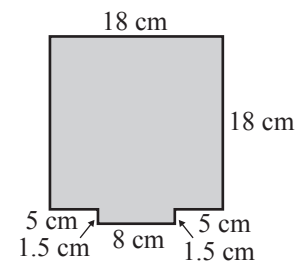
## Applications

Solve.

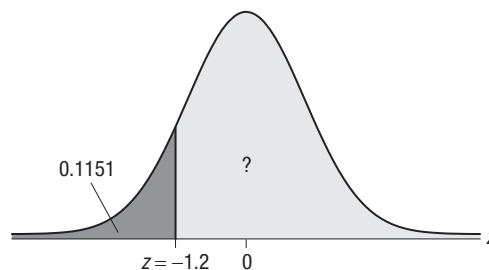
10. **Construction:** The main stage at a theater is in the shape of a trapezoid. The owner of the theater is planning to install a new specially designed flooring system on the stage. The stage is 12 feet wide in the front and 15 feet wide in the back. The stage is 10 feet deep. How much flooring will the manager need?



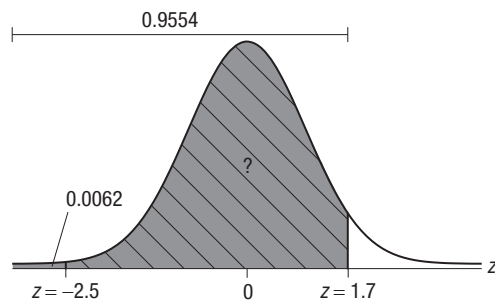
11. **Technology:** A square electronics circuit board is 18 centimeters on each side. On the center of one of the edges is an 8 by 1.5 centimeter rectangular lip for plugging in.
- What is the total perimeter of the circuit board, including the lip?
  - What is the area of the circuit board?



12. **Area Under the Standard Normal Curve:** For a standard normal curve, the total area under the curve and above the  $x$ -axis is 1. If the area under the curve and to the left of  $z = -1.2$  is approximately 0.1151, what is the area under the curve and to the right of  $z = -1.2$ ?



13. **Area Under the Standard Normal Curve:** For a standard normal curve, the area under the curve and to the left of  $z = -2.5$  is approximately 0.0062 and the area under the curve and to the left of 1.7 is approximately 0.9554. Find the area under the curve and between  $-2.5$  and 1.7.



## Writing & Thinking

14. Explain why square units are used for labeling areas. Give two examples each of metric area labels and US customary area labels.
15. Explain what the value of  $(b + c)$  represents in the formula for the area of a trapezoid.

## 6.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. If an equation of the form  $ax + b = c$  uses decimal or fractional coefficients, the addition and multiplication principles of equality cannot be used.
2. The first step in solving  $2x + 3 = 9$  is to add 3 to both sides.
3. To solve an equation that has been simplified to  $4x = 12$ , you need to multiply both sides by  $\frac{1}{4}$ , or divide both sides by 4.
4. When solving a linear equation with decimal coefficients, one approach is to multiply both sides in such a way to give integer coefficients before solving.

### Practice

Solve each equation.

---

5.  $3x + 11 = 2$

6.  $-5x + 2.9 = 3.5$

7.  $\frac{2}{5} - \frac{1}{2}x = \frac{7}{4}$

8.  $\frac{y}{3} - \frac{2}{3} = 7$

## Applications

Solve.

---

9. **Music:** The tickets for a concert featuring the new hit band, Flying Sailor, sold out in 2.5 hours. If there were 35,000 tickets sold, solve the equation  $35,000 - 2.5x = 0$  to find the number of tickets sold per hour.
10. **Movies:** All snacks (candy, popcorn, and soda) cost \$3.50 each at the local movie theater. Admission tickets cost \$7.50 each. After a long week, Carlos treats himself to a night at the movies. His movie night budget is \$25 and he spends all his movie money. Solve the equation  $3.50x + 7.50 = 25.00$  to determine how many snacks Carlos can buy.
11. **Standard Score:** Solve the following equation involving the standard score formula for  $x$  given  $z = 2$ ,  $\mu = 135.2$ , and  $\sigma = 11.1$ .

$$z = \frac{x - \mu}{\sigma}$$

12. **Probability:** In probability, the probability of all possible outcomes of an event must add to 1. Suppose there are 6 possible outcomes of an event. One of the outcomes has a probability of 0.25. The other five outcomes all have the same probability. Solve the equation  $5p + 0.25 = 1$  to find the probability of each of the other five outcomes.

**Writing & Thinking**

13. Find the error(s) made in solving each equation and give the correct solution.

$$\frac{1}{3}x + 4 = 9$$

a.

$$3 \cdot \frac{1}{3}x + 4 = 3 \cdot 9$$

$$x + 4 = 27$$

$$x + 4 - 4 = 27 - 4$$

$$x = 23$$

b.

$$5x + 3 = 11$$

$$(5x - 3) + (3 - 3) = 11 - 3$$

$$2x + 0 = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

## 6.R.3 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. When using formulas, typically it does not matter if capital or lower case letters are used:  $A = a$ ,  $C = c$ , etc.
2. If the perimeter and length are known,  $P = 2l + 2w$  can be used to find the width of a rectangle.
3. Rate of interest is stated as an annual rate in percent form.

### Practice

Solve each formula for the indicated variable.

---

4.  $P = a + b + c$ ; solve for  $b$ .
5.  $P = 3s$ ; solve for  $s$ .
6.  $I = Prt$ ; solve for  $t$ .
7.  $A = P(1 + rt)$ ; solve for  $r$ .

## Applications

Solve.

8. **Velocity:** If an object is shot upward with an initial velocity  $v_0$  in feet per second, the velocity  $v$  in feet per second is given by the formula  $v = v_0 - 32t$ , where  $t$  is time in seconds. ( $v_0$  is read “ $v$  sub zero.” The 0 is called a subscript.) An object projected upward with an initial velocity of 106 feet per second has a velocity of 42 feet per second. How many seconds have passed?
9. **Investments:** The total amount of money  $A$  in an account with  $P$  dollars invested in it is given by the formula  $A = P + Prt$ , where  $r$  is the rate expressed as a decimal and  $t$  is time (one year or part of a year). If \$1000 is invested at 6% interest, find the total amount in the account after 6 months.
10. **Cost:** The total cost  $C$  of producing  $x$  items can be found by the formula  $C = ax + k$ , where  $a$  is the cost per item and  $k$  is the fixed costs (rent, utilities, and so on). Find the total cost of producing 30 items if each costs \$15 and the fixed costs are \$580.
11. **Percentile of a Data Value:** The  $P^{\text{th}}$  percentile of a particular value in a data set is given by

$$P = \frac{l}{n} \cdot 100$$

where  $P$  is the percentile rounded to the nearest whole number,  $l$  is the number of values in the data set *less than or equal to* the given value, and  $n$  is the number of data values in the sample. For a particular data set, a data value in the 20<sup>th</sup> percentile has 30 values in the data set less than or equal to it. How many data values are in the sample?

## 8.R.1 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. Equations involving absolute value can only have one solution.
2. If two numbers have the same absolute value, they must be equal to each other.
3. There is no number that has a negative absolute value.
4. If  $|a| = |b|$ , we can only rewrite it as  $a = b$ .

### Practice

Solve each absolute value equation.

---

5.  $|z| = -2.575$

6.  $z = -|-2.33|$

7.  $|-2x + 1| = -3$

$$8. \quad 3 \left| \frac{x}{3} + 1 \right| - 5 = -2$$

$$9. \quad \left| \frac{x}{3} - 4 \right| = \left| \frac{5x}{6} + 1 \right|$$

## 8.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. If the solution is a union, there are two statements or inequalities, both of which must be true.
2. If the solution to a compound inequality is  $-4 < x < 6$ , then the solution is a union.
3. For a number to have absolute value greater than 2, its distance from 0 must be less than 2.
4. The inequality  $|2x + 9| < -2$  has no solution.

### Practice

Solve each of the absolute value inequalities and graph the solution sets. Write each solution using interval notation.

---

5.  $|x| \geq -2$

6.  $|x - 3| > 2$

7.  $|x + 2| \leq -4$

8.  $|3x + 4| - 1 < 0$

9.  $4 \leq |3x + 1| - 6$

10.  $3|4x + 5| - 5 > 10$

In the following questions a set of real numbers is described. **a.** Sketch a graph of the set on a real number line. **b.** Represent each set using absolute value notation. **c.** Represent each set using interval notation. If the set is one interval, state what type of interval it is.

---

11. The set of real numbers between  $-10$  and  $10$ , inclusive

12. The set of real numbers within 7 units of 4

## Applications

A confidence interval is described. **a.** Sketch a graph of the interval on a real number line. **b.** Represent the confidence interval using interval notation.

---

- 13. Confidence Interval:** A confidence interval around a mean of 45 with a margin of error equal to 4.
- 14. Confidence Interval:** A confidence interval where the endpoints of the interval are within 10.2 units of the mean, 302.7.
- 15. Find the Mean Given a Confidence Interval:** If a confidence interval for the mean is given as (14.8, 16.6), what is the mean?

## Looking Ahead

In the following statistics example, you must translate the stated problem into symbolic notation. Here, the key word “underfilled” signals the correct symbol to use for  $H_0$ .

### Example Preview

A manufacturer of potato chips would like to know whether its bag filling machine works correctly at the 417 gram setting. Is there sufficient evidence at the 0.02 level that the bags are underfilled? Assume the population is normally distributed.

State the null and alternative hypotheses for the above scenario.

### Solution

The issue in this problem is whether or not the bags are underfilled. Since  $\mu$  is the true mean filling capacity of the bags and the potato chips manufacturer wants to test if  $\mu$  is less than 417, this information is placed in the alternative hypothesis. The null hypothesis states that  $\mu$  is equal to the standard setting of 417 grams.

Specifically, for the null hypothesis, the true mean filling capacity is equal to 417 and for the alternative hypothesis, the true mean filling capacity is less than 417. Given this information, the null and alternative hypotheses are stated below.

$$H_0: \mu = 417$$

$$H_a: \mu < 417$$

## 10.R.1 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The order in which the values are given is particularly important when working with subtraction and division problems.
2. “More than” and “increased by” are key phrases specifying the operation of subtraction.
3. Division is indicated by the phrase “five less than a number.”
4. Key phrases for parentheses can be used to limit ambiguity in English phrases.

**Practice**

Write the algebraic expressions described by the English phrases. Choose your own variable.

---

5. six added to a number
  
6. twenty decreased by the product of four and a number
  
7. eighteen less than the quotient of a number and two

Translate each pair of English phrases into algebraic expressions. Notice the differences between the algebraic expressions and the corresponding English phrases.

---

8. a. six less than a number  
  
b. six less a number
  
9. a. six less than four times a number  
  
b. six less four times a number

Write the algebraic expression described by the English phrase using the given variables.

---

10. The cost of purchasing a fishing rod and reel if the rod costs  $x$  dollars and the reel costs \$8 more than twice the cost of the rod

Translate each algebraic expression into an equivalent English phrase. (There may be more than one correct translation.)

---

11.  $-9x$
  
12.  $\frac{9}{x+3}$



## 10.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

---

1. The exponent in the number  $1.4 \times 10^4$  indicates that the decimal point should be moved 4 places to the right.
2. The exponent in the number  $2.5 \times 10^{-3}$  indicates that the decimal point should be moved 3 places to the right.
3. The number  $3.53 \times 10^5$  is less than  $8.72 \times 10^{-4}$ .
4. The number 4000 written in scientific notation is  $0.4 \times 10^4$ .

### Practice

Write the following numbers in scientific notation.

---

5. 86,000

6. 0.0000000002368

Write the following numbers in decimal form.

---

7.  $4.2 \times 10^{-2}$

8.  $3.067 \times 10^{10}$

First write each of the numbers in scientific notation. Then perform the indicated operations and leave your answer in scientific notation.

---

9.  $0.0003 \cdot 0.0000025$

10.  $23,400,000,000 \cdot 5,500,000,000$

## Applications

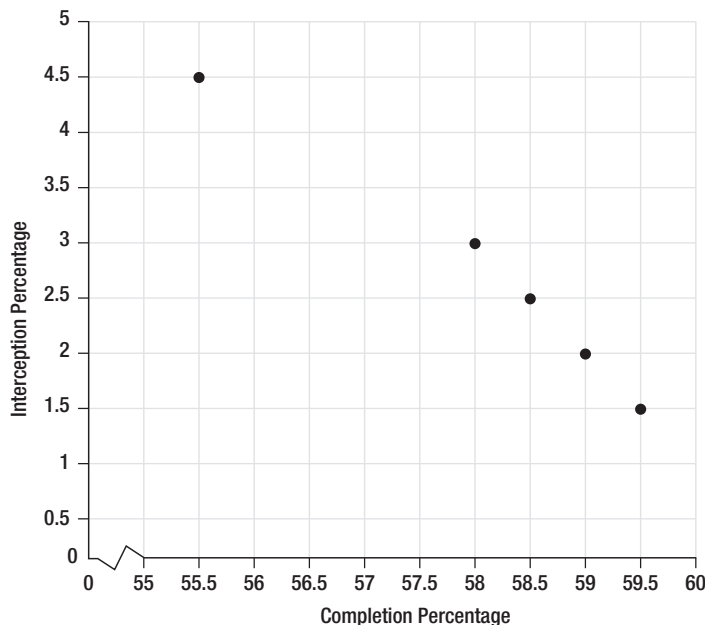
Solve.

---

11. **Speed of Light:** One light-year is approximately  $9.46 \times 10^{15}$  meters. The distance to a certain star is 4.3 light-years. How many meters is this?
  
12. **Atomic Weight:** The mass of an atom of gold is approximately  $3.25 \times 10^{-22}$  grams. What would be the mass of 2000 atoms of gold? Express your answer in scientific notation.
  
13. **p-Value:** A statistical hypothesis test results in a  $p$ -value of  $4.5 \times 10^{-3}$ . The null hypothesis will be rejected if the  $p$ -value is less than 0.05. Convert this  $p$ -value to a decimal and determine if the null hypothesis should be rejected.

**Solution**

- a. Since our  $x$ -axis is labeled “Completion Percentage”, the values given in the table for the row “Completion Percentage” will be our  $x$ -coordinates. Then the corresponding values in the row “Interception Percentage” will be the  $y$ -values of each point. For example, our first point would be (55.5, 4.5), so we find 55.5 on the  $x$ -axis and 4.5 on the  $y$ -axis and place a point at the intersection. This point and the others are shown below.



- b. Looking at the scatter plot from part a., we see that the points form a fairly linear pattern that slopes downward and to the right, i.e., as the completion percentage increases, the interception percentage decreases. Thus, we would say that the data has a negative correlation.

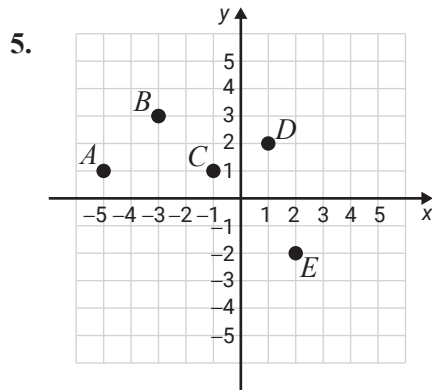
## 12.R.1 Exercises

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- The graph of every ordered pair that has a positive  $x$ -coordinate and a negative  $y$ -coordinate can be found in Quadrant IV.
- To find the  $y$ -value that corresponds with  $x = 2$ , substitute 2 for  $x$  into the given equation and solve for  $y$ .
- If  $(-7, 3)$  is a solution of  $y = 3x + 24$ , then  $(-7, 3)$  satisfies  $y = 3x + 24$ .
- If point  $A = (0, 4)$ , then point  $A$  lies on the  $x$ -axis.

## Practice

List the set of ordered pairs corresponding to the points on the graph.



Plot each set of ordered pairs and label the points.

6.  $\{A(4, -1), B(3, 2), C(0, 5), D(1, -1), E(1, 4)\}$

Determine the missing coordinate in each of the ordered pairs so that the point will satisfy the equation given.

7.  $x - 2y = 2$

a.  $(0, \underline{\quad})$

b.  $(4, \underline{\quad})$

c.  $(\underline{\quad}, 0)$

d.  $(\underline{\quad}, 3)$

Complete the tables so that each ordered pair will satisfy the given equation. Plot the resulting sets of ordered pairs.

8.  $y = 2x - 3$

$x$	$y$
0	
	-1
-2	
	3

Determine which, if any, of the ordered pairs satisfy the given equation.

---

9.  $2x - 3y = 7$

a.  $(1, 3)$

b.  $(\frac{1}{2}, -2)$

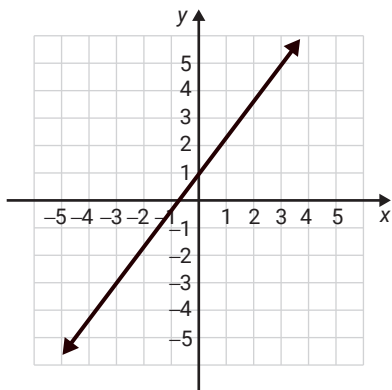
c.  $(\frac{7}{2}, 0)$

d.  $(2, 1)$

The graph of a line is shown. List any three points on the line. (There is more than one correct answer.)

---

10.



## Applications

Solve.

---

11. **Exchange Rates:** At one point in 2017, the exchange rate from US dollars to Euros was  $E = 0.85D$  where  $E$  is Euros and  $D$  is dollars.

a. Make a table of ordered pairs for the values of  $D$  and  $E$  if  $D$  has the values \$100, \$200, \$300, \$400, and \$500.

b. Plot the points corresponding to the ordered pairs.

12. **Temperature:** Given the equation  $F = \frac{9}{5}C + 32$  where  $C$  is temperature in degrees Celsius and  $F$  is the corresponding temperature in degrees Fahrenheit:
- Make a table of ordered pairs for the values of  $C$  and  $F$  if  $C$  has the values  $-20^\circ$ ,  $-10^\circ$ ,  $-5^\circ$ ,  $0^\circ$ ,  $5^\circ$ ,  $10^\circ$ , and  $15^\circ$ .
  - Plot the points corresponding to the ordered pairs.

13. **Bone Density Scatter Plot:** A study on bone density at a local hospital produced the following results.

<b>Age (in Years), <math>x</math></b>	34	43	49	51	65
<b>Bone Density (in <math>\text{mg}/\text{cm}^2</math>), <math>y</math></b>	946	875	804	723	691

- Draw a scatter plot of the given data set.
- Estimate the correlation in words: positive, negative, or no correlation.

## 12.R.2 Exercises

### Concept Check

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

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1. The  $y$ -intercept is the point where a line crosses the  $y$ -axis.
2. The terms ordered pair and point are used interchangeably.
3. A horizontal line does not have a  $y$ -intercept.
4. All  $x$ -intercepts correspond to an ordered pair of the form  $(0, y)$ .

### Practice

Graph each linear equation by locating at least two ordered pairs that satisfy the given equation.

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5.  $x + y = 3$

7.  $y = -3$

6.  $x = 1$

Graph each linear equation by locating the  $x$ -intercept and the  $y$ -intercept.

---

8.  $y = 4x - 10$

9.  $3x - 7y = -21$

## Applications

Solve.

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10. **Chemistry:** The amount of potassium in a clear bottle of a popular sports drink declines over time when exposed to the UV lights found in most grocery stores. The amount of potassium in a container of this sports drink is given by the equation  $y = -30x + 360$ , where  $y$  represents the mg of potassium remaining after  $x$  days on the shelf. Find both the  $x$ -intercept and  $y$ -intercept, and interpret the meaning of each in the context of this problem.
11. **Education:** Mr. Adler has found that the grade each student gets in his Introductory Algebra course directly correlates with the amount of time spent doing homework, and is represented by the equation  $y = 7x + 30$ , where  $y$  represents the numerical score the student receives on an exam (out of 100 points) after spending  $x$  hours per week doing homework. Find the  $y$ -intercept and interpret its meaning in this context.
12. **Birth Length Prediction:** The following equation is the least squares regression line for a set of data listing the birth weight (in pounds),  $x$ , and the birth length (in inches),  $y$ , for newborn babies at a local hospital. The variable  $\hat{y}$  is the birth length the model would predict for a baby based on a birth weight of  $x$ .

$$\hat{y} = 12.343 + 0.814x$$

Predict the birth length of an 8-pound baby.

## Writing & Thinking

13. Explain, in your own words, why it is sufficient to find the  $x$ -intercept and  $y$ -intercept to graph a line (assuming that they are not the same point).
14. Explain, in your own words, how you can determine if an ordered pair is a solution to an equation.

**Solution**

The estimated linear model is given by the following equation.

$$\hat{y} = -7.647 + 0.431x$$

If the value of the independent variable “ $x$ ” is increased by one unit, we have

$$\begin{aligned}\hat{y} &= -7.647 + 0.431(x+1) \\ &= -7.647 + 0.431x + 0.431 \\ &= \text{Previous value of } \hat{y} + 0.431\end{aligned}$$

Thus, the value of  $\hat{y}$  is increased by 0.431, the value of the slope, when  $x$  is increased by one unit.

## 12.R.3 Exercises

**Practice**

Find the slope of the line determined by each pair of points.

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1.  $(1, -2); (1, 4)$

2.  $(-3, 7); (4, -1)$

Determine whether the equation  $x = -3$  represents a horizontal line or a vertical line and give its slope.

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3.  $x = -3$

Write each equation in slope-intercept form. Find the slope and  $y$ -intercept, and then use them to draw the graph.

---

4.  $y = 2x - 1$

5.  $3y - 9 = 0$

Find an equation in slope-intercept form for the line passing through (0,3) with the slope  $m = -\frac{1}{2}$ .

---

6.  $(0,3); m = -\frac{1}{2}$

## Applications

Solve.

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7. **Purchases:** John bought his new car for \$35,000 in the year 2014. He knows that the value of his car has depreciated linearly. If the value of the car in 2017 was \$23,000, what was the annual rate of depreciation of his car? Show this information on a graph. (When graphing, use years as the  $x$ -coordinates and the corresponding values of the car as the  $y$ -coordinates.)
8. **Cell Phones:** The number of people in the United States with mobile cellular phones was about 198 million in 2011 and about 232 million in 2016. If the growth in the usage of mobile cellular phones was linear, what was the approximate rate of growth per year from 2011 to 2016. Show this information on a graph. (When graphing, use years as the  $x$ -coordinates and the corresponding numbers of users as the  $y$ -coordinates.)<sup>1</sup>

<sup>1</sup> Source: <https://www.statista.com/statistics/231612/number-of-cell-phone-users-usa/>

